10.5: Comparison Tests

Learning Objectives. Upon successful completion of Section 10.5, you will be able to. . .

- Answer conceptual questions involving the Comparison Tests.
- Use the Comparison Test or Limit Comparison Tests to determine series convergence or divergence.
- Determine series convergence or divergence using a test of your choice.

The (Direct) Comparison Test

Similar to the Comparison Theorem for Improper Integrals that was introduced in Section 8.9, we can compare a given infinite series to a series that we know to be convergent or divergent. We will usually do comparisons to a p-series or a geometric series.

Comparison Test. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. Then. . .

(1) If $\sum b_n$ converges and $a_n \leq b_n$ for all n, then $\sum a_n$ converges.

(2) If $\sum b_n$ diverges and $a_n \geq b_n$ for all n, then $\sum a_n$ diverges.

✍ **Example.** $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

✍ **Example.** $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k}$

Important Notes on Using Comparison Tests

The Limit Comparison Test

Limit Comparison Test. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where $0 < c < \infty$,

then either $\sum a_n$ and $\sum b_n$ both converge or both diverge.

✍ **Example.** $\sum_{n=1}^{\infty} \frac{n}{2n^3 - 1}$

✍ **Example.** $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^3+n^2}$

✍ **Example.** $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$

✍ **Example.** $\displaystyle\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$