

12.2: Polar Coordinates

Learning Objectives. Upon successful completion of Section 12.2, you will be able to...

- Answer conceptual questions involving polar coordinates.
- Graph with polar coordinates and give alternate polar representations of polar points.
- Convert between polar and Cartesian coordinates.
- Convert between polar equations and Cartesian equations.
- Graph simple polar curves.
- Identify corresponding points given Cartesian and polar graphs of the same curve.

Introduction to Polar Coordinates

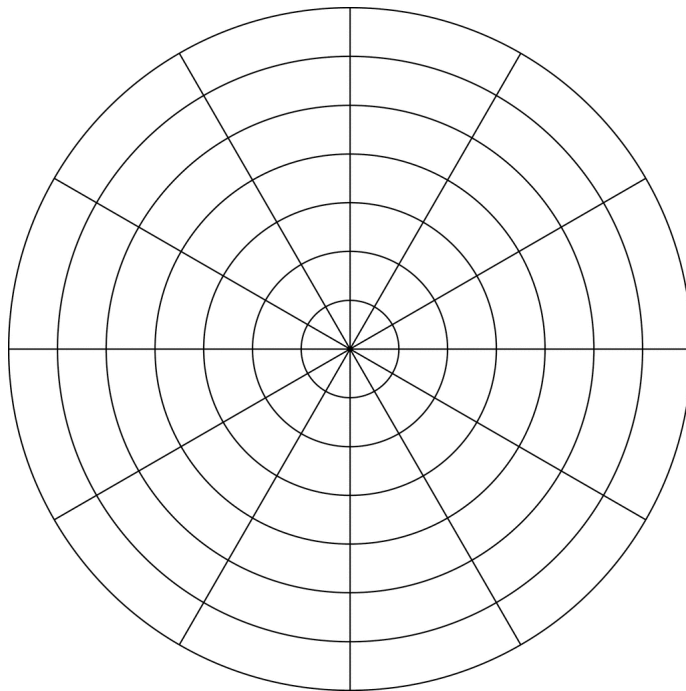
Rectangular coordinates, or **Cartesian coordinates**, describe the position of a point P by its horizontal (x) distance and vertical (y) distance.

Polar coordinates instead describe the position of a point P using...

- a _____, which is a directed distance from the origin to P , and
- an _____, which is a directed angle measured counterclockwise from the polar axis to P .

✎ **Example.** Plot the following polar coordinates.

$$(0, \theta) \qquad \left(2, \frac{2\pi}{3}\right) \qquad \left(5, -\frac{\pi}{4}\right) \qquad \left(-3, \frac{\pi}{6}\right)$$



Note: Polar coordinates do not give a unique representation of a point.

Coordinate Conversion

Let the polar axis coincide with the positive x -axis and the pole with the origin.

- ① A point with polar coordinates (r, θ) has Cartesian coordinates (x, y) where
- ② A point with Cartesian coordinates (x, y) has polar coordinates (r, θ) where

▮ **Example.** Find the Cartesian coordinates of the point with polar coordinates $\left(-4, \frac{3\pi}{4}\right)$.

▮ **Example.** Find polar coordinates (with $r > 0$) of the point with Cartesian coordinates $(-4, 4\sqrt{3})$. Also find polar coordinates with $r < 0$ for this point.

Polar Curves

The **graph of a polar equation** $r = f(\theta)$ consists of all points P that have at least one polar representation (r, θ) whose coordinates satisfy the equation.

Basic Curves

$r = a$, Example: $r = 3$

$\theta = b$, Example: $\theta = 5\pi/6$

▮ **Example.** Convert the following equation to Cartesian coordinates. Describe the resulting curve.

$$r = 6 \cos \theta + 8 \sin \theta$$

Note: $r = 2a \cos \theta + 2b \sin \theta$ describes the circle of radius $r = \sqrt{a^2 + b^2}$ and center (a, b) .

Symmetry in Polar Equations

- We have symmetry about the **x -axis (polar axis)** if (r, θ) is on the graph whenever $(r, -\theta)$ is.
- We have symmetry about the **y -axis ($\theta = \pi/2$)** if (r, θ) is on the graph whenever $(r, \pi - \theta) = (-r, -\theta)$ is.
- We have symmetry about **the origin (the pole)** if (r, θ) is on the graph whenever $(-r, \theta) = (r, \theta + \pi)$ is.

Sketching Polar Curves

✎ **Example.** Sketch the graph of the polar curve $r = 4 + 4\cos\theta$ using a table of values and symmetry.