

Bias Analyses for South Carolina Maps

Two reliable measures of partisan bias are used to assess each map.

Definition

Let V^O be the observed statewide average district vote and let M be the median district vote share. The **median-mean** metric is defined as

$$MM = M - V^O.$$

$MM < 0$ indicates an advantage for Republicans.

Definition

Geometric partisan bias is defined as

$$B_G = \int_0^1 \left| \hat{S}(v) - \hat{S}^{-1}(v) \right| dv,$$

where $\hat{S}(V)$ is the estimated seats-votes curve using a variable partisan swing assumption and $\hat{S}^{-1}(V)$ is this curve inverted around the (0.5, 0.5) midpoint.

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Both **median-mean** and **geometric bias** satisfy the partisan symmetry standard.

Median-mean indicates the **direction** of the bias, while geometric bias characterizes **overall** bias in either party direction.

Though not a measure of partisan symmetry, average district **efficiency gap** was also considered to characterize wasted votes.

Definition

For two Parties D and R , the **efficiency gap** is computed as

$$EG = \frac{(\ell_D + w_D) - (\ell_R + w_R)}{N},$$

where ℓ is the number of votes cast for a losing candidate, w is the number of votes cast for a winning candidate above the threshold of 50% plus one vote, and N is the total number of votes cast. $EG > 0$ indicates that Party D wasted more votes than Party R .

Bias Analyses for South Carolina Maps

- Bias analyses are currently being conducted for Congressional, SC Senate, and SC House districting plans.
- Precinct-level geography and voting data corresponding to the 2020 General Election were utilized for the analysis.
- The U.S. Senate race (Graham vs. Harrison) was utilized as a proxy for statewide voter preferences.
- Comparative analyses were run for the 2011 redistricting plans from last year and the proposed 2021 plans provided by the SC League of Women Voters.

Congressional Maps (7 Districts)

	Original 2011 Map	Proposed 2021 Map
Median-Mean	-0.0238	-0.0336
Geometric Bias	0.0231	0.0352
Efficiency Gap	0.2624	0.1117

Compared to the existing districting plan from 2011, the proposed Congressional map by LWV fares better in terms of efficiency gap, but worse in terms of both partisan bias measures.

SC Senate Map (46 districts)

	Original 2011 Map	Proposed 2021 Map
Median-Mean	-0.0328	-0.0348
Geometric Bias	0.0301	0.0260
Efficiency Gap	0.0685	0.0800

Compared to the existing districting plan from 2011, the proposed State Senate map by LWV fares better in terms of geometric bias. It is worse in terms of efficiency gap, but only slightly worse in terms of median-mean.

SC House Map (124 districts)

	Original 2011 Map	Proposed 2021 Map
Median-Mean	-0.0445	-0.0273
Geometric Bias	0.0408	0.0341
Efficiency Gap	0.0976	0.0903

Compared to the existing districting plan from 2011, the proposed State House map by LWV...

Markov Chain Analysis: Maria Chikina, Alan Frieze, and Wesley Pegden, *Assessing significance in a Markov chain without mixing*, Proceedings of the National Academy of Sciences 114 (2017), no. 11, 2860–2864.

Partisan Symmetry: Jonathan N. Katz, Gary King, and Elizabeth Rosenblatt, *Theoretical foundations and empirical evaluations of partisan fairness in district-based democracies*, American Political Science Review (2019), 1–15.

Geometric Bias: John F. Nagle, *Measures of partisan bias for legislating fair elections*, Election Law Journal 14(2015), no. 4, 346–360.

Thesis (demonstrating equivalent performance of Geometric Bias under outlier analysis): Anna Marie Vagnozzi, *Detecting partisan gerrymandering through mathematical analysis: A case study of South Carolina*, TigerPrints: Theses and Dissertations (2020).