BUSINESS KNOWLEDGE

Most important part of the creating a model is to have a sound business knowledge of the problem you are trying to solve

Types of Research

1. Primary research

   A. Discussions
   Ask questions and gather information from the stakeholders

   B. Dry Run
   If possible take a dry run of problem you are trying to investigate

2. Secondary research

   A. Reports and Studies
   Read reports and studies by government agencies, trade associations or other businesses in your industry

   B. Previous works
   Go through any previous work and findings related to your problem

BUSINESS KNOWLEDGE

Examples

Cart Abandonment Analysis

Problem :

High fractions of your online customer are adding product to their cart but not purchasing it

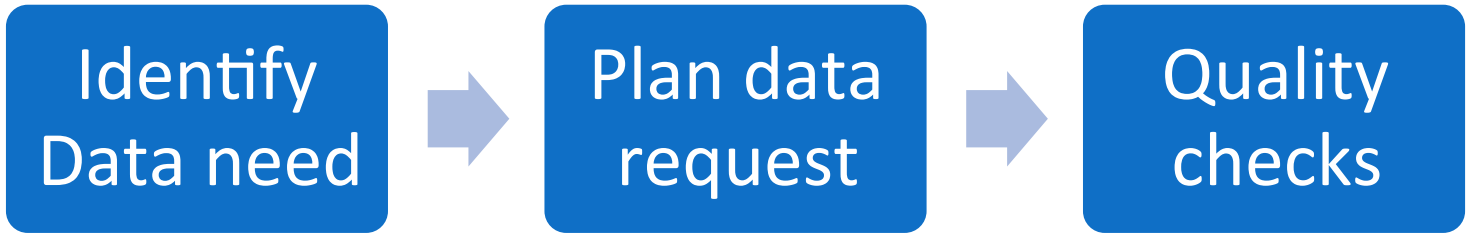
Business Knowledge that will be helpful

1. Discussions with the marketing team
2. Discussions with the product team
3. Dry run of the online purchasing process to understand customer journey
4. Research on industry reports regarding cart abandonment
5. Any previous work in your /other organization regarding cart Abandonment

Data Exploration

Next step should be to use the acquired business knowledge to search for relevant data

Steps



Data Exploration

Next step should be to use the acquired business knowledge to search for relevant data

Data Exploration

1. Internal Data

   Data collected by your organization
   E.g. Usage data, sales data, promotion data

2. External data

   Data acquired from external data sources
   E.g. Census Data, External vendor Data, Scrape data

Data Exploration

Examples

Cart Abandonment Analysis

1. Input from the marketing team –
   Our 50 % comes from email marketing, 30% from organic search and rest 20% from ad word marketing
   -> Gather the source website data for all customers
2. Input from the product team
   We have 3 step purchase process – Cart review, Address/personal detail, Payment
   -> Gather the Cart Abandonment location for all customer
3. Input from industry reports regarding cart abandonment
   Customers tends to put high value item for long duration in their cart
   -> Gather the data about total Cart value of all customers
4. Input from dry run
   Encountered a survey link for rate website experience
   -> Gather survey data for all customers

DATA DICTIONARY

Next step should be to understand the data. You should know variable definition and distribution along with table's unique identifiers and foreign keys

Data Dictionary

A Comprehensive Data Dictionary should include

1. Definition of predictors
2. Unique identifier of each table (or Primary Keys)
3. Foreign keys or matching keys between tables

   https://youtu.be/76Y6Tg1glrQ
4. Explanation of values in case of Categorical variables

DATA DICTIONARY

Examples

Data Dictionary
House Pricing Dataset

The data set contains 506 observations of house prices from different towns. Corresponding to each house price, data of 18 other variables is available on which price is suspected to depend

price	Value of the house
crime_rate	Crime rate in that neighborhood
resid_area	Proportion of residential area in the town
air_qual	Quality of air in that neighborhood
room_num	Average number of rooms in houses of that locality
age	How old is the house construction in years
dist1	Distance from employment hub 1
dist2	Distance from employment hub 2
dist3	Distance from employment hub 3
dist4	Distance from employment hub 4
teachers	Number of teachers per thousand population in the town

DATA DICTIONARY

Examples

Data Dictionary
House Pricing Dataset

The data set contains 506 observations of house prices from different towns. Corresponding to each house price, data of 18 other variables is available on which price is suspected to depend

poor_prop	Proportion of poor population in the town
airport	Is there an airport in the city? (Yes/No)
n_hos_beds	Number of hospital beds per 1000 population in the town
n_hot_rooms	Number of hotel rooms per 1000 population in the town
waterbody	What type of natural fresh water source is there in the city (lake/ river/ both/ none)
rainfall	The yearly average rainfall in centimeters
bus_ter	Is there a bus terminal in the city? (Yes/No)
parks	Proportion of land assigned as parks and green areas in the town

UNIVARIATE ANALYSIS

Univariate analysis is the simplest form of analyzing data. “Uni” means “one”, so in other words your data has only one variable. It doesn’t deal with causes or relationships (unlike regression) and it’s major purpose is to describe; it takes data, summarizes that data and finds patterns in the data.

Univariate Analysis

Ways to describe patterns found in univariate data

1. Central tendency
    1. Mean
    2. Mode
    3. Median
2. Dispersion
    1. Range
    2. Variance
    3. maximum, minimum,
    4. Quartiles (including the interquartile range), and
    5. Standard deviation
3. Count /Null count

EDD (EXTENDED DATA DICTIONARY)

Example

	Age	Name	Score
count	12.000000	12	12.000000
unique	NaN	12	NaN
top	NaN	Rahul	NaN
freq	NaN	1	NaN
mean	32.500000	NaN	73.000000
std	9.209679	NaN	17.653225
min	24.000000	NaN	44.000000
25%	25.750000	NaN	64.000000
50%	29.000000	NaN	74.000000
75%	35.250000	NaN	87.500000
max	51.000000	NaN	99.000000

Missing Value Imputation

Real-world data often has missing values. Data can have missing values for a number of reasons such as observations that were not recorded and data corruption.

Missing Value Imputation

Impact

- Handling missing data is important as many machine learning algorithms do not support data with missing values.

Solution

- Remove rows with missing data from your dataset.
- Impute missing values with mean/median values in your dataset.

Note

- Use business knowledge to take separate approach for each variable
- It is advisable to impute instead of remove in case of small sample size or large proportion of observations with missing values

Missing Value Imputation

Methods

1. Impute with ZERO
   - Impute missing values with zero
2. Impute with Median/Mean/Mode
   - For numerical variables, impute missing values with Mean or Median
   - For categorical variables, impute missing values with Mode
3. Segment based imputation
   - Identify relevant segments
   - Calculate mean/median/mode of segments
   - Impute the missing value according to the segments
   - For example, we can say rainfall hardly varies for cities in a particular State
   - In this case, we can impute missing rainfall value of a city with the average of that state

Outlier Treatment

Outlier is a commonly used terminology by analysts and data scientists, Outlier is an observation that appears far away and diverges from an overall pattern in a sample.

Outlier Treatment

Reasons

- Data Entry Errors
- Measurement Error
- Sampling error etc

Impact

- It increases the error variance and reduces the power of statistical tests

Solution

- Detect outliers using EDD and visualization methods such as scatter plot, histogram or box plots
- Impute outliers

Outlier Treatment

Example

	Without Outlier	With Outlier
Data	6,6,6,4,4,5,5,5,5,7,7	6,6,6,4,4,5,5,5,5,7,7,300
Mean	5.45	30.0
Median	5	5.5
Mode	5	5
Standard deviation	1.04	85.03
Variance	1.08	7230.10

Outlier Treatment

Methods

1. Capping and Flooring
   - Impute all the values above 3* P99 and below 0.3*P1
   - Impute with values 3* P99 and 0.3*P1
   - You can use any multiplier instead of 3, as per your business requirement
2. Exponential smoothing
   - Extrapolate curve between P95 to P99 and cap all the values falling outside to the value generated by the curve
   - Similarly, extrapolate curve between P5 and P1
3. Sigma Approach
   - Identify outliers by capturing all the values falling outside $\boldsymbol{\mu \mp x\sigma}$
   - You can use any multiplier as x, as per your business requirement

Seasonality

Seasonality is the presence of variations that occur at specific regular intervals less than a year, such as weekly, monthly, or quarterly.

Seasonality

Reasons

- Weather,
- Vacation,
- Holidays

Examples

- Ice cream sales
- Christmas sales

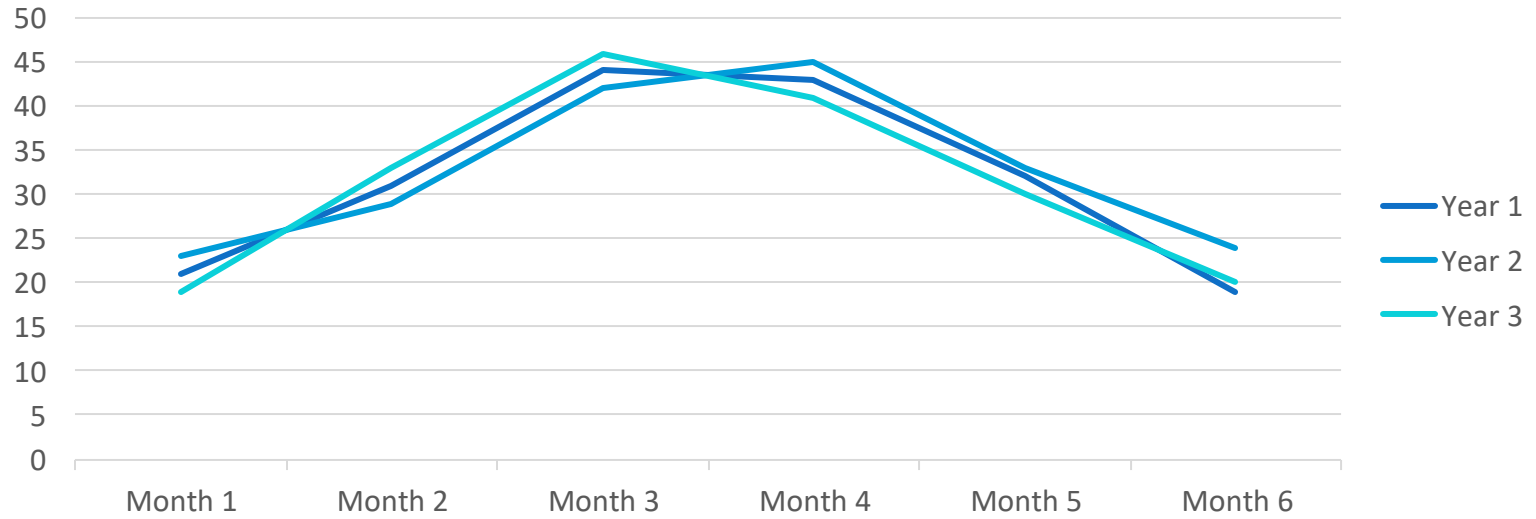
Solution

- Calculate multiplication factor for each month as $m = \mu_{Year} \div \mu_{Month}$
- Multiply each observation with its multiplication factor

Seasonality

Example

	Year 1	Year 2	Year 3
Month 1	21	23	19
Month 2	31	29	33
Month 3	44	42	46
Month 4	43	45	41
Month 5	32	33	30
Month 6	19	24	20



Seasonality

Example

	Year 1	Year 2	Year 3	Factor
Month 1	21	23	19	1.521164
Month 2	31	29	33	1.030466
Month 3	44	42	46	0.72601
Month 4	43	45	41	0.742894
Month 5	32	33	30	1.008772
Month 6	19	24	20	1.521164

	Year 1	Year 2	Year 3
Month 1	31.94	34.99	28.9
Month 2	31.94	29.88	34.01
Month 3	31.94	30.49	33.4
Month 4	31.94	33.43	30.46
Month 5	32.28	33.29	30.26
Month 6	28.90	36.51	30.42

Bivariate Analysis

Bivariate analysis is the simultaneous analysis of two variables (attributes). It explores the concept of relationship between two variables, whether there exists an association and the strength of this association, or whether there are differences between two variables and the significance of these differences.

Creating new Variables

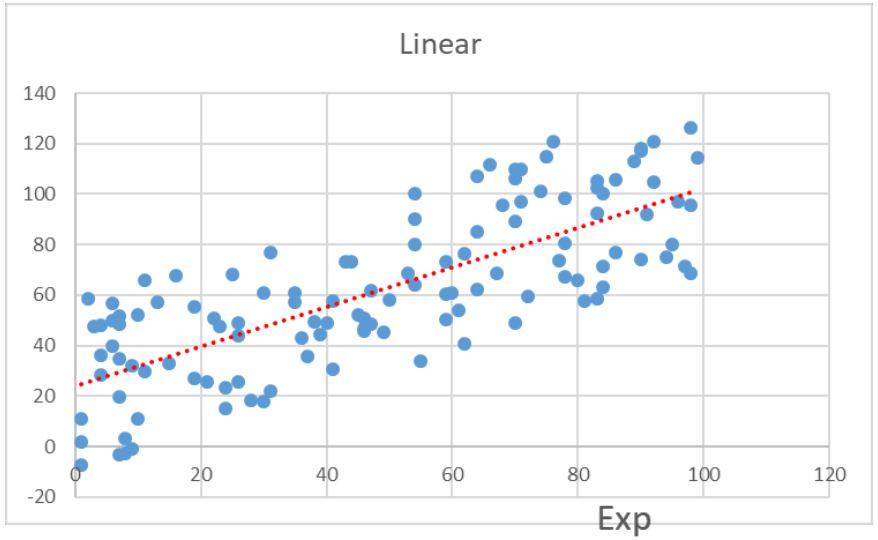
Scatter Plot

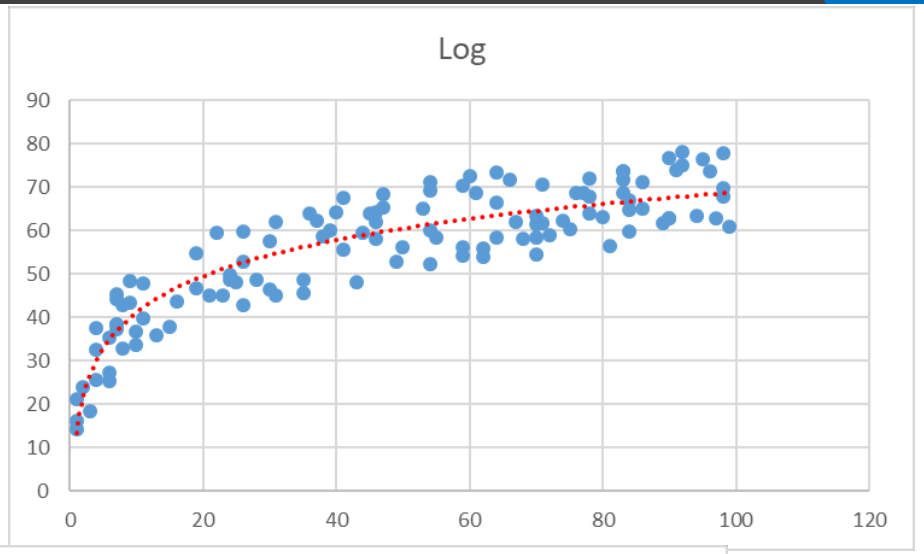
- Scatter indicates the type (linear or non-linear) and strength of the relationship between two variables
- We will use Scatter plot to transform variables

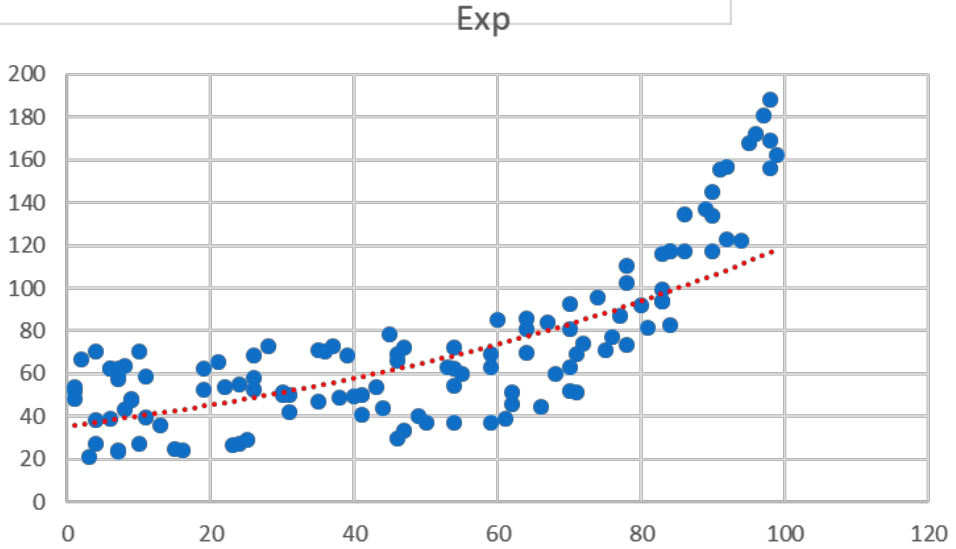
Correlation

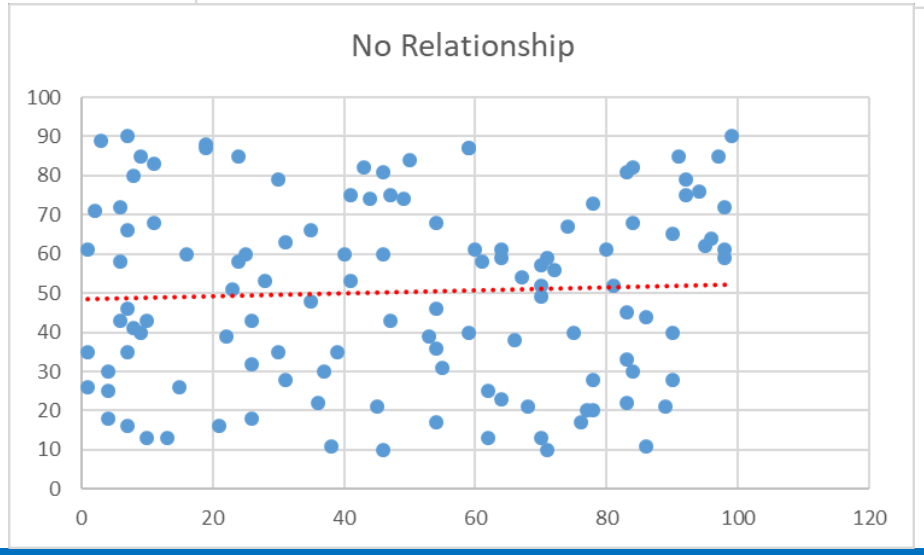
- Linear correlation quantifies the strength of a linear relationship between two numerical variables.
- When there is no correlation between two variables, there is no tendency for the values of one quantity to increase or decrease with the values of the second quantity.
- Correlation is used to drop Non Usable variables

Scatter plots









Variable Transformation

Transform your existing variable to extract more information out of them

Creating new Variables

Identify

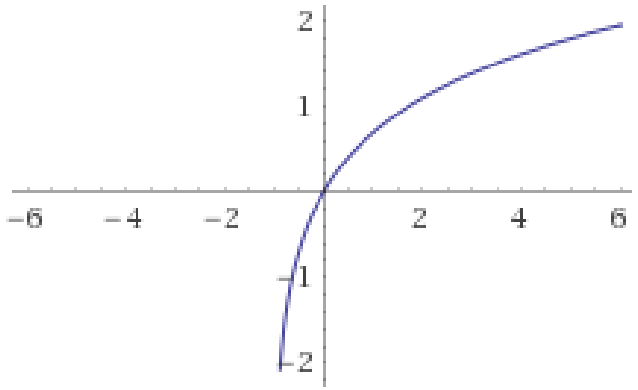
- Using your business knowledge and bivariate analysis to modify variable

Methods

- Use Mean/Median of variables conveying similar type of information
- Create ratio variable which are more relevant to business
- Transform variable by taking log, exponential, roots etc.

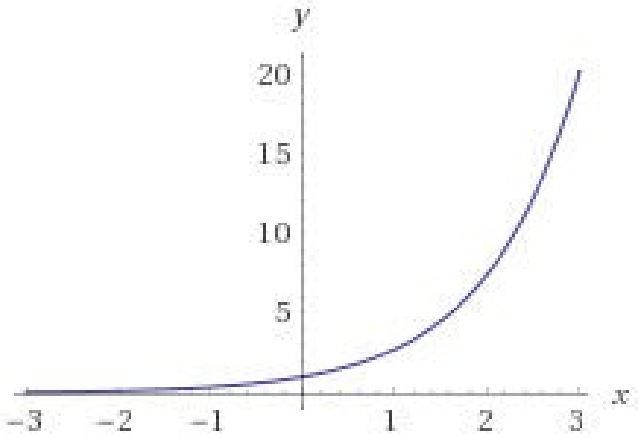
Transformation

If



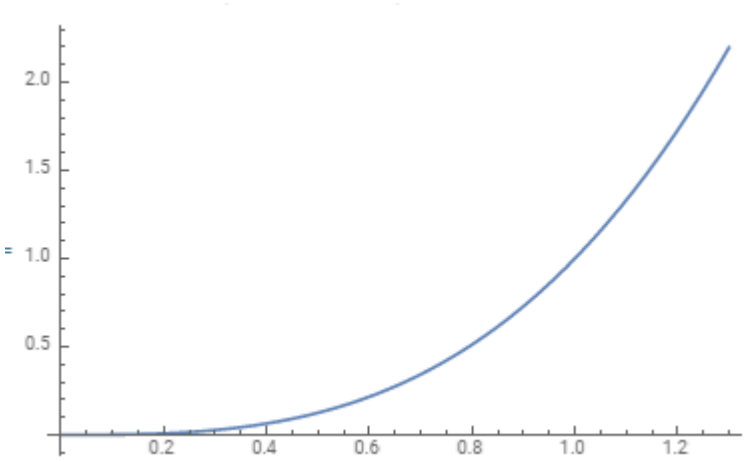
Take e^x instead of x

If



Take log(1+x) instead of x

Transformation

If 

Take $\sqrt{x}$ or $\sqrt[n]{x}$ instead of x

Non Usable Variables

Identify the non usable variables to reduce the dimension of your dataset

Non Usable Variables

1. Variables with single unique value
2. Variables with low fill rate
3. Variables with regulatory issue
4. Variable with no business sense

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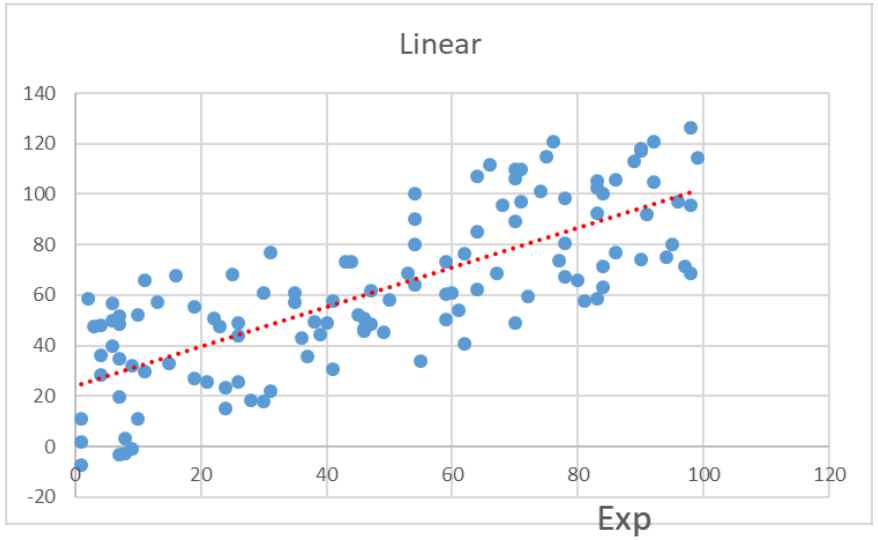

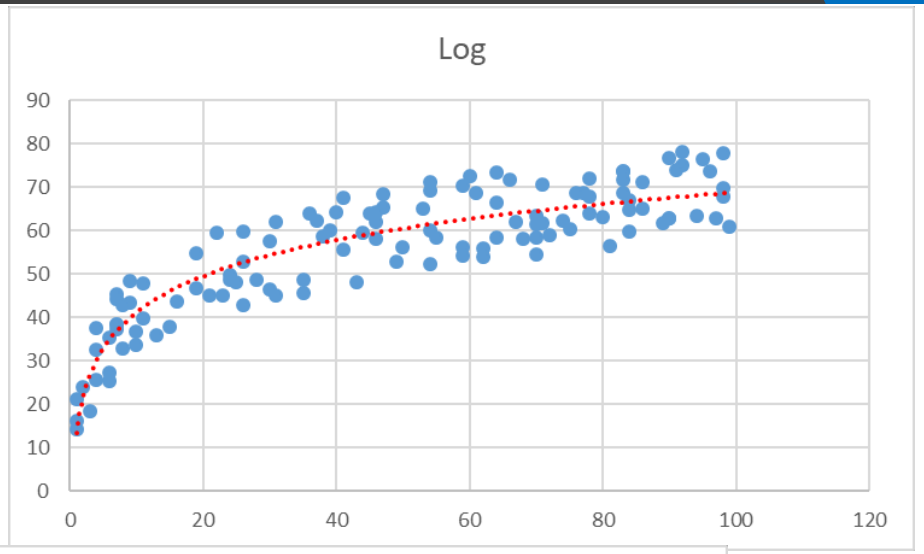

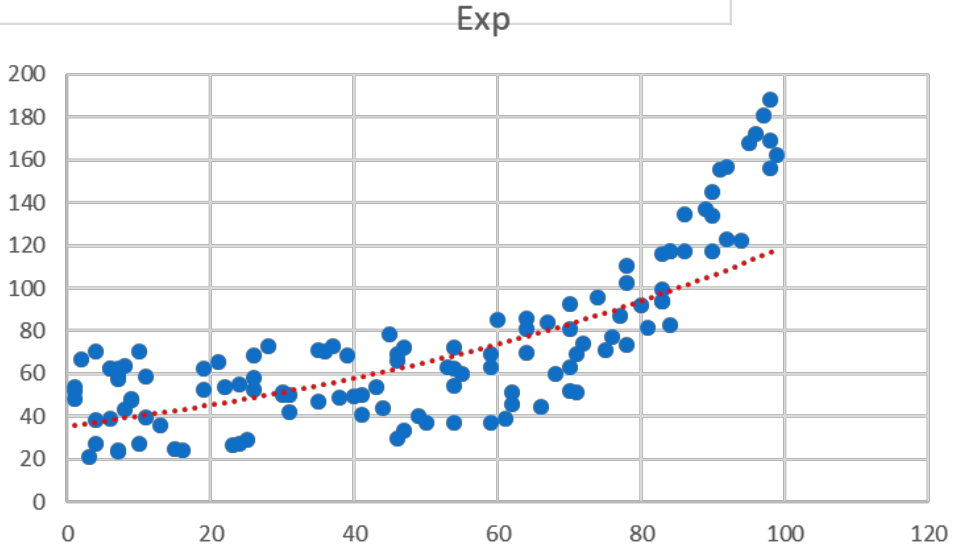

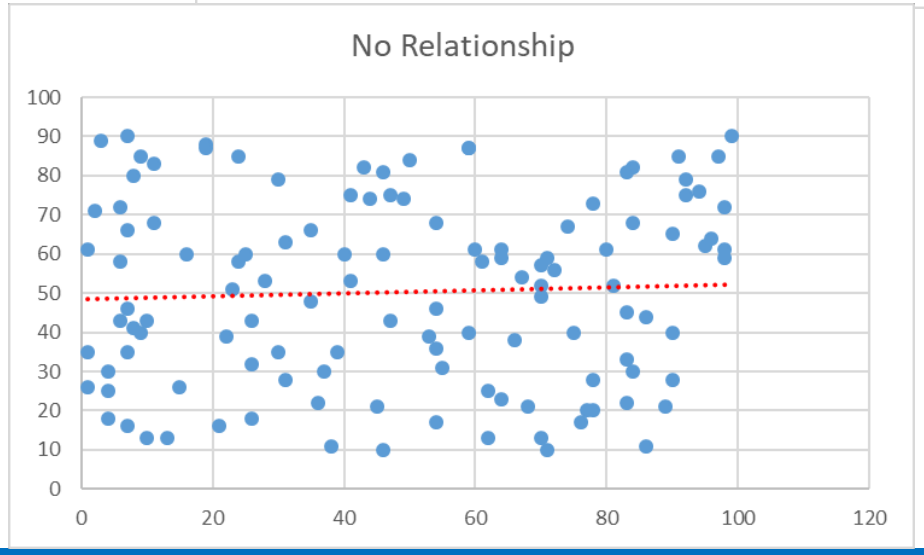

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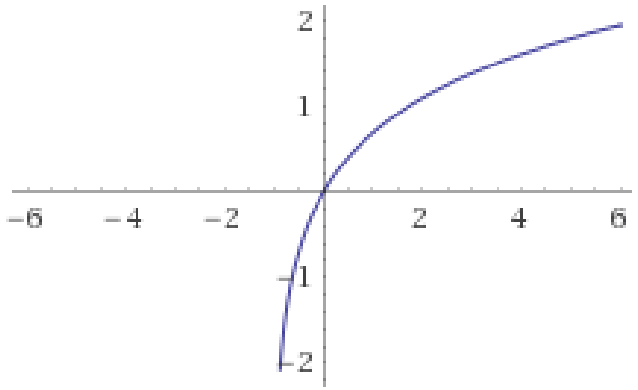

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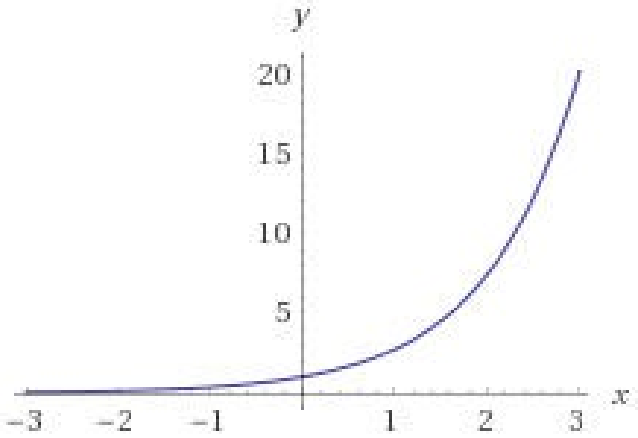

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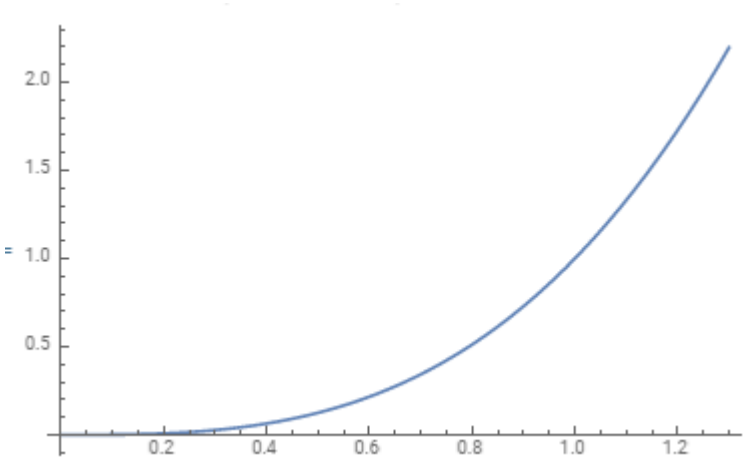
If 

Take $\sqrt{x}$ or $\sqrt[n]{x}$ instead of x

Correlation

Correlation is a statistical measure that indicates the extent to which two or more variables fluctuate together. A positive correlation indicates the extent to which those variables increase or decrease in parallel; a negative correlation indicates the extent to which one variable increases as the other decreases.

Correlation

Examples

Some examples of data that have a high correlation:

- Your caloric intake and your weight.
- The amount of time your study and your GPA.

Some examples of data that have a low correlation (or none at all):

- A dog's name and the type of dog biscuit they prefer.
- The cost of a car wash and how long it takes to buy a soda inside the station.

The Correlation Coefficient

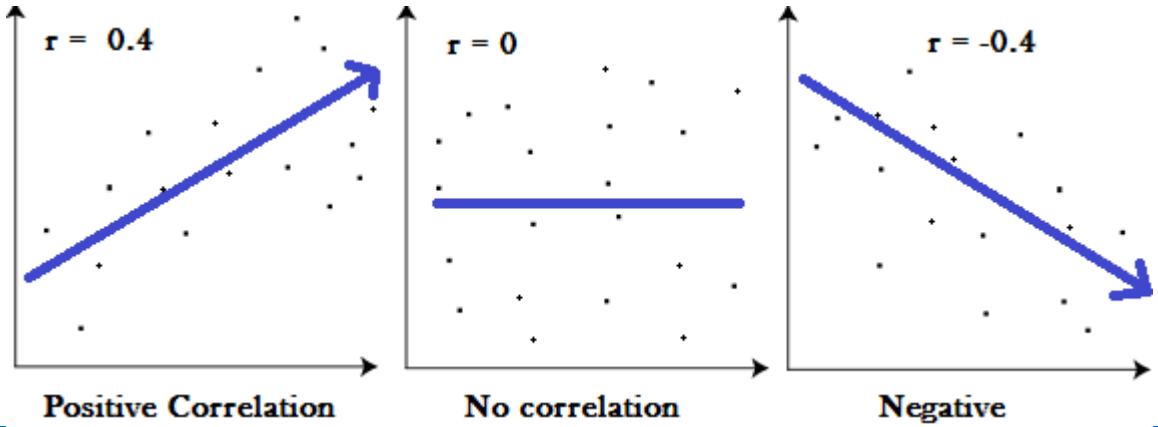
Correlation is a statistical measure that indicates the extent to which two or more variables fluctuate together. A positive correlation indicates the extent to which those variables increase or decrease in parallel; a negative correlation indicates the extent to which one variable increases as the other decreases.

Correlation Coefficient

Definition

- A correlation coefficient is a way to put a value to the relationship.
- Correlation coefficients have a value of between -1 and 1.
- A “0” means there is no relationship between the variables at all,
- While -1 or 1 means that there is a perfect negative or positive correlation

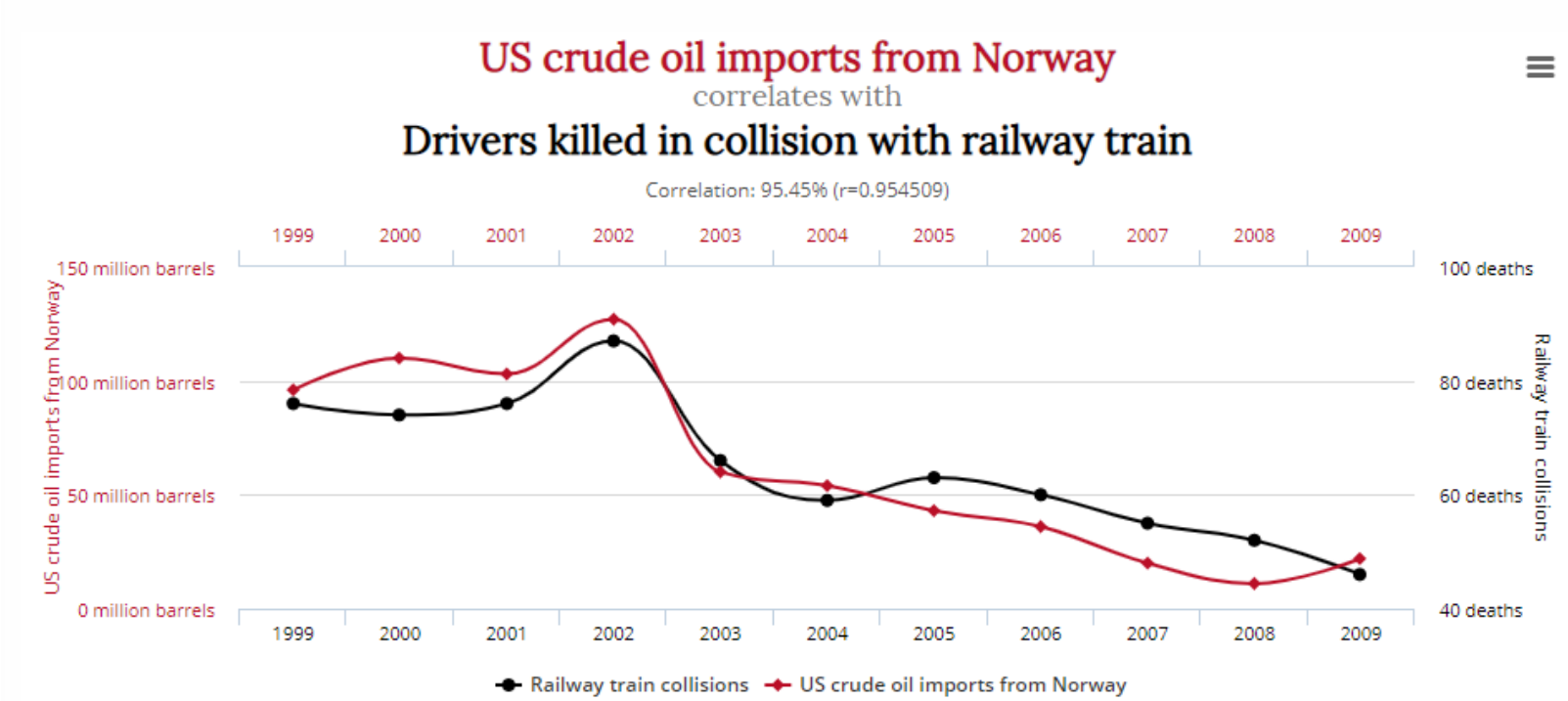
Example



Correlation vs Causation

Causation : The relation between something that happens and the thing that causes it . The first thing that happens is the cause and the second thing is the effect .

Correlation
vs
Causation



Source :http://www.tylervigen.com/spurious-correlations

The Correlation Matrix

Correlation Matrix

Definition

- A correlation matrix is a table showing correlation coefficients between variables.
- Each cell in the table shows the correlation between two variables.
- A correlation matrix is used as a way to summarize data, as an input into a more advanced analysis, and as a diagnostic for advanced analyses.

Example

	Always to vote in elections	Never to try to evade taxes	Always to obey laws	Keep watch on action of govt
Always to vote in elections	1.00	.94	.94	.94
Never to try to evade taxes	.94	1.00	.97	.95
Always to obey laws	.94	.97	1.00	.96
Keep watch on action of govt	.94	.95	.96	1.00

Application

- To summarize a large amount of data where the goal is to see patterns.
- To Identify collinearity in the data

Multicollinearity

Multicollinearity

Definition

- Multicollinearity exists whenever two or more of the predictors in a regression model are moderately or highly correlated.

Effects

- Multicollinearity results in a change in the signs as well as in the magnitudes of the partial regression coefficients from one sample to another sample.
- Multicollinearity makes it tedious to assess the relative importance of the independent variables in explaining the variation caused by the dependent variable.

Solution

- Remove highly correlated independent variables by looking at the correlation matrix and VIF

Dummy Variable

A Dummy variable or Indicator Variable is an artificial variable created to represent an attribute with two or more distinct categories/levels.

Dummy Variable

Why

- Regression analysis treats all independent (X) variables in the analysis as numerical.
- Nominal variables, or variables that describe a characteristic using two or more categories, are commonplace in regression research, but are not always useable in their categorical form.
- Dummy coding is a way of incorporating nominal variables into regression analysis

How

- We can make a separate column, or variable, for each category.
- This new variables can take value 0 or 1 depending on the value of the categorical variable

Dummy Variable

A Dummy variable or Indicator Variable is an artificial variable created to represent an attribute with two or more distinct categories/levels.

Dummy Variable Example

Student	Favorite class	Science	Math
1	Science	1	0
2	Science	1	0
3	English	0	0
4	Math	0	1

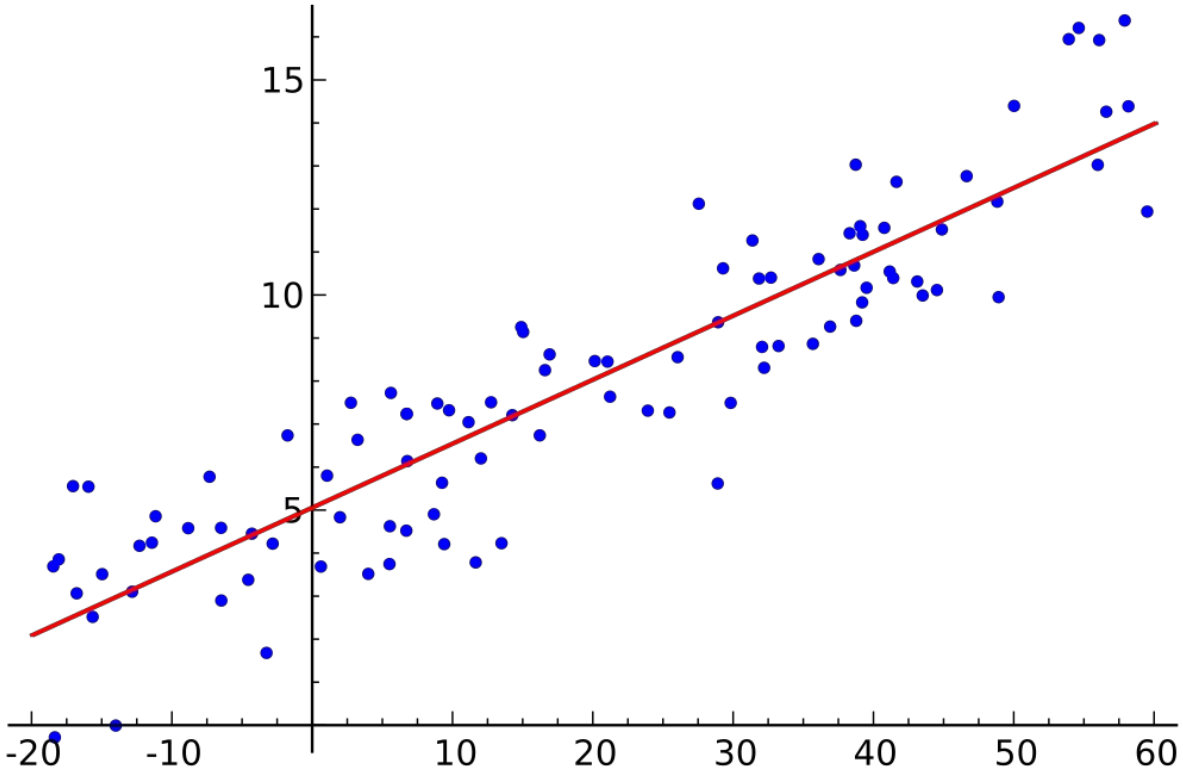
Things to keep in mind

- The number of dummy variables necessary to represent a single attribute variable is equal to the number of levels (categories) in that variable minus one.
- We cannot code variables like science = 1, math = 2, and English = 3. As, we can see that there is no such thing as an increase in favorite class – math is not higher than science, and is not lower than language either. And even if there is increase , we cannot quantify that increase

Linear Regression

linear regression is a linear approach to modelling the relationship between a dependent variable and one or more independent variables

Introduction



Linear Regression

price	crime_rate	resid_area	air_qual	room_num	age	dist1	dist2	dist3	dist4	teachers	poor_prop	airport	n_hos_beds	n_hot_rooms	waterbody	rainfall	bus_ter	parks
24	0.00632	32.31	0.538	6.575	65.2	4.35	3.81	4.18	4.01	24.7	4.98	YES	5.48	11.192	River	23	YES	0.04935
21.6	0.02731	37.07	0.469	6.421	78.9	4.99	4.7	5.12	5.06	22.2	9.14	NO	7.332	12.1728	Lake	42	YES	0.04615
34.7	0.02729	37.07	0.469	7.185	61.1	5.03	4.86	5.01	4.97	22.2	4.03	NO	7.394	101.12	None	38	YES	0.04576
33.4	0.03237	32.18	0.458	6.998	45.8	6.21	5.93	6.16	5.96	21.3	2.94	YES	9.268	11.2672	Lake	45	YES	0.04715
36.2	0.06905	32.18	0.458	7.147	54.2	6.16	5.86	6.37	5.86	21.3	5.33	NO	8.824	11.2896	Lake	55	YES	0.03947
28.7	0.02985	32.18	0.458	6.43	58.7	6.22	5.8	6.23	5.99	21.3	5.21	YES	7.174	14.2296	None	53	YES	0.04591
22.9	0.08829	37.87	0.524	6.012	66.6	5.87	5.47	5.7	5.2	24.8	12.43	YES	6.958	12.1832	River	41	YES	0.05217
22.1	0.14455	37.87	0.524	6.172	96.1	6.04	5.85	6.25	5.66	24.8	19.15	NO	5.842	12.1768	Lake	56	YES	0.05707
16.5	0.21124	37.87	0.524	5.631	100	6.18	5.85	6.3	6	24.8	29.93	YES	5.93	12.132	None	55	YES	0.0563
18.9	0.17004	37.87	0.524	6.004	85.9	6.67	6.55	6.85	6.29	24.8	17.1	YES	9.478	14.1512	River	45	YES	0.05073
15	0.22489	37.87	0.524	6.377	94.3	6.65	6.31	6.55	5.88	24.8	20.45	NO	6	11.12	Lake	29	YES	0.05778
18.9	0.11747	37.87	0.524	6.009	82.9	6.27	5.93	6.51	6.19	24.8	13.27	NO	9.278	13.1512	Lake and Riv	23	YES	0.05524
21.7	0.09378	37.87	0.524	5.889	39	5.76	5.14	5.58	5.33	24.8	15.71	YES	5.534	10.1736	Lake and Riv	57	YES	0.05742

Questions

Here are a few important questions that we might seek to address:

1. Prediction Question
   How accurately can I predict the price of a house , given the values of all variables
2. Inferential Question
   How accurately can we estimate the effect of each of this variables on the house price

Simple Linear Regression

Simple linear regression is an approach for predicting a quantitative response Y on the basis of a single predictor variable X. It assumes that there is approximately a linear relationship between X and Y .

Introduction

Model Equation

$$Y \approx \beta_0 + \beta_1 X$$

β_0 is known as Intercept

β_1 is known as slope

Together β_0 and β_1 known as the model *coefficients* or *parameters*.

For House Price data

- X will represent Room_num
- Y will represent Price

$$\text{Price} \approx \beta_0 + \beta_1 \times \text{Room\_num}$$

From our training data we will get β_0 and β_1

Simple Linear Regression

Estimating the Coefficients

- Our goal is to obtain coefficient estimates β_0 and β_1 such that the linear model fits the available data well
- Total number of rows (Data Point) ⇒ n = 506
- Data ⇒ $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots\ldots\ldots\ldots\ldots\ldots (x_{506}, y_{506})$
- Lets call calculated y value as y

$$\hat{y}_1 = \beta_0 + \beta_1 x_1$$

$$\hat{y}_2 = \beta_0 + \beta_1 x_2$$

$$\hat{y}_{506} = \beta_0 + \beta_1 x_{506}$$

- The difference between residual the *i*th observed response value and the *i*th response value that is predicted by our linear model is known as residual

$$e_i = \hat{y}_i - y_i$$

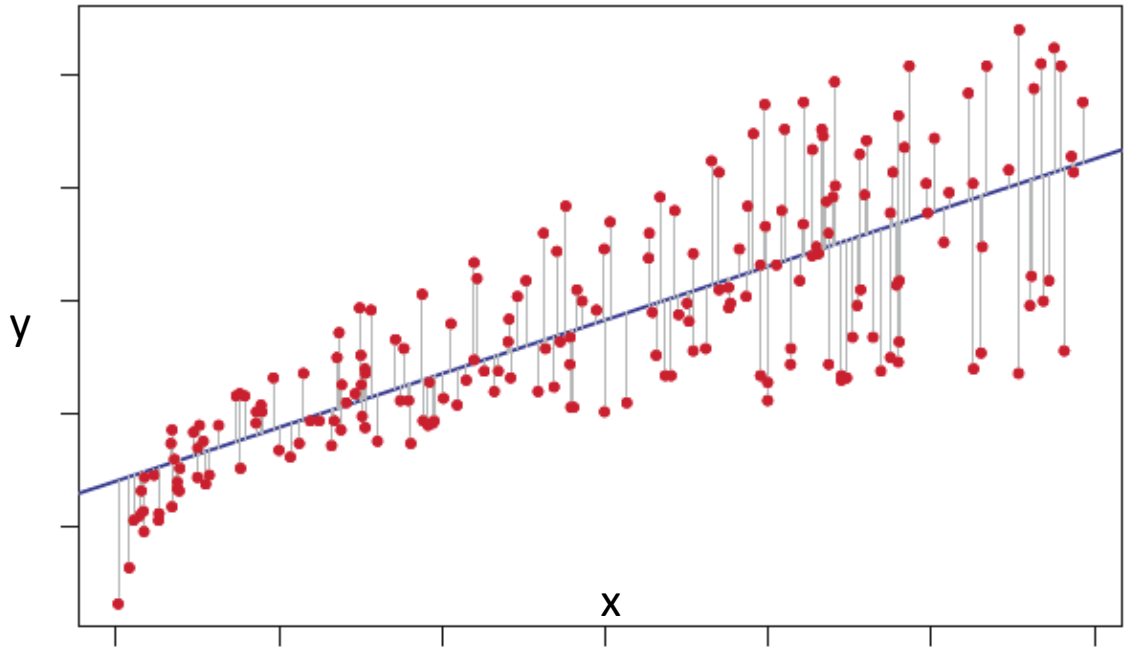
Simple Linear Regression

Residual

Residual –

The difference between residual the *i*th observed response value and the *i*th response value that is predicted by our linear model is known as residual

$$e_i = y_i - y_i$$



Simple Linear Regression

RSS

Residual sum of squares (RSS)

$$RSS = e_1^2 + e_2^2 \ldots \ldots . + e_n^2$$

$$\text{RSS} = (y_1 - \hat{\beta}_0 - \hat{\beta}_1 x_1)^2 + (y_2 - \hat{\beta}_0 - \hat{\beta}_1 x_2)^2 + \ldots + (y_n - \hat{\beta}_0 - \hat{\beta}_1 x_n)^2.$$

The least squares approach chooses β_0 and β_1 to minimize the RSS

Using some calculus, one can show that the minimizers are

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2},$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x},$$

Simple Linear Regression

Model

For our Model

```
Residuals:
    Min      1Q  Median      3Q     Max
-23.336  -2.425   0.093   2.918  39.434

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -34.6592     2.6421  -13.12   <2e-16 ***
room_num      9.0997     0.4178   21.78   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.597 on 504 degrees of freedom
Multiple R-squared:  0.4848,    Adjusted R-squared:  0.4838
F-statistic: 474.3 on 1 and 504 DF,  p-value: < 2.2e-16
```

Simple Linear Regression

we assume that the true relationship between X and Y takes the form Y = f(X) + ε for some unknown function f, where ε is a mean-zero random error term.

Assessing the Accuracy

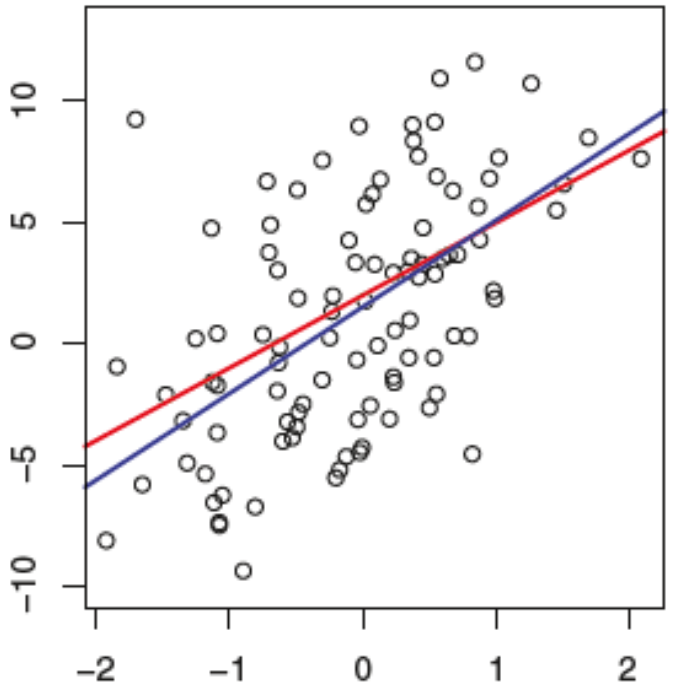
If *f* is to be approximated by a linear function, then we can write this relationship as

$$\boldsymbol{Y} = \beta_0 + \beta_1 X + \varepsilon$$

β_0 is known as Intercept
β_1 is known as slope
ε is an error term

Population regression line

Sample regression line



Simple Linear Regression

Standard error In Coefficients

$$\mathrm{SE}(\hat{\beta}_0)^2 = \sigma^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right] \qquad \mathrm{SE}(\hat{\beta}_1)^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2},$$

$$\sigma^2 = Var(\varepsilon)$$

$\sigma 2$ is not known, but can be estimated from the data. This estimate is known as the *residual standard error (RSE)*

$$\mathrm{RSE} = \sqrt{\mathrm{RSS}/(n-2)}.$$

There is approximately a 95% chance that the interval

$$\left[\hat{\beta}_1 - 2 \cdot \mathrm{SE}(\hat{\beta}_1),\ \hat{\beta}_1 + 2 \cdot \mathrm{SE}(\hat{\beta}_1)\right]$$

will contain the true value of β_1

Simple Linear Regression

Hypothesis tests

Is there any relationship between X and Y

$$\boldsymbol{Y} = \beta_0 + \beta_1 X$$

- If β_1 is zero, it means there is no relationship

Ho : There is no relationship between X and Y

Ha : There is some relationship between X and Y

$H : \beta_1 = 0$

$Ha : \beta_1 \neq 0,$

Simple Linear Regression

Hypothesis tests

- To disapprove Ho, we calculate T statistics $$t = \frac{\hat{\beta}_1 - 0}{\mathrm{SE}(\hat{\beta}_1)}$$

- We also compute the probability of observing any value equal to $|t|$ or Larger
- We call this probability the *p-value*
- *A* small p-value means there is an association between the predictor and the response (typically less than 5% or 1 %)

```
Residuals:
    Min      1Q  Median      3Q     Max
-23.336  -2.425   0.093   2.918  39.434

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -34.6592     2.6421  -13.12   <2e-16
room_num      9.0997     0.4178   21.78   <2e-16
```

Simple Linear Regression

Quality of Fit

$\boldsymbol{RSE}$

The quality of a linear regression fit is typically assessed using two related quantities: the *residual standard error* (RSE) and the R^2 statistic.

Residual Standard Error

$$\text{RSE} = \sqrt{\frac{1}{n-2}\text{RSS}} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}.$$

- RSE is the average amount that the response will deviate from the true regression line
- RSE is also considered as a measure of lack of fit of the model to the data

```
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Multiple R-squared:  0.4848,    Adjusted R-squared:  0.4838
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```

Simple Linear Regression

Quality of Fit
R^2

The RSE provides an absolute measure of lack of fit of the model to the data.

R^2

- R^2 is the proportion of variance explained
- R^2 always takes on a value between 0 and 1,
- R^2 is independent of the scale of Y.

$$R^2 = \frac{\text{TSS} - \text{RSS}}{\text{TSS}} = 1 - \frac{\text{RSS}}{\text{TSS}}$$

- *TSS - total sum of squares*
- *RSS - residual sum of squares*

```
Residual standard error: 6.597 on 504 degrees of freedom
Multiple R-squared:  0.4848,    Adjusted R-squared:  0.4838
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```

Multiple Linear Regression

In Multiple linear regression more than one predictor variables are used to predict the response variable

Multiple Linear Regression

Relationship for Multiple linear regression can be written as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon,$$

β_0 is known as Intercept
p is the number of predictors
ϵ is an error term

For our Model,
The equation is

$$\boldsymbol{Price} = \beta_0 + \beta_1 \text{ Crime\_rate} + \boldsymbol{\beta_2} \text{ poor\_pop} \ldots \ldots \ldots \boldsymbol{\beta_{16}} \text{ avg\_dist}$$

Multiple Linear Regression

Estimating Regression Coefficients

$$\text{RSS} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

```
Coefficients:
                            Estimate Std. Error t value Pr(>|t|)
(Intercept)                -6.498625   5.264076  -1.235   0.2176
crime_rate                  0.009710   0.348185   0.028   0.9778
resid_area                 -0.040875   0.057585  -0.710   0.4782
air_qual                  -15.897400   4.003793  -3.971 8.24e-05 ***
room_num                    4.019017   0.426606   9.421  < 2e-16 ***
age                        -0.005715   0.013606  -0.420   0.6747
teachers                    1.007001   0.122098   8.247 1.50e-15 ***
poor_prop                  -0.577271   0.052695 -10.955  < 2e-16 ***
airportYES                  1.131516   0.454266   2.491   0.0131 *
n_hos_beds                  0.329221   0.152239   2.163   0.0311 *
n_hot_rooms                 0.091868   0.082174   1.118   0.2641
waterbodyLake               0.264086   0.641963   0.411   0.6810
`waterbodyLake and River`  -0.687556   0.714023  -0.963   0.3361
waterbodyRiver             -0.291319   0.546656  -0.533   0.5943
rainfall                    0.016119   0.017839   0.904   0.3667
avg_dist                   -1.218640   0.188933  -6.450 2.68e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.925 on 490 degrees of freedom
Multiple R-squared:  0.7208,    Adjusted R-squared:  0.7123
F-statistic: 84.34 on 15 and 490 DF,  p-value: < 2.2e-16
```

Multiple Linear Regression

In Multiple linear regression more than one predictor variables are used to predict the response variable

Multiple Linear Regression

Relationship for Multiple linear regression can be written as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon,$$

β_0 is known as Intercept
p is the number of predictors
ϵ is an error term

For our Model,
The equation is

$$\boldsymbol{Price} = \beta_0 + \beta_1 \text{ Crime\_rate} + \boldsymbol{\beta_2} \text{ poor\_pop} \ldots \ldots \ldots \boldsymbol{\beta_{16}} \text{ avg\_dist}$$

Multiple Linear Regression

Estimating Regression Coefficients

$$\text{RSS} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

```
Coefficients:
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```

Multiple Linear Regression

F statistics

```
Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
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crime_rate               0.009710   0.348185   0.028   0.9778
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F-statistic: 84.34 on 15 and 490 DF,  p-value: < 2.2e-16
```

$H_0 : \beta_1 = \beta_2 = \cdots = \beta_p = 0$

$H_a :$ at least one β_j is non-zero.

Multiple Linear Regression

F statistics

Coin is biased if I get 5 consecutive heads in 5 tosses

Probability of Head	1/2	1/2	1/2	1/2	1/2

Probability of classifying a fair coin as a biased coin = (1/2)^5 = 0.03125

If 100 coins are tossed 5 times each ,
What is the probability of getting all heads in at least one of the coin

$$1 - \left(1 - \frac{1}{32}\right)^{100} \approx 95\%$$

Multiple Linear Regression

F statistics

For our model

	β	β	β	β	β
Probability of Wrongly classifying B as significant	5%	5%	5%	5%	5%

If number of variables is large, there is very high chance that one of the B is wrongly classified

$$F = \frac{(\mathrm{TSS} - \mathrm{RSS})/p}{\mathrm{RSS}/(n-p-1)},$$

Multiple Linear Regression

Categorical Variables

airportYES	waterbodyLake	waterbodyLake and River	waterbodyRiver
1	0	0	1
0	1	0	0
0	0	0	0
1	1	0	0
0	1	0	0
1	0	0	0
1	0	0	1
0	1	0	0
1	0	0	0

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i = \begin{cases} \beta_0 + \beta_1 + \epsilon_i & \text{If Airport is present} \\ \beta_0 + \epsilon_i & \text{If Airport is not present} \end{cases}$$

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
airportYES      1.131516   0.454266   2.491   0.0131 *
```

Multiple Linear Regression

Categorical Variables

airportYES	waterbodyLake	waterbodyLake and River	waterbodyRiver
1	0	0	1
0	1	0	0
0	0	0	0
1	1	0	0
0	1	0	0
1	0	0	0
1	0	0	1
0	1	0	0
1	0	0	0

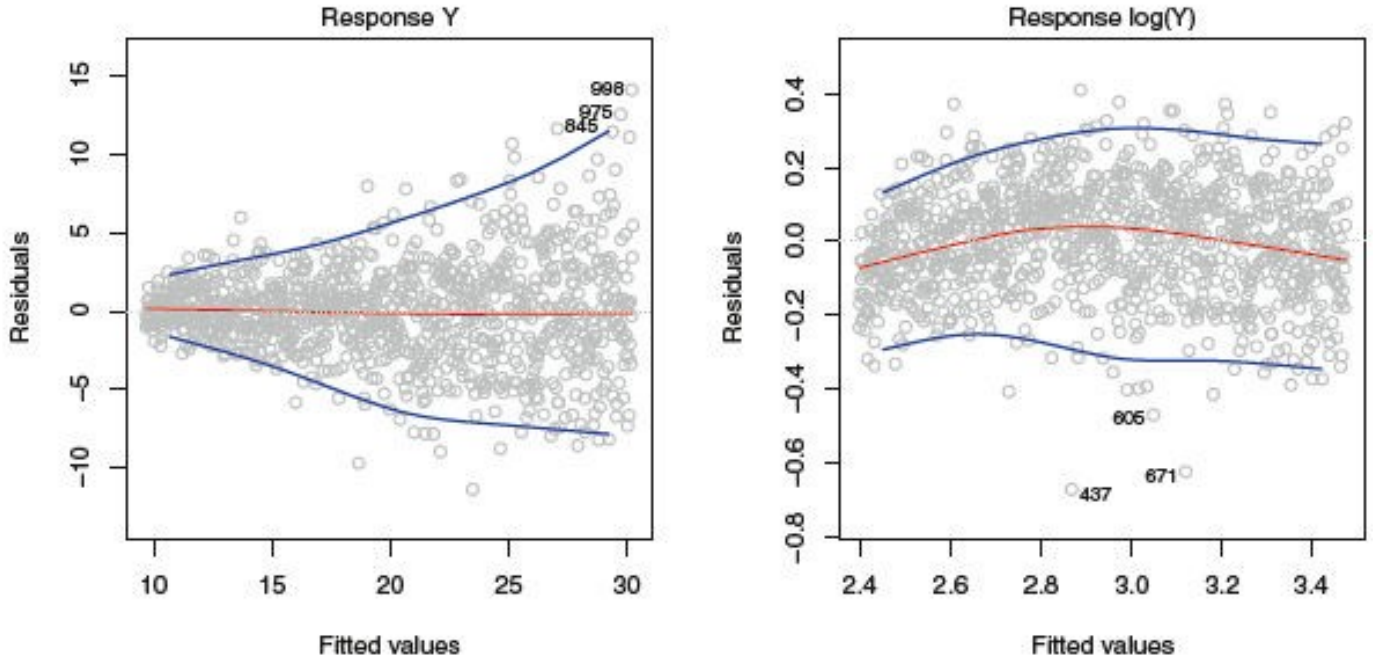
```
Coefficients:

                          Estimate Std. Error t value Pr(>|t|)
waterbodyLake             0.264086   0.641963   0.411   0.6810
`waterbodyLake and River` -0.687556   0.714023  -0.963   0.3361
waterbodyRiver            -0.291319   0.546656  -0.533   0.5943
```

Multiple Linear Regression

heteroscedasticity.

$$\mathrm{Var}(\epsilon_i) = \sigma^2.$$

Assumption : Variance of error term is independent of values of Y



Solution : Scaled down Y variable by using log(y) or $\sqrt{y}$ instead

Multiple Linear Regression

Categorical Variables

airportYES	waterbodyLake	waterbodyLake and River	waterbodyRiver
1	0	0	1
0	1	0	0
0	0	0	0
1	1	0	0
0	1	0	0
1	0	0	0
1	0	0	1
0	1	0	0
1	0	0	0

```
Coefficients:

                          Estimate Std. Error t value Pr(>|t|)
waterbodyLake             0.264086   0.641963   0.411   0.6810
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```

Linear Regression

Other Linear Models

The standard linear model

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \epsilon$$

Ordinary least Square method, minimize RSS

$$RSS = e_1^2 + e_2^2 \ldots \ldots . + e_n^2$$

Why look for alternatives

1. Prediction Accuracy
2. Model Interpretability

Linear Regression

Other Linear Models

We will be discussing two types of methods

1. Subset selection
   Instead of training model on all variables, we will be using a subset of available variables

2. Shrinkage method
   We will try to regularize i.e. shrink the coefficient of some variables towards zero

Other Linear Regression

Subset Selection

This approach involves identifying a subset of the *p* predictors that we believe to be related to the response.

We will be discussing these three techniques of Subset selection

1. Best Subset Selection
2. Forward Step-wise selection
3. Backward Step-wise selection

Other Linear Regression

Best Subset Selection

In Best Subset selection technique, we will be training the model on all possible subsets of p variables. For p variables the total number of subsets are 2^p

1. Let *M*0 denote the *null model* , which contains no predictors. This model simply predicts the sample mean for each observation.
2. For *k* = 1, 2, . . .*p*:
   a) Fit all $\binom{p}{k}$ models that contain exactly *k* predictors.
   b) Pick the best among these $\binom{p}{k}$ models, and call it *Mk*. Here *best* is defined as having the smallest RSS, or equivalently largest *R*^2.
3. Select a single best model from among *M*0, . . . ,*Mp* using adjusted *R*^2.

Other Linear Regression

Forward Step-wise selection

In Forward stepwise selection technique, we start training the model with no predictors and continue to add predictors one by one till we have added all of them.

For p variables the total number of models are $1+p(p+1)/2$

1. Let *M*0 denote the *null* model, which contains no predictors.
2. For *k* = 0, . . . , *p* − 1:
    A. Consider all *p* − *k* models that augment the predictors in *Mk* with one additional predictor.
    B. Choose the *best* among these *p* − *k* models, and call it *Mk*+1. Here *best* is defined as having smallest RSS or highest *R*^2.
3. Select a single best model from among *M*0, . . . ,*Mp* using adjusted *R*^2.

Other Linear Regression

Backward Step-wise selection

In Backward stepwise selection technique, we start training the model with all the predictors and continue to remove them one by one till we have removed all of them.

For p variables the total number of models are $1+p(p+1)/2$

1. Let *Mp* denote the *full* model, which contains all *p* predictors.
2. For *k* = *p*, *p* − 1, . . . , 1:
    1. Consider all *k* models that contain all but one of the predictors in *Mk*, for a total of *k* − 1 predictors.
    2. Choose the *best* among these *k* models, and call it *Mk*−1. Here *best* is defined as having smallest RSS or highest *R*^2.
3. Select a single best model from among *M*0, . . . ,*Mp* using adjusted *R*^2.

If n is less than p backward stepwise selection will not work , onlt forward selection technique can work in this case

Other Linear Regression

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This approach involves identifying a subset of the *p* predictors that we believe to be related to the response.

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    2. Choose the *best* among these *k* models, and call it *Mk*−1. Here *best* is defined as having smallest RSS or highest *R*^2.
3. Select a single best model from among *M*0, . . . ,*Mp* using adjusted *R*^2.

If n is less than p backward stepwise selection will not work , onlt forward selection technique can work in this case

Other Linear Regression

Shrinkage method

This approach involves fitting a model involving all p predictors. However, the estimated coefficients are shrunken towards zero relative to the least squares estimates.

We will be discussing these two techniques of Shrinkage method

1. Ridge Regression
2. The Lasso

Other Linear Regression

Ridge Regression

In Ridge regression, we will be trying to shrink the coefficients of variable towards zero by adding shrinkage penalty

$$\text{RSS} = \sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij}\right)^2$$

$$\sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij}\right)^2 + \lambda\sum_{j=1}^{p}\beta_j^2 = \text{RSS} + \lambda\sum_{j=1}^{p}\beta_j^2$$

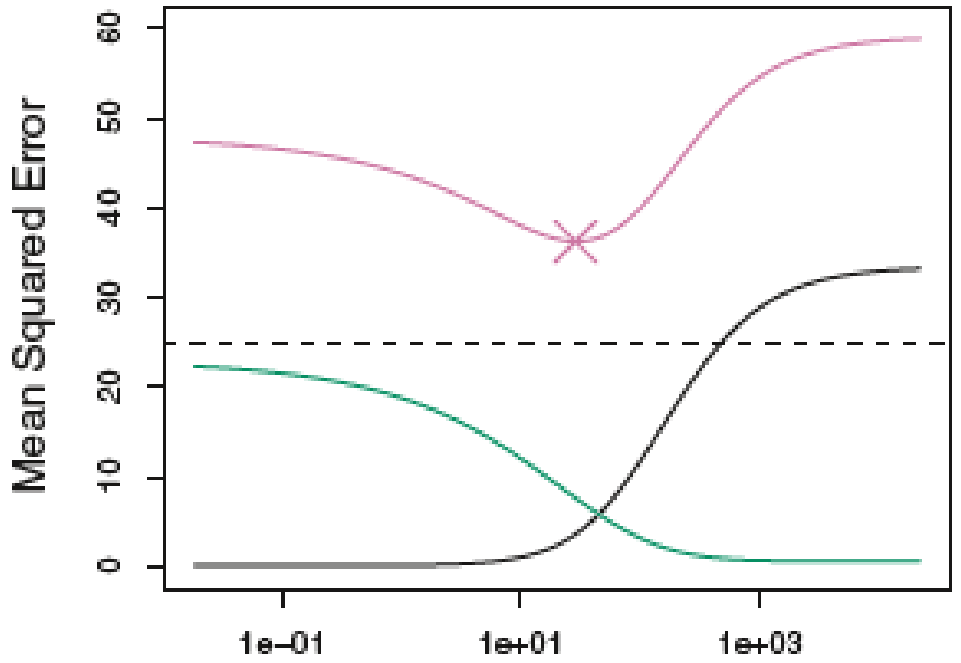
Shrinkage Penalty

λ – Tuning Parameter

Because of this shrinkage penalty, Ridge regression varies with scale of independent variable, therefore, we need to standardized the values of these variables

Other Linear Regression

Ridge Regression



Other Linear Regression

Lasso Regression

In Lasso regression, we will be trying to shrink the coefficients of variable towards zero by adding shrinkage penalty

$$\sum_{i=1}^{n}\left(y_i-\beta_0-\sum_{j=1}^{p}\beta_j x_{ij}\right)^2+\lambda\sum_{j=1}^{p}|\beta_j| \;=\mathrm{RSS}+\lambda\sum_{j=1}^{p}|\beta_j|$$

Shrinkage Penalty
λ – Tuning Parameter

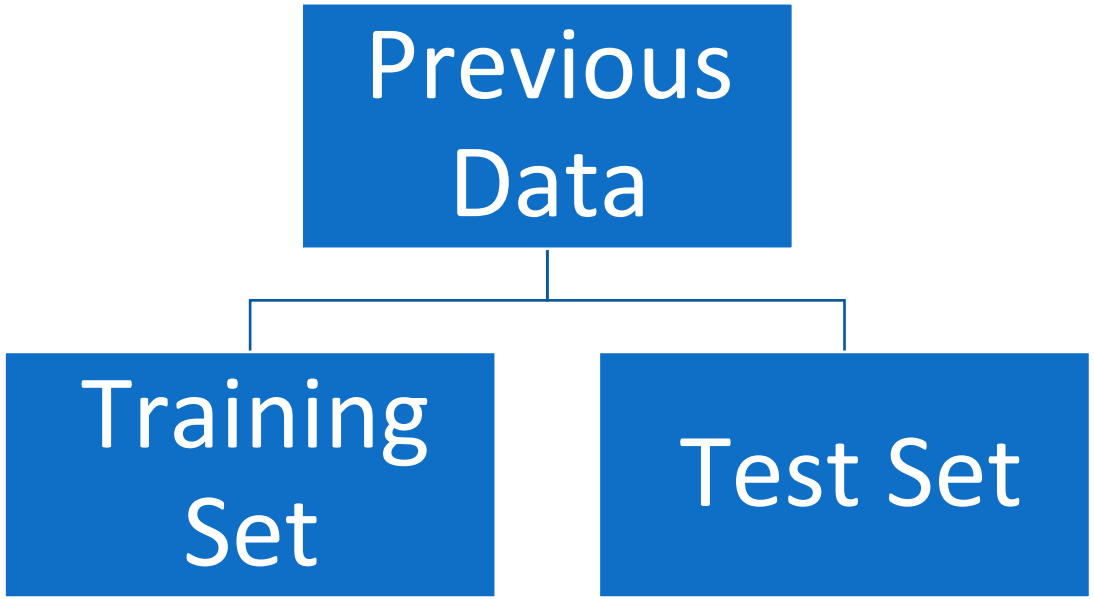
In the Lasso technique, for sufficiently large value of λ, several coefficient will actually become zero, resulting in variable selection

Linear Regression

Test-Train Split

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{f}(x_i))^2$$

- Training error – Performance of model on the previously seen data
- Test error – Performance of model on the unseen data



Linear Regression

Test-Train Split

Training Set - $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$

Model is trained

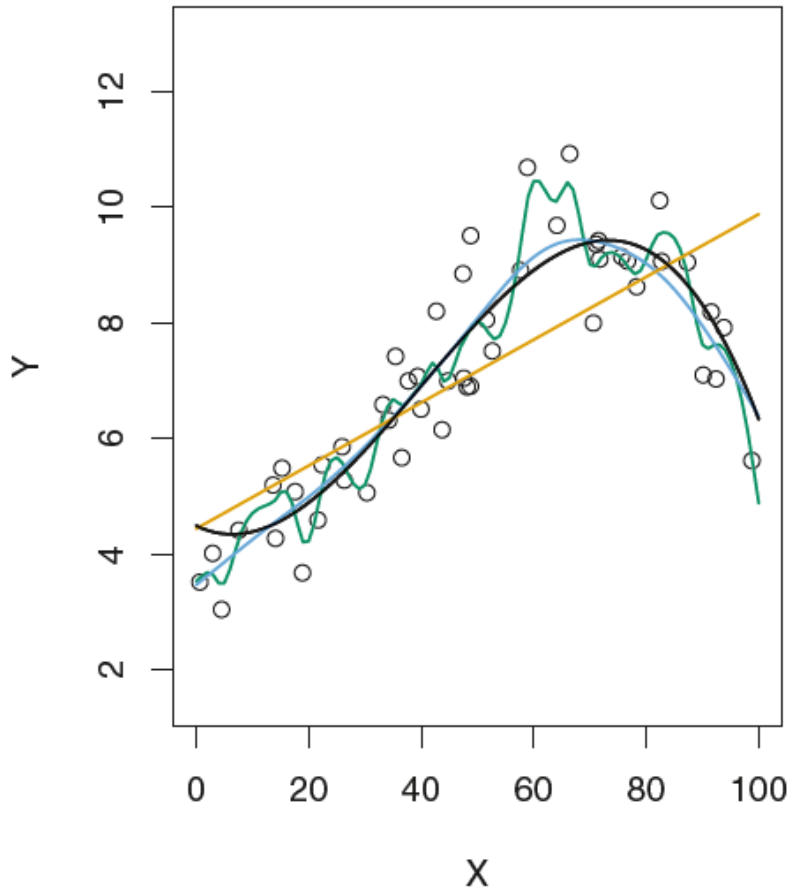
$$y = f(x)$$

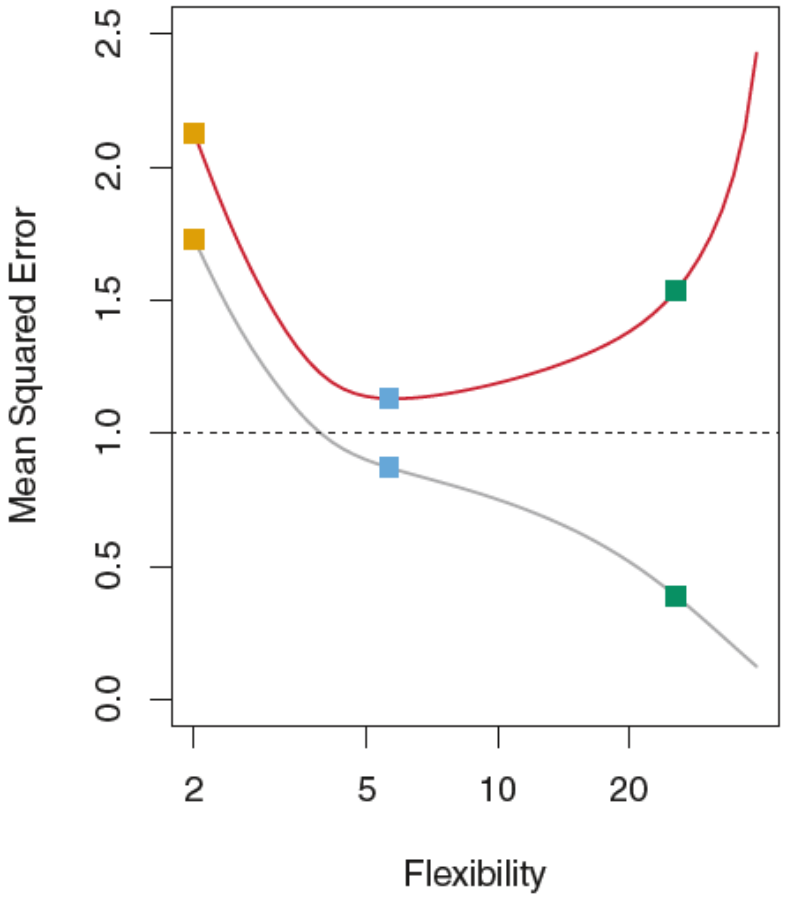
Test Set - Previously unseen data (x_0, y_0)

Test MSE - $\text{Ave}(\hat{f}(x_0) - y_0)^2$

Other Linear Regression

Test-Train Split





Linear Regression

Test-Train Split Techniques

1. Validation set approach
    - Random division of data into two parts
    - Usual split is 80:20 (Training : Test)
    - When to use – In case of large number of observations
2. Leave one out cross validation
    - Leaving one observation every time from training set
3. K-Fold validation
    - Divide the data into k set
    - We will keep one testing and K-1 for training

Linear Regression

Bias-Variance Trade-Off

$$Expected\ test\ error = E(Bias^2) + E(Variance) + E(\epsilon)$$

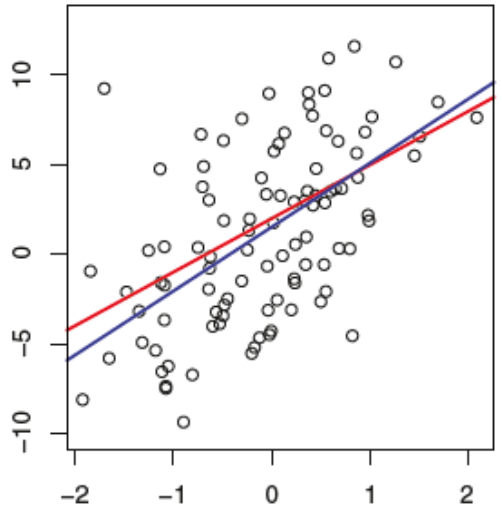
$E(\epsilon)$
Variance of error, Irreducible

$E(Variance)$
Amount by which predicted function will change if we change training dataset

$E(Bias^2)$
Error due to approximation of complex relationship as a simpler model such as linear model

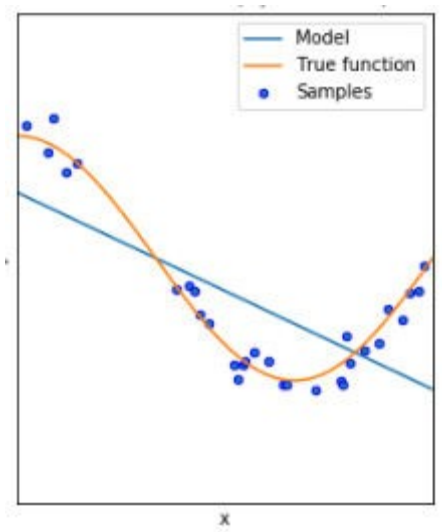
Linear Regression

Variance



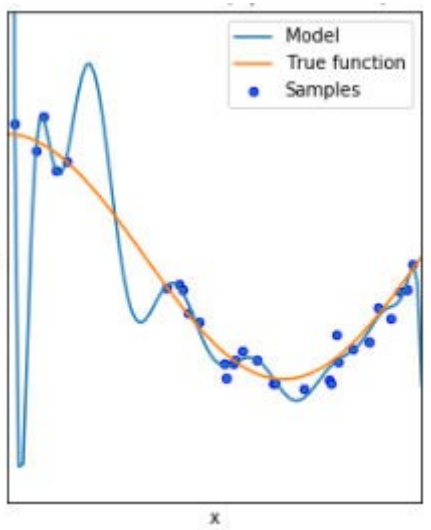
Population regression line

Sample regression line



Less Flexible method

More Flexible method

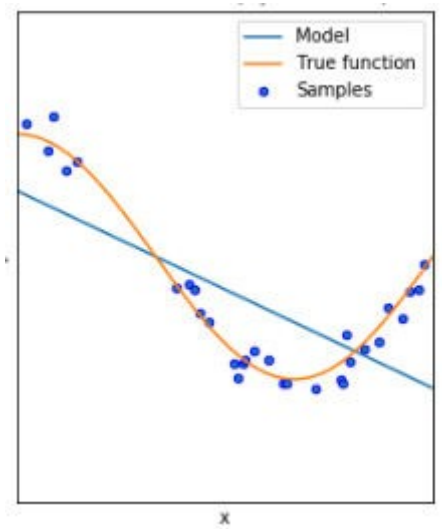


Linear Regression

Bias

$E(Bias)$

Error due to approximation of complex relationship as a simpler model such as linear model



Linear Regression

Bias-Variance Trade-Off

The Tradeoff
If we try to decrease one by changing model flexibility, other one increases