Redistricting Model

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1 Introduction

The purpose of this write-up is to propose a redistricting model for use in South Carolina's redistricting process. Because previous iterations of SC redistricting guidelines have advised (or outright outlawed) computer-generated maps, we will be using a multi-objective model to generate a wide variety of maps on the Pareto Frontier. From this selection of maps, we hope that one will be chosen for implementation in South Carolina.

2 Possible Objectives and/or constraints

The following is a non-exhaustive list of possible objectives and/or constraints that can be used in the model:

- Compactness
  - Geometry-based (Polsby-Popper, Reock, etc.)
  - Population density-based
  - Graph Theory-based
- Fairness/Competitiveness
- Population Equality
- Maintenance of city boundaries (i.e. we do not wish for cities to exist inside two different districts)
- Maintenance of county boundaries (i.e. we do not wish for counties to be split between two districts)
- Maintenance of Civil Rights Act districts
- Contiguity
- Maintenance of Communities of Interest (COIs)

3 Multiobjective Model

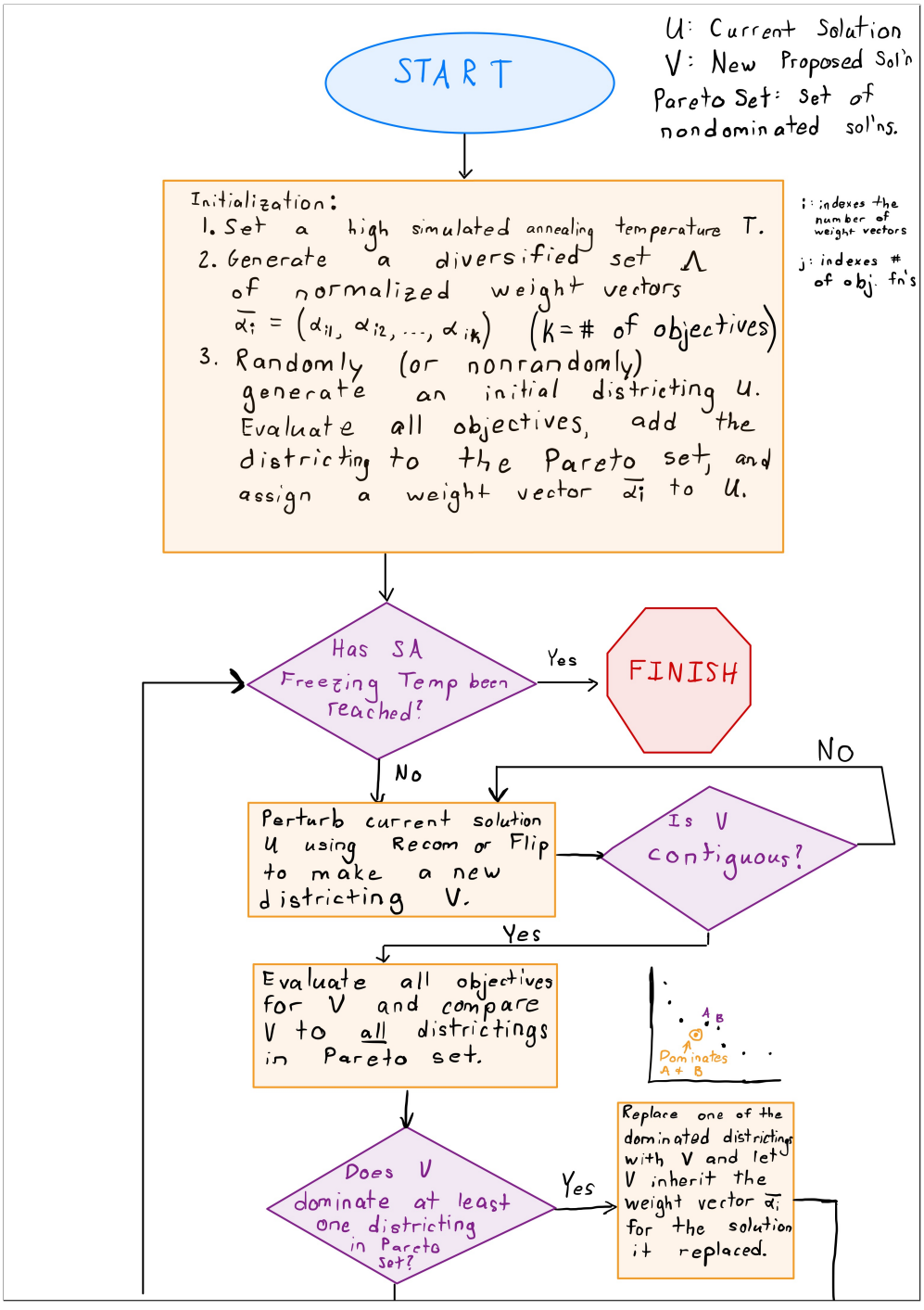
The following model is based on the concept used in Rincón-Garcia et al. in 2013. Let K represent the number of objective functions.

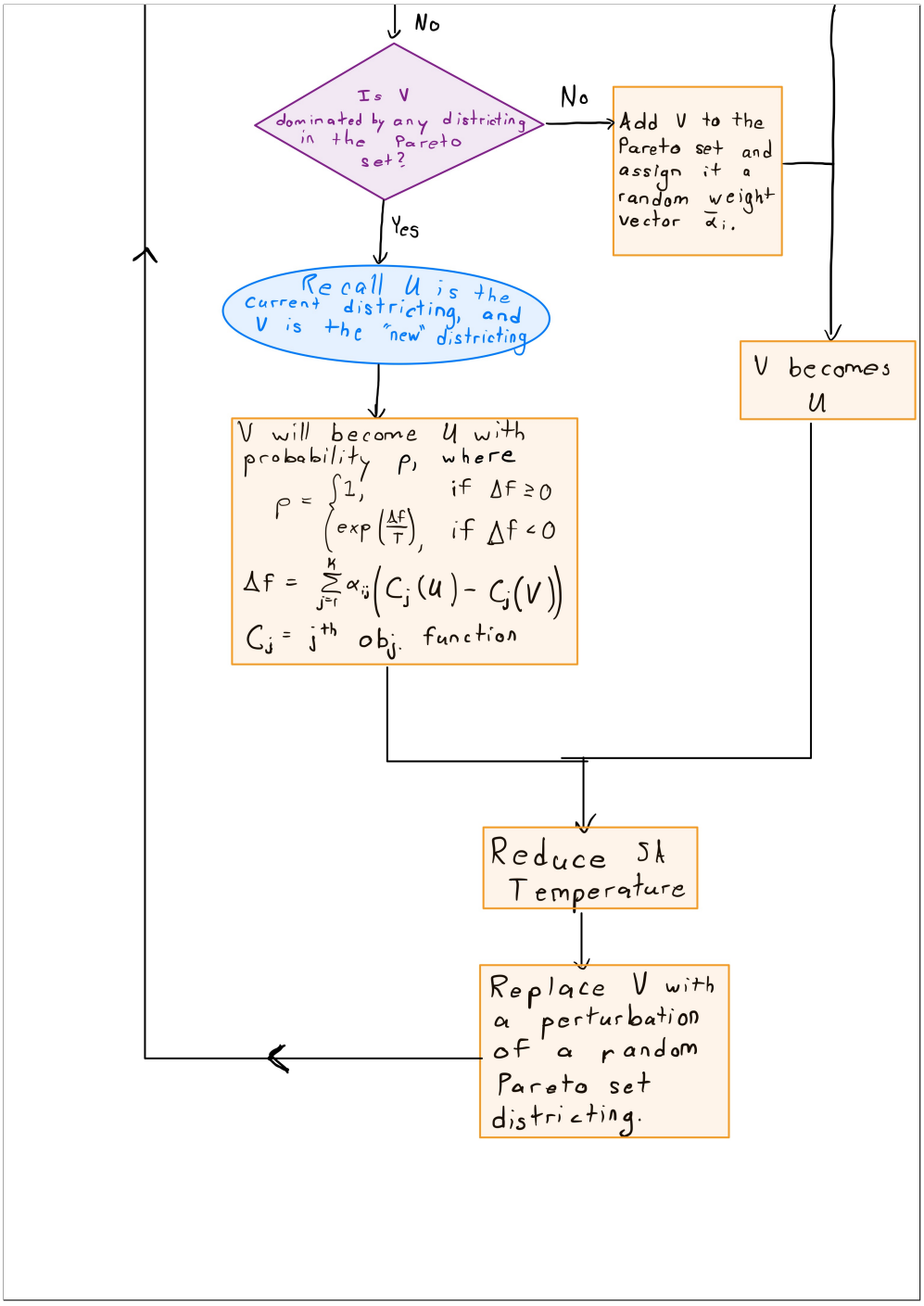
Algorithm 1: Multiobjective Redistricting Algorithm

Initialization:
Set a high simulated annealing temperature T;
Generate a wide diversified set Λ of normalized weight vectors $\overline{\alpha_i} = (\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{iK})$;
Generate an initial districting (either randomly or nonrandomly). Let this be the current solution U;
Evaluate all objective functions for U;
Save current solution into an archive (set of Pareto solutions) and assign it a random weight vector $\overline{\alpha_i}$;
while *Simulated annealing temperature is above freezing* **do**
  Apply either ReCom or Swap to generate a new districting V;
  while V *is discontiguous* **do**
    Apply either ReCom or Swap again to generate a new districting V;
  end
  Evaluate all objective functions for V and compare V to all solutions in the Pareto set;
  if V *dominates at least one districting in the Pareto set* **then**
    Replace one of the dominated districtings with V and let V inherit the weight vector $\overline{\alpha_i}$ for the solution it replaced;
    $U := V$;
  else if V *is NOT dominated by any districting in the Pareto set* **then**
    Add V to the Pareto set and assign it a random weight vector $\overline{\alpha_i}$;
    $U := V$;
  else if V *is dominated by any districting in the Pareto set* **then**
    Let $\Delta f = \sum_{j=1}^{K} \alpha_{ij}(C_j(U) - C_j(V))$, where C_j is the jth objective function;
    $U := V$ with probably ρ, where

$$\rho = \begin{cases} 1, & \text{if } \Delta f \geq 0 \\ \exp\left(\frac{\Delta f}{T}\right), & \text{if } \Delta f < 0 \end{cases}$$

  Reduce the simulated annealing temperature T;
end


U: Current Solution
V: New Proposed Soln
Pareto Set: set of nondominated sol'ns.
START
Initialization:
1. Set a high simulated annealing temperature T.
2. Generate a diversified set Λ of normalized weight vectors $\overline{\alpha_i} = (\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{ik})$ (k = # of objectives)
3. Randomly (or nonrandomly) generate an initial districting U. Evaluate all objectives, add the districting to the Pareto set, and assign a weight vector $\overline{\alpha_i}$ to U.
i: indexes the number of weight vectors
j: indexes # of obj. fn's
Has SA Freezing Temp been reached?
Yes
FINISH
No
Perturb current solution U using Recom or Flip to make a new districting V.
Is V contiguous?
No
Yes
Evaluate all objectives for V and compare V to all districtings in Pareto set.
A
B
Dominates A + B
Does V dominate at least one districting in Pareto set?
Yes
Replace one of the dominated districtings with V and let V inherit the weight vector $\overline{\alpha_i}$ for the solution it replaced.


No
Is V dominated by any districting in the Pareto set?
No
Add V to the Pareto set and assign it a random weight vector $\vec{\alpha}_i$.
Yes
V becomes U
Recall U is the current districting, and V is the "new" districting
V will become U with probability ρ, where
$\rho = \begin{cases} 1, & \text{if } \Delta f \geq 0 \\ \exp\left(\frac{\Delta f}{T}\right), & \text{if } \Delta f < 0 \end{cases}$
$\Delta f = \sum_{j=1}^{k} \alpha_{ij}\left(C_j(U) - C_j(V)\right)$
C_j = j^{th} obj. function
Reduce 5k Temperature
Replace V with a perturbation of a random Pareto set districting.

4 Goal

After running this algorithm, the end result will be a set of high-quality maps that are on the Pareto Frontier.