Betting Systems in Blackjack

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Introduction

Blackjack, also known as twenty-one, is recognized worldwide as the main casino card game. Many players are choosing to gamble by playing it and this is why there have been many studies about which are the best responses to all situations. On top of that, after finding the best way to play, there were questions like who is having the edge, the player or the house (casino)? And by how much? Without any surprise, casino had the edge and so, there were new studies regarding ways to beat that edge. There are many books relating to this subject including guide of how to play and even how to behave when you are in a casino. In the following sections, we are not going to present anything similar to that. In fact, we are going to test a strategy which is well known for games such as roulette or craps. This strategy involves changing your bets after a win or lose and it is commonly known as a betting system. Roulette does not have many other strategies rather than changing the way you bet. Blackjack, on the other hand, has many other strategies that can improve the way you play. Therefore, studies about betting systems are quite rare in blackjack.

In the following pages, basic concepts of blackjack will be discussed which will be used in simulations. On these simulations there shall be tested many different popular betting systems. All these betting systems became famous while they were used on other games, however, we will use them on blackjack. Furthermore, there will be an introductory of all betting systems which will be tested. Before investigating the betting systems, we are going to produce results about the game. Given these results, we shall make some assumptions of what to expect. Finally, we will try out the betting systems on the game and we will make adjustments wherever it is needed with the view to getting better results. All final outcomes will be presented and discussed.

In the end, we will know if using betting systems on blackjack gives us an advantage or not and what is the deviation between using them and not using them. On the simulations there are no approximations since the main purpose is to be identical with the real game conditions.

1 Blackjack Basics

Let's start by explaining the game. Anyone from one to seven player can play the game of blackjack at a time. Players are not fighting against each other, but they are playing against the dealer or "the house". Each player is dealt two cards as well as the dealer having one in plain view and the other face down (also known as the hole card). Every player sums the value of his cards, each card has the value of its number, faces have 10 value and aces have either 1 or 11 depending on the other cards. For instance, the cards King and Seven have a total value 17, while an Ace and a Five have a total value of either 6 or 16. After being dealt the cards, the player must make a decision. His main options are to Hit (player asks for another card to try and improve the value of his hand), Stand (player retains his existing hand and does not opt for an additional card), Split (player splits his hand into 2 hands if he has 2 cards of the same value, each hand has an individual bet and is played separately) and Double (player doubles his original bet and gets another card; he has to stand once he gets the card). Two extra options being offered at the beginning are to Surrender (player quits after being dealt the original hand and retains half his original wager) and Insurance (insurance is offered when the dealer shows an ace and you place extra one half of your bet, if the dealer happens to have blackjack you are paid 3 times that extra bet and the game stop, if he doesn't have blackjack, the game continues and you lost the extra bet). The decision making gives the opportunity to develop a strategy. For example, players tend to Double when their hand is 10 and the dealer's is Six.

After players stand, dealer's turn begins. Dealer makes no decisions; he must draw cards until he reaches a value of 17 or higher. As you can imagine, player's strategy is based on dealer's no decision-making game. In case player's card value is higher than 21, then he busts and dealer wins. In case player's cards value is lower than 21 but the dealer's are not, then dealer busts and player wins. In case they both have lower than 21, then the one with the higher value wins.

As a result, these are the decisions which a player can make, but what about betting? Players bet before the dealer deals the cards. If they win the dealer pays them the same amount with the bet, otherwise they lose their bet. There are some cases where you can bet while having dealt the cards. When you choose to double, you double your bet and receive one extra card and then stand. In addition, when you split, you place the amount of your bet to split the cards. In both cases, if you win you get paid twice of your original bet. One more way to win more than the usual payout is when you form Blackjack. Blackjack is formed when your two cards are a 10 in value and an ace and in this case, you get paid 3:2 of your original bet.

1.1 European Blackjack Rules

There are many variations of blackjack usually differing from one casino to another. European Blackjack is one of the most popular blackjack variations played at most casinos across Europe. In addition, it's one of the most commonly played Blackjack variants that is played at casinos online as well. Just like other Blackjack games, European Blackjack is played against the dealer. The game generally uses 6 decks so as not to shuffle in every game. Nevertheless, it's also possible to find variants with 4 or even 8 decks. In European Blackjack, the dealer takes his hole card after he has dealt the player's cards meaning that he cannot check for blackjack if his first card is an Ace or a 10-value card. Therefore, the player has no way of knowing if the dealer has a blackjack until his hand is actually over. The impact on a player's strategy is obvious. The goal, however, remains the same which is to beat the dealer. The card values in European Blackjack are essentially the same as those in any other Blackjack games. As per usual, both 10s and face cards are worth 10 points and Aces are worth either 1 or 11. Whenever a player has at least one ace his hand is called soft since he can decide whether it will be 1 or 11. Number cards are also worth the same numerical value.

In European Blackjack, you can get the same payout as well if you win. For different hand types there are different payouts. Blackjack has a payout of 3:2, however, Blackjack formed after a Split has a payout of 1:1 and a Hand of 21 formed by more than two cards has a payout of 1:1 as well. Other Winning Hands have also a payout of 1:1. Insurance has a payout of 2:1.

European Blackjack Rules summed up:

- This variant of Blackjack is generally played with 6 decks. However, there are variants of the game which are played with 2-8 decks.
- The dealer cannot peek for blackjack.
- The dealer's blackjack is an automatic winner over a player's hand with a value of 21.
- The game is pushed if both the player and dealer have blackjack hands.
- The dealer must hit for a hand value of 16.
- The dealer must stand on a soft 17.
- The player may split only once per hand.
- An insurance bet can be taken out by the player whenever dealer show an ace, if he wishes to.
- Surrender is never allowed during this game.

Payouts on wins:

- Even money pays out at 1:1.
- Blackjack pays out at 3:2.
- Insurance bet pays out at 2:1.

1.2 Basic Strategy

The fact that the dealer makes no decision at all means that the player can develop an optimal strategy which is going to be used in every game. This strategy will produce the optimal decision at any cards situation making the edge which the dealer has to become almost zero. There is a hand calculator here in which you can calculate the expected value of every decision at any cards situation. In general, the Basic Strategy which a player must follow is summed up in the board bellow.

Horizontally there are the possible plain cards the dealer may show while vertically there are possible sums of the cards you can have. When you have two cards you should follow the whole board whereas when you have more cards you should follow the Hard or Soft section and at the Double decisions you should Hit since doubling is no longer allowed.

European Blackjack

Player	Dealer's card									
Hard	2	3	4	5	6	7	8	9	10	A
5 to 8	H	H	H	H	H	H	H	H	H	H
9	H	D	D	D	D	H	H	H	H	H
10	D	D	D	D	D	D	D	D	H	H
11	D	D	D	D	D	D	D	D	H	H
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
16	S	S	S	S	S	H	H	H	H	H
17 to 21	S	S	S	S	S	S	S	S	S	S
Soft	2	3	4	5	6	7	8	9	10	A
13 to 17	H	H	H	H	H	H	H	H	H	H
18	S	S	S	S	S	S	S	H	H	H
19 to 21	S	S	S	S	S	S	S	S	S	S
Pairs	2	3	4	5	6	7	8	9	10	A
2,2	P	P	P	P	P	P	H	H	H	H
3,3	P	P	P	P	P	P	H	H	H	H
4,4	H	H	H	P	P	H	H	H	H	H
5,5	D	D	D	D	D	D	D	D	H	H
6,6	P	P	P	P	P	H	H	H	H	H
7,7	P	P	P	P	P	P	H	H	H	H
8,8	P	P	P	P	P	P	P	P	H	H
9,9	P	P	P	P	P	S	P	P	S	S
10,10	S	S	S	S	S	S	S	S	S	S
A,A	P	P	P	P	P	P	P	P	P	H

H	Hit
S	Stand
D	Double if allowed, otherwise hit
P	Split

Figure 1: Basic Strategy

1.3 Card Counting

Over the years there have been many strategies - methods aiming to beat the house, meaning that they will have a positive expected value by following this strategy. The most known method is the card counting. Blackjack is a game which depends from its previous outcome. Since the cards are not shuffled after every game, the player gains more and more information about the rest of the cards while the game goes on. For instance, when at a game the cards 3,5,7,8,8,10,A are used, the player knows that these cards are going to be missing from the rest of the cards which are due to be dealt and since the player is aware of the fact that at the beginning there were 6(or 8) decks he can know the exact cards left. As a result, he can reorganize his strategy in order to make it optimal for the current cards left to be dealt. The basic strategy which we viewed previously applies when all the cards are in the deck to be dealt. Many simulations have shown that when there are many high value-cards in the deck the player has the advantage and thus he should bet more. Betting more when you have the advantage results in positive expected value, therefore the player beats the house.

As you can understand, the less cards are left to be dealt the more information you have about them. Casinos are aware of this method, so they have found many ways to overcome the edge which the player gains in these situations. The most known way is by using as many decks as possible (8 in most cases) and put in the half of the total deck a stop card. What this card does is when it is draw it means that the cards have to be shuffled. This way the player has not enough information to gain edge and all the information he had gathered are eliminated. As a result, not only is counting cards difficult but it offers almost nothing as well. Not to mention how difficult this method is to master and also not to get caught applying it, since casinos are not in favor of this kind of methods.

2 Betting Systems

One other well-known method (or myth?) for beating the house is applying a betting system. To be more exact, player changes his bet every time after a game is played with the view to gaining more value, instead of placing the same bet every time. Betting placements usually depend on the outcome of the previous games (wins or losses) and the payouts offered at that time. In our case, we will apply the Basic Strategy (best response) so as to have an as close to 50-50 situation as possible and

thus we will use betting systems which are specialized in this kind of situations. However, we must take into account the Table Betting Limits which are the Minimum and Maximum bet allowed at that table. Maximum bets are usually equal to a hundred times the Minimum bet in casinos or they can even be two thousand times greater than the Minimum bet in online blackjack. In the following pages there are presented some betting systems which we are going to be analyze in later sections.

2.1 Martingale System

One of the most popular betting system is the Martingale System. It is the only system to have a mathematically speaking linear increase to your outcome when being applied, however, it has a very high risk of going broke since it requires an extremely huge bankroll during losing streaks. How this system works is whenever player loses, he doubles up his next bet until he wins, thereafter he puts his original bet. In this way, player gains his losses back when he wins or better yet, should he hit blackjack, he will win even more, depending on the losing streak. In general, after every win player bets just one betting unit. The only thing is that his bet size can grow extremely fast during losing streaks resulting in going broke when many losses in a row occur. Nevertheless, huge bankroll is not the solution to this problem due to the betting limits of the table, therefore, even if player starts this system from the minimum bet, he can run into the table limit with the first high losing streak happening.

Table 1:Example of Martingale System

Bet	Result	Earnings
1	W	1
1	L	0
2	W	2
1	L	1
2	L	-1
4	W	3
1	W	4
1	L	3
2	L	1
4	L	-3
8	W	5

In this example Martingale System is applied. It is can be clearly seen that if there was no betting system applied the final earnings would be -1. Instead, with the Martingale being applied, the final earning is +5!

2.2 Reverse Martingale System

The Martingale betting system can be so much deadly at times, that people have seriously started considering playing the reverse of it. That is, you keep on doubling your bet after winning and place the original bet after losing. It is obvious that you must set a winning point in which you must leave the table or else you will just linearly be losing your money over time. The main advantage of this system is that it does not require a huge bankroll.

Table 2: Example of Reverse Martingale System

Bet	Result	Earnings
1	L	-1
1	W	0
2	L	-2
1	L	-3
1	L	-4
1	W	-3
2	L	-5
1	L	-6
1	W	-5
2	W	-3
4	W	1

In this example Reverse Martingale System is applied. It is obvious that if we would be flat betting, the final earnings would have been -1. Instead, with the Reverse Martingale System the final earnings are +1.

2.3 Climbing Betting System

Climbing betting system is a simple betting strategy, designed for normal bankrolls. Player starts with an original bet; we will call it 1 betting unit. He keeps on playing with 1 unit until he wins three times in a row. Thereafter, he bets 2 units, in case he wins two more in a row, he bets 3 unit for the next two games (if he is still winning). On the occasion of losing, he starts betting 1 unit again. This loss is going to be paid by the previous wins. As you can understand, applying this system is slightly better than playing with a constant bet and thus we will not discuss further about it.

2.4 Oscar's System

Oscar's system is a rather complex system, it is contained of a series of bets. The main purpose of each series is to win 1 betting unit. In this system, when player loses, he does not change his bet whereas when he wins, he adds 1 unit to his bet. However, it is essential that the player not place a bet with which he will earn more than 1 unit from his series. For instance, given the fact that you just won 5-unit bet and you are just 2 units below the +1 for your series, what you should do is to place a 3-unit bet instead of the excepted 6-unit one.

Table 3: Example of Oscar's System

Bet	Result	Earnings
1	L	-1
1	W	0
1	L	-1
1	W	0
1	L	-1
1	L	-2
1	W	-1
2	L	-3
2	W	-1
2	L	-3
2	W	-1
2	W	1

In this example Oscar's System is applied. It is clear that but for the betting system, the final earning would be 0. Instead, with the Oscar's system being applied, the final earnings are +1.

2.5 D'Alembert System

D'Alembert betting system is similar to Martingale betting system since you increase your bet after every loss. More specifically, you start by betting 1 betting unit (usually the minimum) and after every lose you increase your bets by 1 unit while after every win you decrease them by 1, unless it is already 1 unit. This system can only work when odds are close enough to 50-50. The logic behind this system is that after a full cycle of equal wins and losses, every win with betting above 1 unit will counter up every loss above 1 unit, leaving the loss at 1 unit and the win at the peak number of betting units to counter each other. This results in a positive value even though an equal number of wins and losses would result in zero value.

Table 4: Example of D'Alembert System

Bet	Result	Earnings
1	W	1
1	L	0
2	L	-2
3	W	1
2	L	-1
3	W	2
2	W	4
1	L	3
2	W	5
1	L	4
2	W	6

In this example D'Alembert System is applied. It is can be clearly seen that were it not for the betting system the final earning would be 1. Instead, thanks to D'Alembert System, the final earnings are +6!

3 Blackjack Games in Simulation

Choosing the best response to all kind of situations in blackjack is fairly easy to calculate. By doing so, we created the Basic Strategy, which consists of the best strategy to follow against all combinations of cards you may have versus the dealer. Nonetheless, it is quite difficult to calculate the exact probabilities of winning or losing a game against the dealer. It is even harder to predict what kind of win or lose is this going to be, is it going to be blackjack? Or will you lose double? etc. In addition, there are more questions to be asked such as what is the probability to win 2 or 3 or 4 games in a row? Therefore, we are going to run a simulation of many blackjack games. To make this simulation as real as possible, we shuffle 8 decks of cards and play until the 4 decks are dealt. This way, it would be useless to count cards which is not our goal anyways. Player will always play the Basic Strategy and the rules are going to be the European Blackjack rules as discussed above.

3.1 Blackjack Statistics

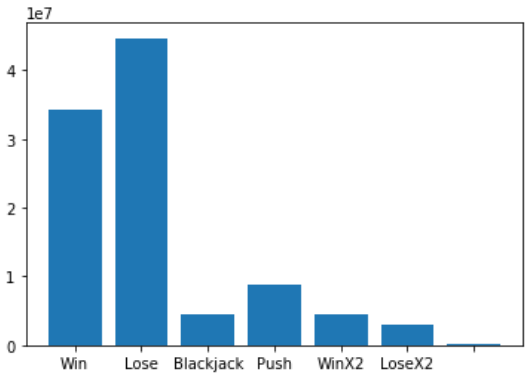
There are many claims about which are the probabilities of whether winning or losing a game in blackjack. In fact, there are so many claims that you cannot be sure what is true or not. As a result, we will run a blackjack simulation in order to find out ourselves. After playing 100 million blackjack games, we came up with the below results:

Table 5: Blackjack Probabilities

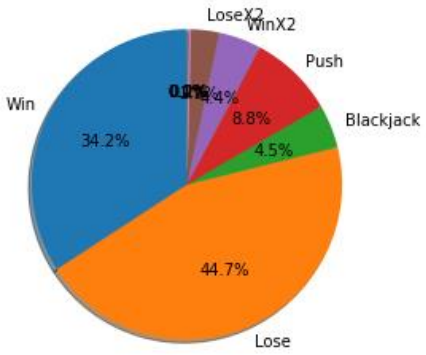
Result of the game	# of times achieved	Final % of the result
Win	34181191	34.181191%
Lose	44689921	44.689921%
Blackjack	4523767	4.523767%
Push	8826586	8.826586%
WinX2	4428585	4.428585%
LoseX2	2939655	2.939655%
WinX3	221484	0.2214845%
LoseX3	144462	0.144462%
WinX4	31099	0.031099%
LoseX4	13250	0.01325%

Push is when you draw with the dealer. Win/LoseX2 is when you win/lose with split or double, Win/LoseX3 is when you win/lose with split and one of the hands being doubled, Win/LoseX4 is when you win/lose with split and double both your splits.

Bellow we can see the results of 100 million games of blackjack in a bar graph and a pie chart:



Graph 1: Bar graph of the 100M games



Graph 2: Pie chart of the 100M games

Taking these points into consideration, player's edge over the house is actually negative, it is -0.4856%. As expected, house has the advantage when playing flat blackjack against dealer. This is why betting systems were invented.

One more critical statistic which we must calculate is the probabillity of having win/lose streaks. Due to the fact that there are more than one kind of wins/loses, winning a game with the original bet and another with the original bet being doubled in a row will result in 3 wins streak (add the winning factors of wins in a row). To make things simplier, blackjack wins will be added as one.

That is to say, here are some examples:

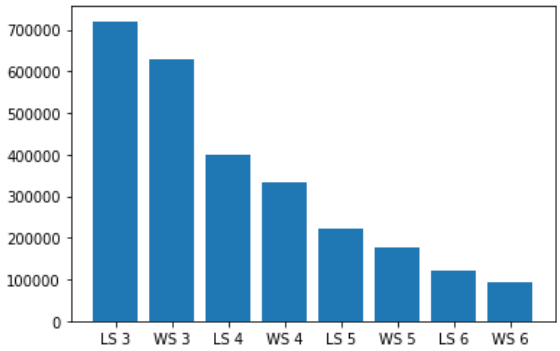
i. W, L, L, LX2, L, W will be a 5 loss streak
ii. L, W, BJ, W, L will be a 3 win streak
iii. L, W, D, WX2, W, L will be a 4 win streak
iv. W, L, LX3, L, D, L, W will be a 6 loss streak

In addition, as lose streak 3 we calculate all 3 or above losses in a row, so that if we want to calculate the probability of having exactly 3 losses in a row, we must divide lose streak 4 from lose streak 3. After playing 10 million blackjack games, we came up with the below results:

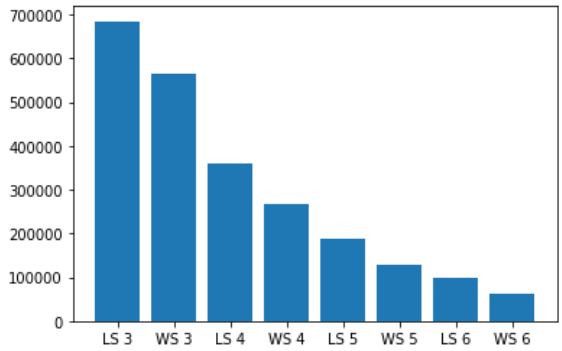
Table 6: Blackjack Win/Lose Streaks

Streak	# of times achieved	Final % of the result
LoseStreak 3	790374	7.90374%
WinStreak 3	690289	6.90289%
LoseStreak 4	438304	4.38304%
WinStreak 4	364828	3.64828%
LoseStreak 5	242849	2.42849%
WinStreak 5	192845	1.92845%
LoseStreak 6	134188	1.34188%
WinStreak 6	102241	1.02241%

Bellow we can see the results of 10 million games of blackjack in two bar graphs, the one illustrates the streaks while adding up the win or lose factor whereas the other one shows the streaks if you count all kind of wins or losses as one:



Graph 3: Blackjack Streaks, adding up



Graph 4: Blackjack Streaks, all wins/losses = 1

These two graphs are very similar, the only difference is that the first one counts some more streaks than the second.

3.2 Betting System Expected Results

Now that we have some results about crucial probabilities of the game, we can make some assumptions about the results of the betting systems. However, finding the results will still be complex and thus we will make some approximations wherever we can. In later sections, results shall be clearer through simulations.

3.2.1 Martingale System

Let's say that our initial bet is α. So according to Martingale System we bet α after every win and we double our bet after every loss. That is, in case we have 5 loses in a row and then one win this is what happens:

Our expenses: $\alpha + 2\cdot\alpha + 4\cdot\alpha + 8\cdot\alpha + 16\cdot\alpha$

Our winnings: $32\cdot\alpha$

We observe that: $\alpha + 2\cdot\alpha + 4\cdot\alpha + 8\cdot\alpha + 16\cdot\alpha = \alpha + 2\cdot\alpha + 2^2\cdot\alpha + 2^3\cdot\alpha + 2^4\cdot\alpha$

Which is a geometric series with start term α and common ratio r=2. Therefore, it is equal with $\alpha\cdot\frac{r^n-1}{r-1} = \alpha\cdot(2^5-1) = 32\cdot\alpha - \alpha$ which the winning minus the starting bet α. As a result, at every win observed in martingale, we add α to our budget leading to its linear increase. Nevertheless, in order for us to win α, we have to make bets, reducing our budget by $\alpha\cdot\frac{r^n-1}{r-1}$ which is a rather large amount. So, in the long-run, it is possible

that we will run out of budget in a high lose streak, for instance, there is a 1.34% probability of having 6 loses in a row, which means $\alpha(2^6-1) = 64\cdot\alpha - \alpha$ in expenses for chasing to win α!

3.2.2 Reverse Martingale System

In contrast to the Martingale System, the reverse of it linearly decrease our budget. In this case losing streaks do not hurt us that much as with the martingale system, just like the winning streaks do not give as any particular profit (compared to flat betting). Therefore, there is no need for a big budget. The real profit with this betting system comes to win streaks, but if a loss occurs after the win streak, we lose all profit. This is why there is no reason to talk about the long run but instead we can have a different strategy with this system. We can set a win limit and a loss limit such that when we reach them, we will stop and start over. By doing so, we will stop and collect the winning of a win streak before a loss comes. So, we can to the following thoughts:

If we play 100 games, it is sure that we will have a 6 wins streak since it happens with probability 1.02241%. In those 100 games, our losses will be 44.689921 + 2 * 4.428585 ≈ 51 while our winning will be $2^7 - 1 = 63$. Therefore, if we set a winning limit of 59 and a losing limit of 55 (losing limit is actually pointless since the betting systems starts over after every win) we will have profits over the long run.

3.2.3 Oscar's System

Oscar's System is too complicated to be analyzed on a piece of paper. There is an obvious goal when completing series and that is to win one betting unit. Further conclusions cannot be made; Therefore, we will test many strategies during the simulations, and we will examine all the results.

3.2.5 D'Alembert System

Assuming our initial bet is 1, after 10 games are played with 5 wins and 5 losses this is what the outcome will be:

For example: L W L L W W L W L W
Our bets will be: 1 2 1 2 3 2 1 2 1 2
And our earnings: -1 1 0 -2 1 3 2 4 3 5

While there should have been 0 (5 wins – 5 losses). That is because of a pyramid effect. Let's make a simpler example to demonstrate it:

Game: L L L L L W W W W W

Bets: 1 2 3 4 5 6 5 4 3 2

As we can clearly see in the second game, we lost 2 betting unit which we gain back at the tenth game (which we have won). Likewise, in the third game we lost 3 which we gained back at the nineth game. Crossing out the losses and wins with the same bet we are left with the first game where we lost 1 and the sixth game where we won 6 and that is how we ended up winning 5 instead of 0. In conclusion, when a full cycle closes (same number of wins and losses) all wins and losses cross out instead of the lowest bet loss and the highest bet win whose difference is our profit. In general, every time that our bettings are oscillating, we have a profit which is equal to the local maximum bet of the oscillation minus the local minimum bet of the oscillation, provided that the cycle will be completed. Having said all that, we must have the same number of wins and losses, which we do not in blackjack. What is more, win or lose streaks boost the profits or losses of the system and so since lose streaks are more possible this system fails to that as well. However, it is worthy of trying some strategies at the simulations with the view to seeking for a profit after all.

4 Running Simulations for the Betting Systems

In this section, we are going to run simulations of many blackjack games, applying betting systems so as to get results. When you play blackjack, you usually play up to a hundred games. Nonetheless, in a hundred games it cannot be clear if you are winning because of the betting system or luck. Assuming that you will play these hundred games a thousand times, then the outcome will be due to the betting system. Consequently, we are going to make our simulations run up to a million games. This does not mean that you will ever play that many games, but it will illustrate the betting system's tendencies. In addition, after playing one million games, we are going to test if the house has edge around -0.4856% ± 0.4856%, which is the expected one. In this way, should the games be in our favor, we will play them again so that the results will be as realistic as possible and not to depend on luck.

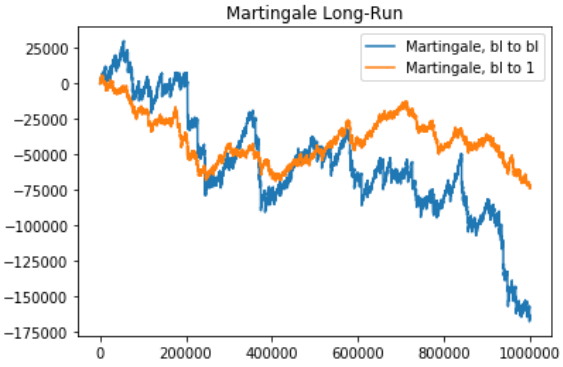
On the other hand, we must say that blackjack game has many payouts. We cannot predict at which time a payout will occur, but we do have an idea of its possibilities to happen. As a result, it is not clear how our outcome will come up even if we play millions of games. Playing with betting systems is even worse since there will be

different bettings every time. For this reason, in most cases, we are going to repeat the process of playing many games up to a thousand times and print the results in the same graph in which condense areas are the most possible outcomes.

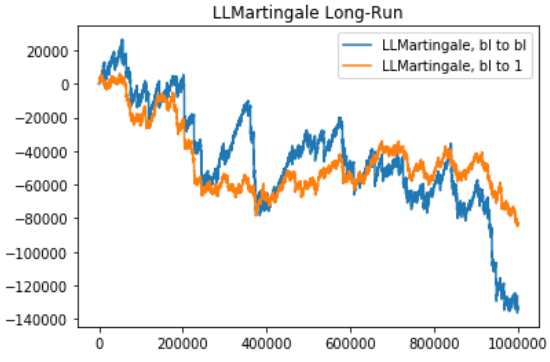
In general, what we are looking for is a profitable system (having a positive outcome) with the minimum risk (not to have extremely low values to the way to a positive outcome). For instance, a gainful outcome with an incredibly low minimum value is worse than a less gainful outcome but with a not so low minimum value. Thus, in our simulations we are going to check the values at which they end up and the minimum value at which they reach to make conclusions.

4.1 Martingale System

To begin with, we must introduce a new variation to this system which is adapted to the blackjack game. In a normal game where you just win the bet you placed or lose it, by every win you gain plus one betting unit (Martingale System). Nevertheless, in blackjack there are many different ways of winning or losing and therefore by doubling your bet after every loss can lead to situations where if you win you will result in something else rather than the plus one that we want. For this reason, we are going to try another betting system in which after every loss we will place a bet such that it will overcome all the debt of the losses plus one, and, as usual, we will bet one unit after winning. Therefore, we will try which system is the best, but before we do, we should solve another problem, the table betting limit. When our bets reach to the betting limit, should we bet the betting limit? or should we start over and bet 1 again? We will attempt both ways in both normal and adapted Martingale System. We will set the betting limit equal to 2000 since some online blackjacks set their table max bet 2000 times the min, and so we shall find out which is the best:



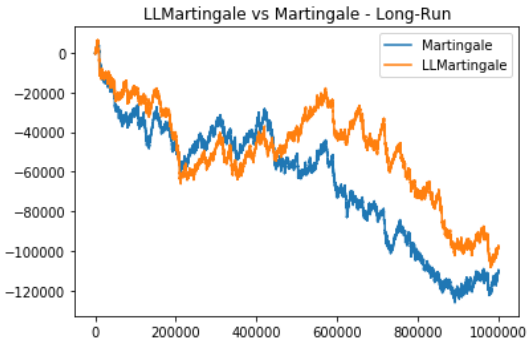
Graph 5: Comparing betlimit to betlimit (bl to bl) vs betlimit to 1 (bl to 1)



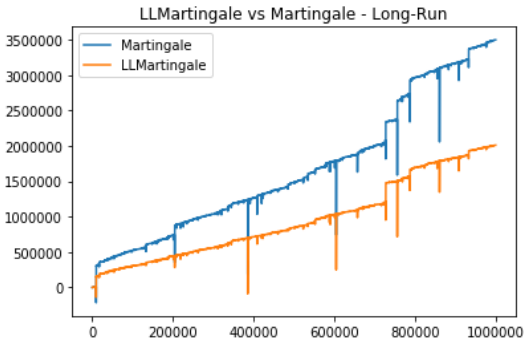
Graph 6: Comparing betlimit to betlimit (bl to bl) vs betlimit to 1 (bl to 1)

Firstly, we must say that we call the adapted Martingale as LLMartingale. We can see that it is not clear enough which is better, however, in graphs 5 & 6, going to 1 after reaching betlimit is slightly better. Staying at the betlimit might be more profitable at times but going to 1 is much safer and ends up higher so, at the simulations to come we will go to 1 when reaching the betlimit.

Accordingly, we are going to test which system is better in the long run considering the above results.

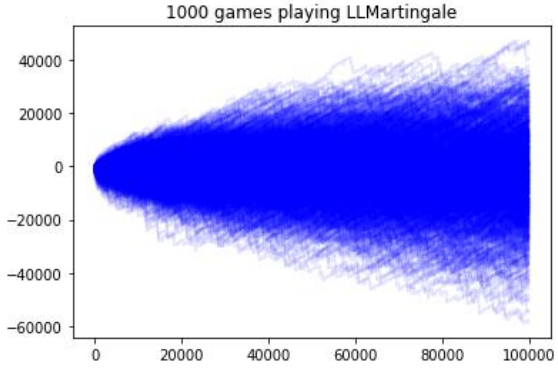


Graph 7: Martingale vs adapted Martingale

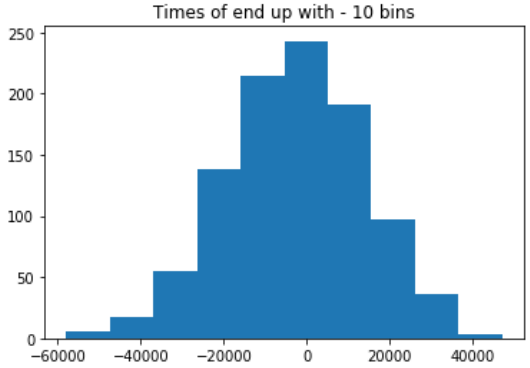


Graph 8: Martingale vs adapted Martingale (no max bet limit)

The graph 7 compares the two systems in the long run, it is hard to point out which is better since they have many similarities. Adapted Martingale earnings are higher during most of the games than the normal one and as a result, it is slightly better. By comparing them without setting a limit, in graph 8, we can see that Adapted Martingale provides lower profit, but it has higher minimum value as well. In particular, for Martingale minimum value was -215094 and we did end up with 3496838, while for LLMartingale minimum value was -138784 and we did end up with 2009513. These values suggest that even though Martingale will have 1.74 times more income, it dramatically falls to 1.55 times higher debt. That is why we prefer LLMartingale to Martingale, even though the numbers might not be accurate. Therefore, we are going to play a hundred thousand games a thousand time with LLMartingale to get accurate results, since it is clearly the best system of the above tested.



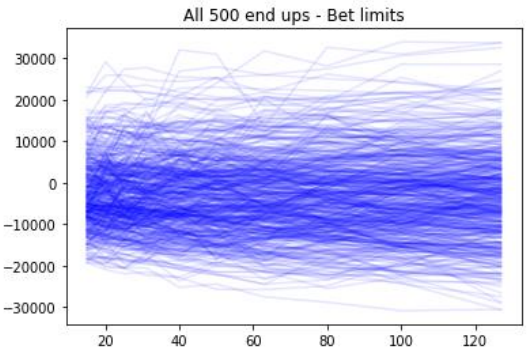
Graph 9: Results from earnings over the games



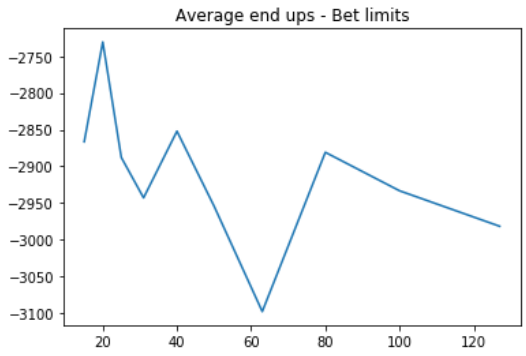
Graph 10: Histogram of earnings end up with

Turning to graph 9, it can be clearly seen that adapted Martingale has an extremely wide range of outcomes, making it significantly unpredictable. Final outcomes range from losing -60% per game to winning 40% per game. In the histogram we are certain that losing is slightly more often than winning, in fact, it turns out to be losing -2.94% a game.

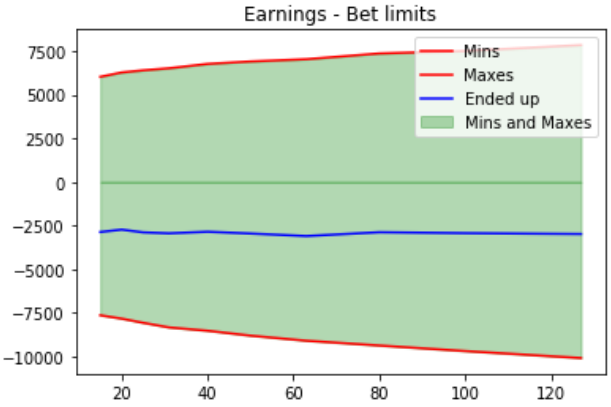
Furthermore, there is one more way with which we can approach LLMartingale System and that is setting a limit of debt at which we will start over. We will run many games setting various limit values until we will find which is the best. This will be done by testing different games with loss limit values which are related to a power of 2 since total losses at n lose streak are 2^n-1. Therefore, we will test loss limits as the following: 15, 20, 25, 31, 40, 50, 63, 80, 100, 127. This limit is the max loss that we will have during a lose streak. We are going to test it five hundred different times playing a hundred thousand games each time and print them all as well as the average end up values and minimum values in the following graphs:



Graph 11: Earnings we end up with different lose bet limits over the games



Graph 12: Average earnings we end up with different lose bet limits



Graph 13: Average min and max values and earnings we end up with different lose bet limits

Regarding graph 11 we can say that at lower limits line are more compact which means that at higher limits there is a wider range of end ups. End ups from low limits diverge from -20000 to 25000 while in higher limits from -30000 to 30000. Thus, we can say lower limits provide more predictable results, although, results are still extensive enough. Looking at results in average, they fluctuate around -2900 (-2.9%) with limits 15, 20 and 40 having the best conclusions.

Finally, we are going to play games in more realistic conditions since one million games produce huge numbers. We will play games until we reach various numbers of winning betting units (20, 50, 100, 200). After doing so, we will keep in track the total number of games as well as the minimum value reached. After that, we are going to establish which winnings are the best.

Table 7: Playing until a number of winning betting units

/	Winning at	No loss limit	Loss limit = 15	Loss limit = 20	Loss limit = 40
Games	20	35	262	236	183
Min Value	20	-106	-119	-141	-146
Games	50	167	644	698	518
Min Value	50	-278	-239	-276	-341
Games	100	216	700	991	616
Min Value	100	-490	-364	-382	-517
Games	200	793	1535	1306	1138
Min Value	200	-2125	-759	-748	-706

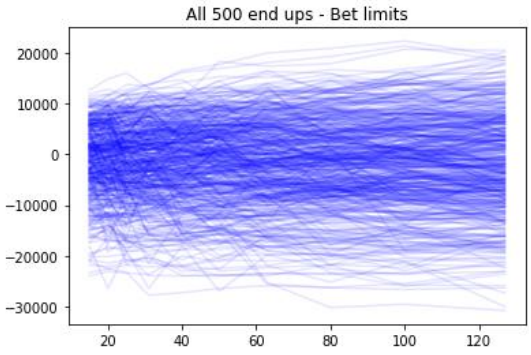
By observing the table above, we can see that the relationship between the games needed to reach the winnings and the value of the winnings is not linear, as the system suggests. To be more exact, at the column in which loss limit is 20, it requires 236 games in average to reach +20 while it requires 698≠236·2.5 games in average to

reach +50. Or in the column where there is no limit, it needs 216 games in average to get to +100 while it needs 793≠216·2 games in average to get to +200. Minimum values on the other hand, are gradually declining over games, except for winnings at 200 where the rapidly fall. Taking both total games and minimum value into account, it seems that winning at 100 is best option. Nonetheless, we can see that it takes too many games to hit small amounts of winnings. Not to mention that huge values of games has been excluded.

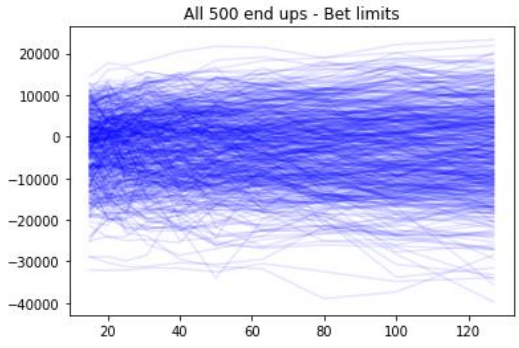
In conclusion, although Martingale System is linearly increasing our outcome, there are high loss streak which lead us to bankrupt. However, even without them, max bet limits do not allow the linear increase to appear, instead, there appears to occur a 2.9% loss per game being worse than playing flat!

4.2 Reverse Martingale System

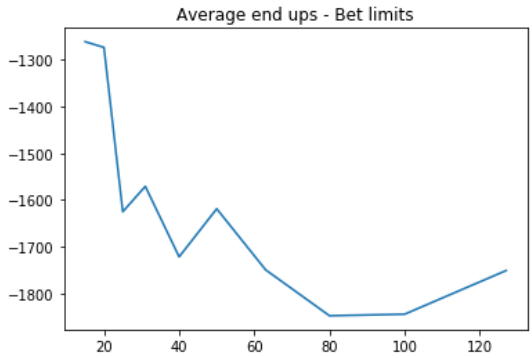
Just like the Martingale System, we will do almost the same process. As established before, the Reverse Martingale system does not work in the long run, for this reason we will set a winning limit at which we are going to start over. Therefore, there is no need to test what we should do when we reach the max bet limit, it is pointless. On the other hand, we do have to look for the adapted Reverse Martingale System (after winning bet to lose 1 in total) to check if it works better than the normal one. So, we will test different games with win limit values that are related to a power of 2 because winning n games in a row results in 2^n-1 betting units won. Therefore, we will test the following win limits: 15, 20, 25, 31, 40, 50, 63, 80, 100, 127. When reaching the win limit value, we will bet 1 betting unit and save the winnings from the win streak. After playing a hundred thousand games five hundred different times and print them all as well as the average end up values and minimum values for both adapted and normal one in the following graphs:



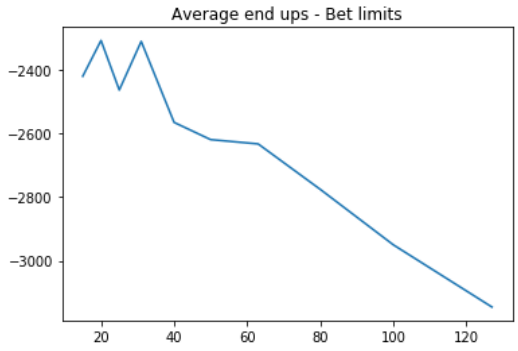
Graph 14: Earnings we end up with different win bet limits over the games (adapted Reverse Martingale)



Graph 15: Earnings we end up with different win bet limits over the games (Reverse Martingale)

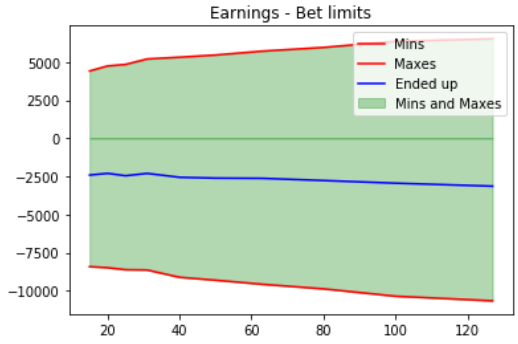


Graph 16: Average earnings we end up with different win bet limits (adapted Reverse Martingale)



Graph 18: Average min and max values and earnings we end up with different win bet limits (Reverse Martingale)

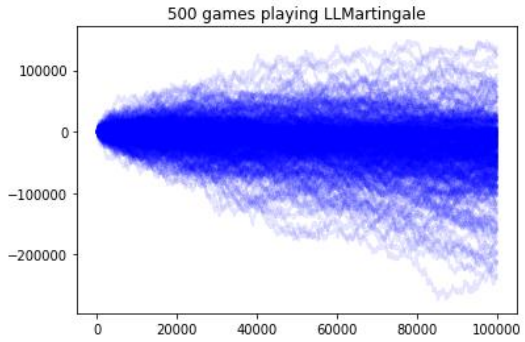
Graph 17: Average min and max values and earnings we end up with different win bet limits (adapted Reverse Martingale)



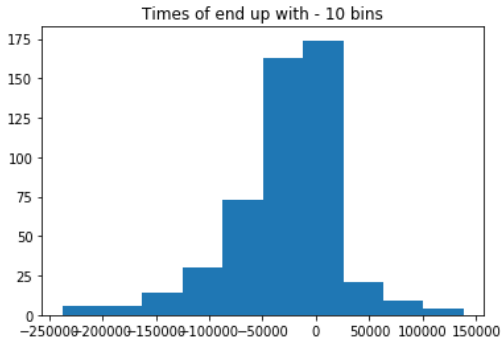
Graph 19: Average earnings we end up with different win bet limits (Reverse Martingale)

In both cases, lower limit lines are slightly more compact meaning that at higher limits there is a larger variety of end ups. End ups at adapted Reverse Martingale from low limits vary from -23000 to 14000 while in higher limits from -30000 to 20000. On the other hand, end ups at Reverse Martingale from low limits differ from -300000 to 15000 while in higher limits from -40000 to 23000. Thus, we can say that Reverse Martingale provides a wider range of outcomes. By observing graphs 16, 17, 18, and 19, it is clear that somewhat better results in low limits. Both systems end up higher when playing with low win limits and therefore they are preferable. The best results are when playing the adapted Reverse Martingale System with win limits 15 or 20, leading to about – 1.25% loss.

By observing Reverse Martingale System and Martingale System, we realize that Reverse Martingale System acts (we do not just bet 1 unit) only after winning. By contrast, Martingale System acts only after losing, so why not try playing with both systems on. When winning we will play the Adapted Reverse Martingale System with win limit 15, while when losing we will play the Adapted Martingale System with loss limit 20:



Graph 20: Results from earnings over the games (LLMartingale combined with Reverse LLMartingale)



Graph 21: Histogram of earnings end up with

Looking at graph 20 we can see a thick declining line. In overall the final result is completely lossy with about -30% losses per game.

Lastly, we are going to play games until we reach various numbers of winning betting units (20, 50, 100, 200). After keeping in track the total number of games and the minimum value reached, we get the results below:

Table 8: Playing until a number of winning betting units

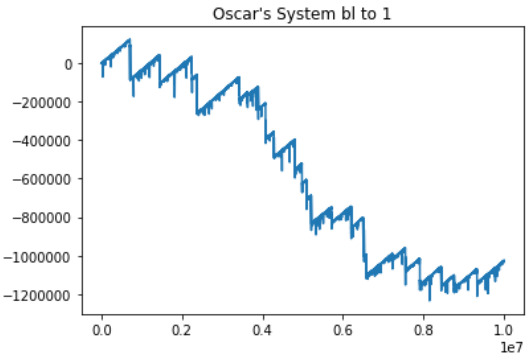
/	Winning at	No win limit	Win limit = 15	Win limit = 20	Win limit = 31
Games	20	206	234	223	209
Min Value	20	-150	-228	-211	-185
Games	50	256	433	375	389
Min Value	50	-218	-267	-244	-294
Games	100	486	829	682	778
Min Value	100	-377	-591	-569	-541
Games	200	760	1276	1138	1047
Min Value	200	-1103	-499	-485	-414

By observing the table above, we can see that setting no win limit makes it faster to reach to winnings in all cases. When setting a win limit, 20 and 31 seem hardly better than the 15 while minimum values do not seem very low. Thus, the risk is smaller, but it still takes too many games to reach to a small winning point. One reason that setting no win limit is better might be that high values of games have been excluded making it need less games.

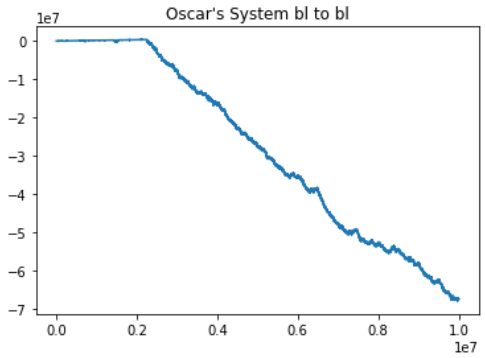
In summary, in contrast to Martingale System, Reverse Martingale with win limits does surprisingly well in the long run. In fact, it has higher minimum value (less risky) and ends up to greater earnings leading to a loss of 1.25% per game, which is better than Martingale System but still worse than playing flat!

4.3 Oscar's System

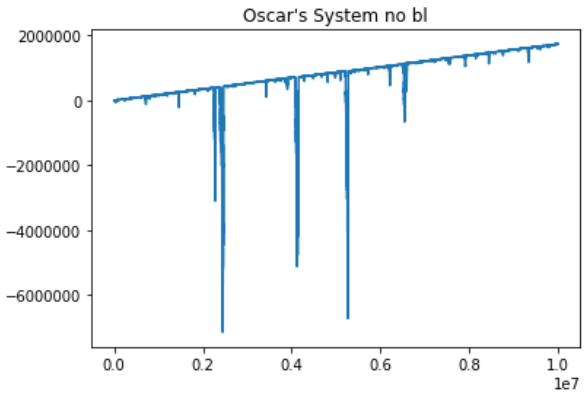
Since Oscar's System is complex, we will run many simulations to observe its behavior. Firstly, we will test it on the long run. We are going to set the maximum table bet limit to 2000 and apply it to 10M games. In one case we will test it with starting over after reaching the bet limit and on the other we will keep on betting on the bet limit. Placing the table bet limit constantly, will not end the series (getting the plus one) and so we expect to see a failure. To get a better picture of what is the purpose of this system, we are going to test it without a table bet limit as well, just to see its original behavior. By doing both of the tests we get the following results:



Graph 22: Oscar's System, betlimit to 1



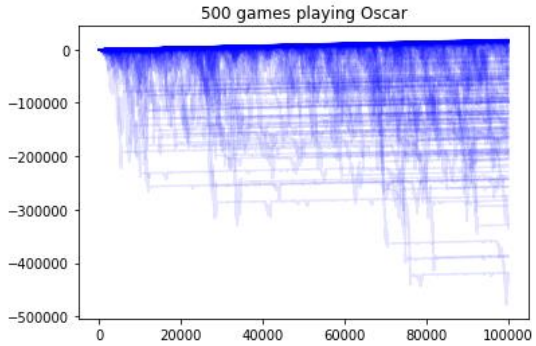
Graph 23: Oscar's System, betlimit to betlimit



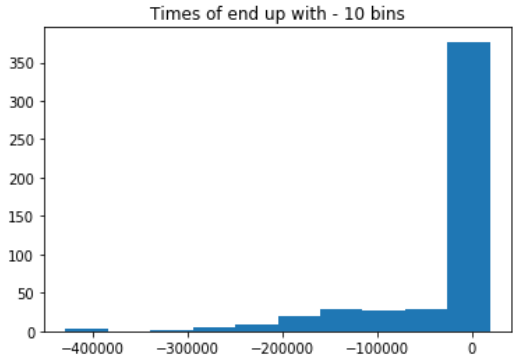
Graph 24: Oscar's System, no betlimit

Turning to graph 24, it is clear that there is a linear increase with around 17% profit at every game, however, there are five dramatic falls, three of them drop to almost -7160000, which is too large debt to manage. Even if there was a high enough bankroll to manage these falls, betlimits would not allow it. With regards to table bet limit, by

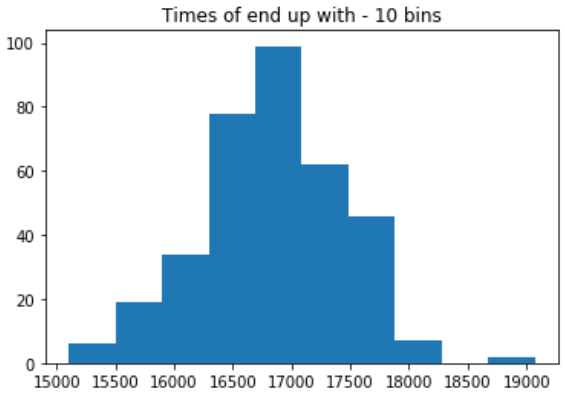
looking at graphs 22 & 23, reaching to the betlimit and starting over from that point is certainly better. Even so, there appears a decline, ending up with -1025866. Playing a hundred thousand games five hundred times we get the results below:



Graph 25: Results from earnings over the games (Oscar's System)



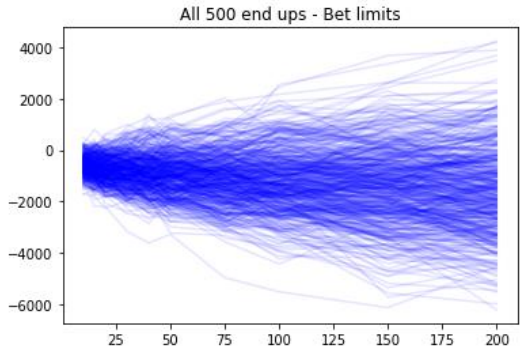
Graph 26: Histogram of earnings we end up with



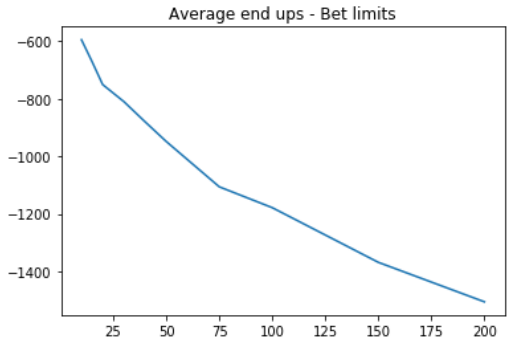
Graph 27: Histogram of earnings we end up with (only for value > 15000)

We observe that most of the times the games end up at positive value, however, many of them seem to be reaching extremely low minimum value. There are many games which reached low minimum value and then their bet reached the betlimit, so they had no chance to return to winnings. These are the once with the completely low ends up. An average of all games is losing about 22% per game. Therefore, much as it seems to be winning most of the times, it is actually reaching to very low minimum values or it can even stay that low and end up with it.

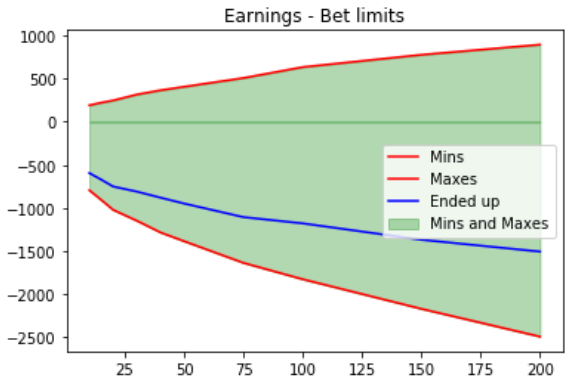
Now we can try another strategy, which we have already tried to other systems, and that is the setting of win or loss limits. Setting win limits is actually pointless, because after ending the series we end up with +1 every time so we would have to invade in the series and stop them when setting win limit. So, we will test many loss limits being 10, 15, 20, 30, 40, 50, 75, 100, 150 and 200.



Graph 28: Earnings we end up with different loss bet limits over the games



Graph 29: Average earnings we end up with different loss bet limits



Graph 30: Average min and max values and earnings we end up with different loss bet limits

It can be clearly seen that setting loss limits is extremely safe. Particularly, the smaller the loss limit the better the outcome as well as the minimum value. In graph 25, in low loss limits end ups range from -2000 to 1500 whereas in high loss limits they range from -6000 to 4000. In both cases the results are quite predictable, in low limits specifically, we are almost certain with an average of losing 0.6% per game.

At last, we are going to play games to a winnings goal. Every time we reach to this winnings goal, we will track the total number of games as well as the minimum value reached. Then, we are going to declare which winnings are the best. After doing so:

Table 9: Playing until a number of winning betting units

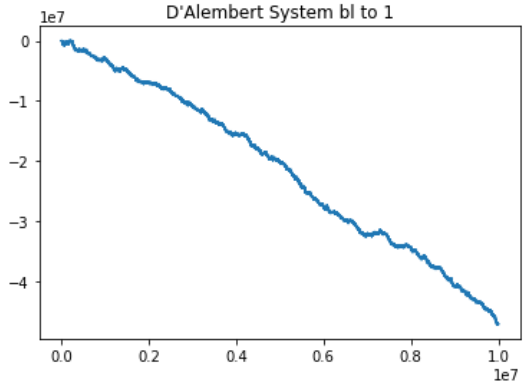
/	Winning at	No lose limit	Lose limit = 15	Lose limit = 20	Lose limit = 31
Games	20	131	551	308	509
Min Value	20	-158	-28	-15	-36
Games	50	306	3531	1366	1731
Min Value	50	-328	-61	50	-53
Games	100	901	-	-	-
Min Value	100	-1013	-	-	-
Games	200	1195	6354	33139	6746
Min Value	200	-738	-16	-290	-34

By testing the system to a winnings goal, we produced the numbers in the above table, however, they are not at all accurate because we reached the winnings only a few times and then kept losing forever. In some cases, it never reached the winning goal, so it fails to win a standard amount of profits. In the no limits strategies, it is slightly better. In winning goal = 20, for the first 283000 games there appears an average of 131 games but after that it never goes back to winnings. In winning goal = 50, the average of 306 games is for the first 365000 games and then it keeps losing. In both cases, it is likely that we reached the bet limit and started over, leaving a high dept to overcome which can almost never happen. Since 300000 games are a lot, we can consider these results as valid. In winning goal = 100, the average of 901 is completely accurate because we kept reaching it throughout 1M games. In the winning goal = 200 though, after playing 50000 games it gradually kept on losing. As a result, lose limits make the system avoid high min values but also winning values, whereas with no limit we have more probabilities to reach a winning goal but if bet limit is being reached, we lose a huge amount.

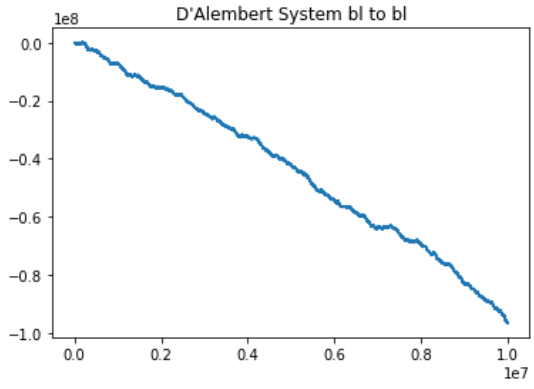
In an overall, Oscar's System is made to have a constant increase, however, max bet limits do not allow it. Instead, there appears an almost constant decline due to that. Setting win limit may eliminate that, nevertheless, there can be some significant and sudden falls to our bankroll. Last but not least, setting both a win and a loss limit shows a slight linear decrease as well, which is similar to playing with flat betting.

4.4 D'Alembert System

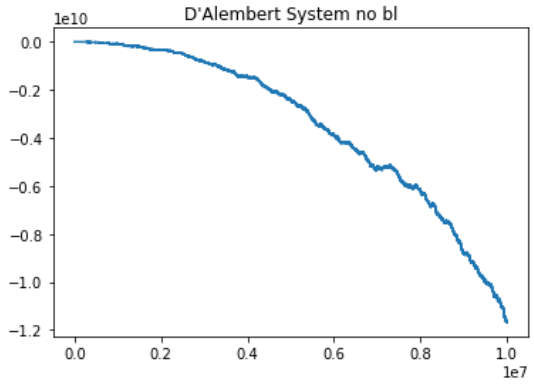
D'Alambert System is a quite useful system, but it requires almost the same number of losses and wins. Just like other systems, we can modify some of its features so as to adapt it to the blackjack payoffs. This can be done by adding 2 if you lost X2 or subtract 3 if you won X3 etc. In this way you may not need so many wins as losses and since winX2 are greater than lossX2, it is in your advantage. We are going to test it in the long run as well as with win and loss limits. For starters, we are going to test what is the best response when reaching the betlimit. After playing 10M games we get the following three lines:



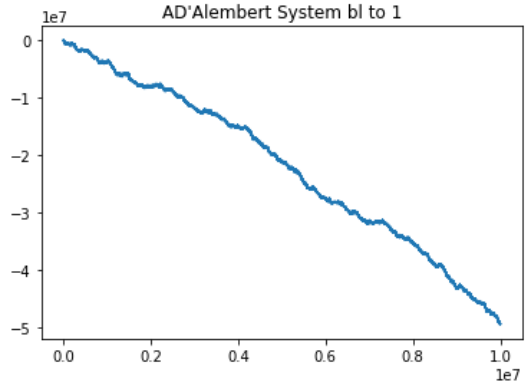
Graph 31: D'Alambert System betlimit to 1



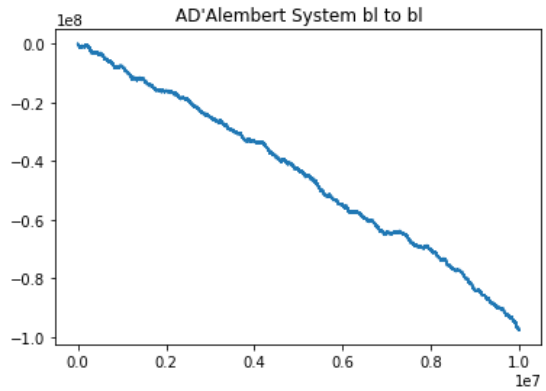
Graph 32: D'Alambert System betlimit to betlimit



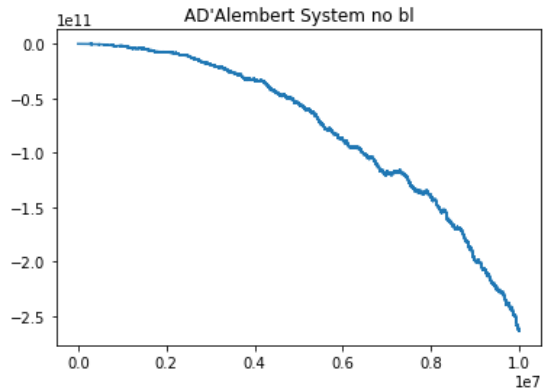
Graph 33: D'Alambert System no betlimit



Graph 34: Adapted D'Alambert System betlimit to 1



Graph 35: Adapted D'Alambert System betlimit to betlimit

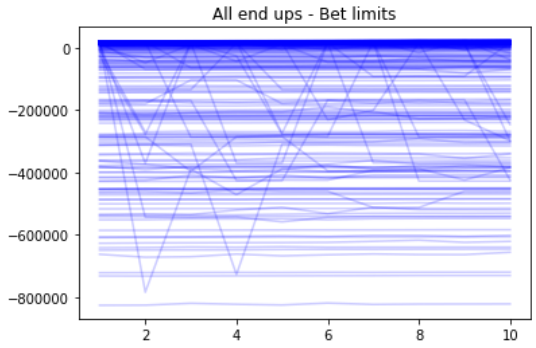


Graph 36: Adapted D'Alambert System no betlimit

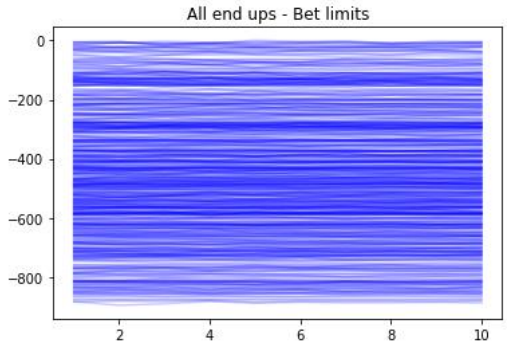
As we can see, both adapted and normal system are extremely lossy at the long run. Apart from the no betlimit long run, there are many similarities in the way the earnings behave over the games and the value they end up at. A critical observation is that when there is no limit, the betting systems go much worse and thus setting win or loss limits should be profitable.

At first, we are going to set win limits which shall stop the procedure of lowering your bets after wins since we will reach the win limts. If we would set high win limits, there would have been no difference between them because once the profits of a "full

cycle" (equal amount of wins and lose) are paid, we start over and so setting a win limit greater than that profit would be pointless. Therefore, we will set limits from 1 to 10 to both normal and adapted D'Alembert.

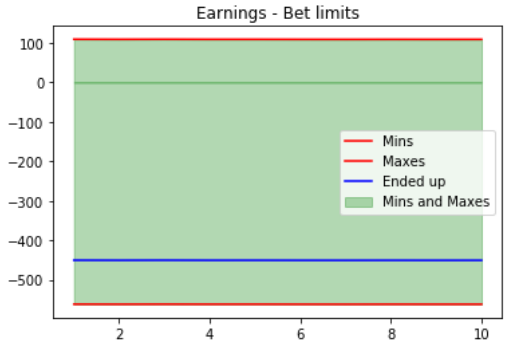


Graph 37: Earnings we end up with different win bet limits over the games (D'Alembert System)

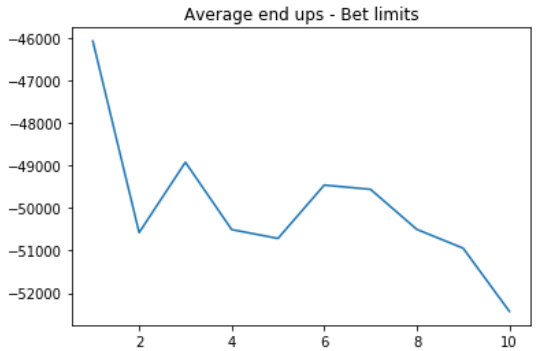


Graph 40: Earnings we end up with different win bet limits over the games (adapted D'Alembert System)

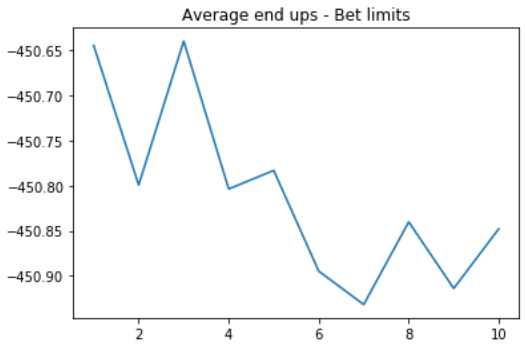
Graph 38: Average min and max values and earnings we end up with different win bet limits (D'Alembert System)



Graph 41: Average min and max values and earnings we end up with different win bet limits (adapted D'Alembert System)



Graph 39: Average earnings we end up with different win bet limits (D'Alembert System)

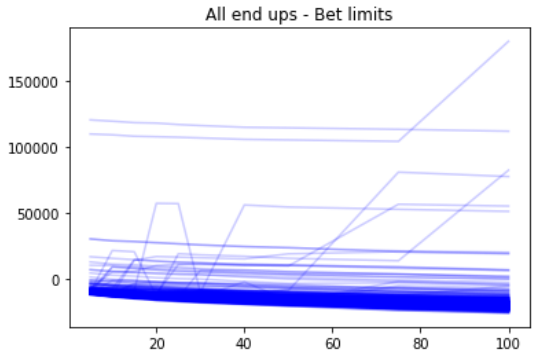


Graph 42: Average earnings we end up with different win bet limits (adapted D'Alembert System)

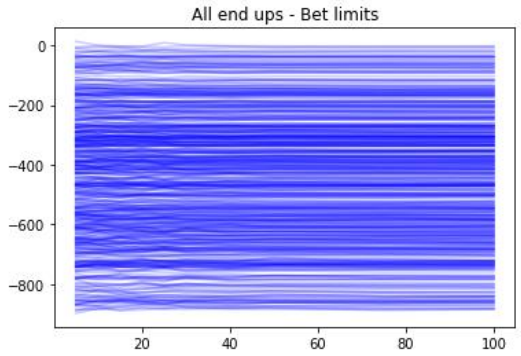
Comparing normal and adapted D'Alembert system with win limits, it is certain that the adapted is much better. It varies from about -1000 to 0 and so it ends up losing a tiny amount being around -0.45% per game. Graphs 40-42 indicate that it is a safe system but with little odds of winning. On the other hand, D'Alembert System as we know it, diverges from -800000 to about 10000. In general, by applying it we lose half

of a betting unit every game which is actually terrible. Both systems seem to end up with the same earnings through the win limits with the win limit = 1 being the highest.

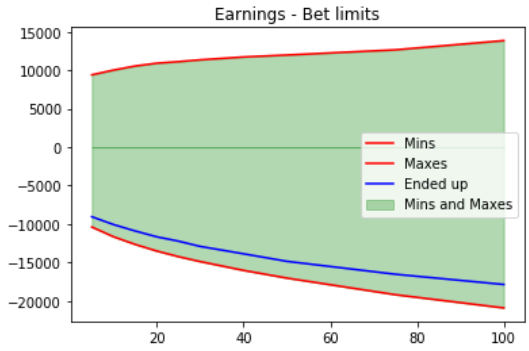
After testing win limits, we are going to test loss limits as well, in both systems. The limits will diverge from 5 to 100. In particular, they will be 5, 10, 15, 20, 25, 30, 40, 50, 75, 100.



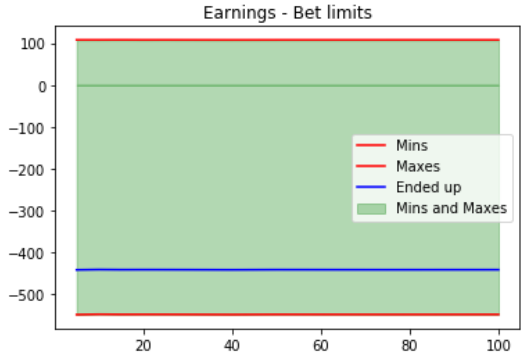
Graph 43 Earnings we end up with different loss bet limits over the games (D'Alembert System)



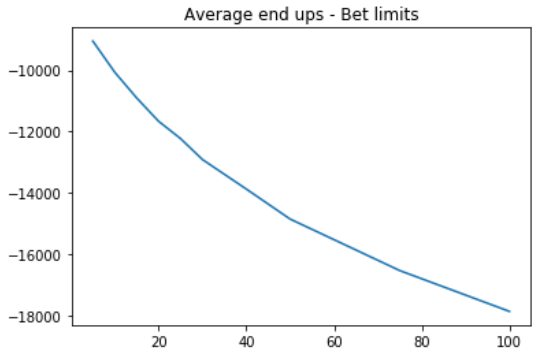
Graph 46: Earnings we end up with different loss bet limits over the games (adapted D'Alembert System)



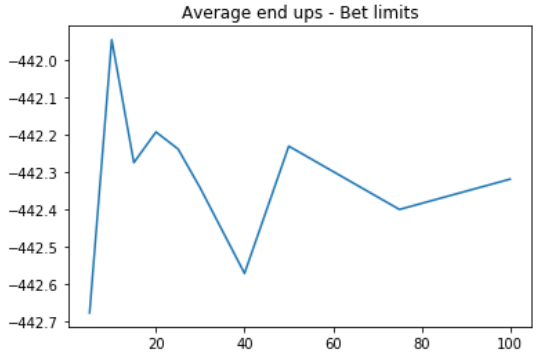
Graph 44: Average min and max values and earnings we end up with different loss bet limits (D'Alembert System)



Graph 47: Average min and max values and earnings we end up with different loss bet limits (adapted D'Alembert System)



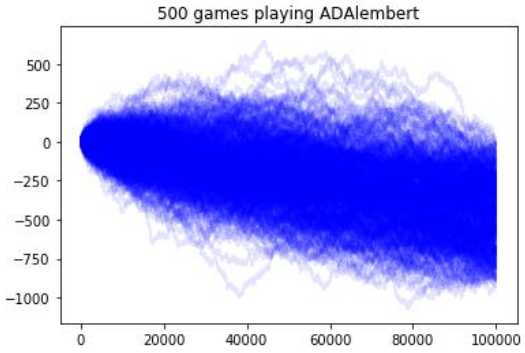
Graph 45: Average earnings we end up with different loss bet limits (D'Alembert System)



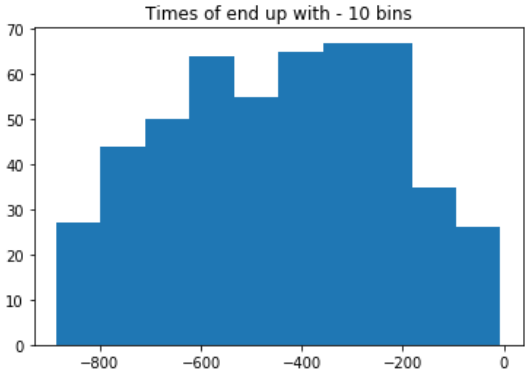
Graph 48: Average earnings we end up with different loss bet limits (adapted D'Alembert System)

By comparing normal and adapted with win limits, adapted D'Alembert system is still better. In fact, it looks very much like that with win limits, varying from about -1000 to 0 and it also ends up losing a small amount which is around -0.45% per game. Turning to the other two graphs, we can see that it is a safe system as well, but with not much odds of winning. By contrast, normal D'Alembert System diverges from -18000 to about 40000 with some cases being up to 150000. In general, by applying it we lose around 10% of betting units per game. Both systems also seem to end up with the same earnings through the lose limits with the lose limit = 10 being the highest.

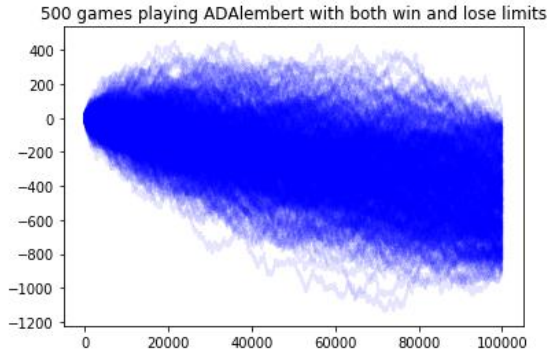
Taking the above graphs into consideration, the best to play D'Alembert System is to adapt it to blackjack payouts and set a lose limit = 10. To illustrate the earning of it thought the games we are going to test it by playing a hundred thousand games five hundred times. Additionally, we will test a combination of lose and win limits. Lose limit shall be 10 and win limit = 1, which are the best limits as we saw above.



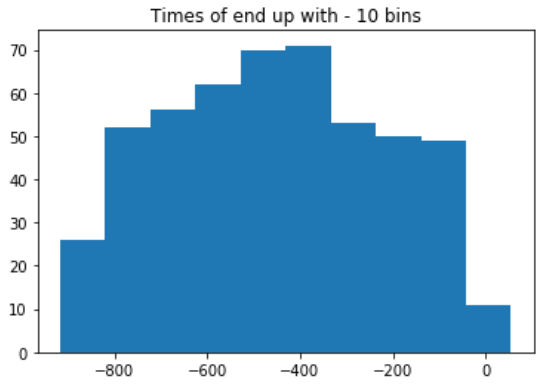
Graph 49: Results from earnings over the games (lose limit = 10)



Graph 50: Histogram of earnings we end up with



Graph 51: Results from earnings over the games (lose limit = 10 and win limit = 1)



Graph 52: Histogram of earnings we end up with

Using linear regression to both ways, for lose limit = 10 the equation is y = -0.004433x while for win limit = 1 and lose limit = 10 it is y = -0.00451x. As we can see they are both rather close with set just lose limit = 10 being better. Linear regression suggests that we shall lose 0.4433% of bet units per games. This suggestion is rather

accurate because coefficient of determination is close enough to 1. We should expect that since the range of the end up value is more or less 900. Looking at the histograms, the earnings we end up with are between – 900 to 0 with their being at about – 450.

In closing, we are going to play games until we reach various numbers of winning betting units (20, 50, 100, 200). After keeping in track the total number of games and the minimum value reached, we get the results below:

Table 10: Playing until a number of winning betting units

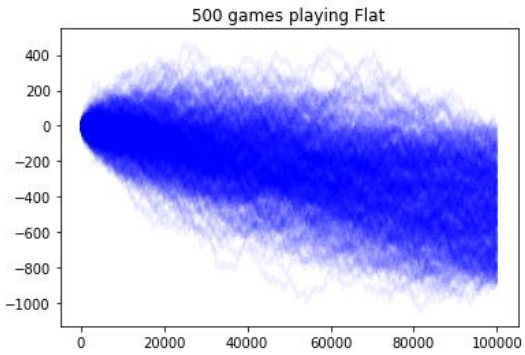
/	Winning at	No lose limit	Lose limit = 15	Lose limit = 20	Lose limit = 31
Games	20	430	429	430	430
Min Value	20	-14	-14	-14	-14
Games	50	16519	16491	16491	16498
Min Value	50	-67	-75	-76	-64
Games	100	-	-	-	-
Min Value	100	-	-	-	-
Games	200	25342	25342	25338	25345
Min Value	200	-161	-163	-160	-162

These results are completely in accurate. The winning goal was reach at most five times. Thereafter, we kept losing by a tiny rate but never came back to profits. This is because this system results in little fluctuations around a linear loss with gradient equal to about -0.005

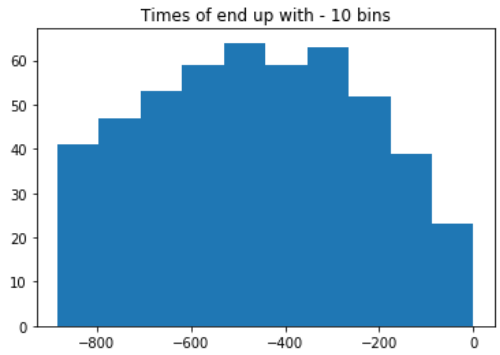
All in all, D'Alembert system is ideal for same number of wins and losses. In blackjack that is not the cases and therefore it dramatically leads to a huge loss in the long run. By setting loss limits

4.5 Conclusions

All the above were the most known systems in the gambler's society. After testing them, we realized that adapting them to the blackjack's payouts would improve them. Even better, when we comprehended that they are dramatically losing in some cases, we set limits in order to prevent this. Results were getting better and better, however, the outcome was still a loss. By comparing them to the edge of the house they were always performing slightly worse. So how is it to play flat all along? To illustrate this, we are going to play a hundred thousand games five hundred times.



Graph 53: Results from earnings over the games by playing flat



Graph 54: Histogram of earnings we end up with

In terms of playing flat, we can undoubtedly see a linear decline. Final earnings are from around -1000 to 0, which is a short range related to one hundred thousand games. In average, we lose 0.4685% of bet units per game which is, of course, the house's edge. As for the winning goals, we get the results above:

Table 11: Playing until a number of winning betting units

/	Winnings = 20	Winnings = 50	Winnings = 100	Winnings = 200
Games	2499	3617	7494	10851
Min Value	- 35	- 35	- 43	- 33

By excluding cases where there were loses that could not get back to profits, we get a high number of games so as to reach a winning goal. For winnings 100 and 200 results are not valid enough because we excluded cases many times.

Taking into consideration this fact, although betting systems seem to win more than playing flat in most of the cases (combination of wins and losses), they drastically lose in some other cases. Thus, they perform poor in the long run since a deadly combination of wins and losses is more likely to occur. In the short term the betting systems might be profitable at times, however, playing them many times results in the long-term results which are worse than playing flat. In this way, flat betting is actually the best system, nevertheless, it is still harmful.