Team Name: ____________________ Group Members: ____

1. Let the continuous random variable X represent the amount of time (in minutes) it takes for Doc and Marty to travel back to the future in their time machine. Doc has optimized the time machine so it will never take longer than a minute to travel. Suppose that X has the pdf

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Verify that the total area under the pdf is equal to 1. Show **integral notation** with the **correct limits**, include the **function** in the integral (do not just write $f(x)$), and write the final **answer** you compute.

   (b) What is the probability that on a given trip, Doc and Marty will take **less than** 15 seconds (0.25 minutes) to reach the future? Include a probability statement, integral notation, and your final answer.

   (c) What is the probability that on a given trip, Doc and Marty will take **exactly** 15 seconds (0.25 minutes) to reach their destination? (Hint: Consider the properties of continuous probability distributions.)

   (d) Find the **expected** amount of time it should take Doc and Marty to time travel. Include the appropriate symbol, integral notation with the correct limits and function, and your final answer with units.

   (e) What is the **standard deviation** of travel times for Doc and Marty? Include the appropriate symbol, integral notation with the correct limits and function, and your final answer with units.

2. Find Harry and Hermione's relative standing by calculating the z-score for each of their scores. Show your work and round your z-scores to two decimal places.

3. Find the percentile rank of both Harry and Hermione (i.e. the proportion of students they scored above). Your work should include:

   - A **sketch** with the correct area shaded and appropriate labels
   - A **probability statement** using the random variable Z
   - **Integral notation** with the proper limits
   - The **standard normal pdf** in the integral (Formula 7.5)
   - Your final **answer** from the standard normal table

For these problems, you may use either the **normal** or the **standard normal distribution**. For each problem, you should provide (1) proper probability notation, (2) an appropriate integral, and (3) a related sketch. Round probabilities to four places and z-scores to two places.

4. The mean height of hobbits, μ_h, is estimated to be approximately 42 inches, with a standard deviation of $\sigma_h = 2.7$ inches. The mean height of elves, μ_e, is estimated to be 75 inches, with a standard deviation of $\sigma_e = 3.5$ inches. The heights of both races are approximately normally distributed. Use this information to answer the following questions.

   (a) Suppose Frodo, a hobbit, is 46 inches tall, and suppose Legolas, an elf, is 80 inches tall. Which character is taller relative to the heights of the rest of their race?

   (b) Let X be the height of hobbits. Find the proportion of hobbits who are shorter than Frodo.

   (c) Let Y be the height of elves. Find the proportion of elves who are taller than Legolas.

   (d) What is the probability that a randomly-selected hobbit is between 40 and 45 inches tall?

(e) Find the minimum height an elf who falls in the **tallest** 25% of elves. (Show your work. You do not need to use integral notation here.)

5. The webpage `http://lotrproject.com/statistics/` includes statistics from Middle Earth based on *The Lord of the Rings* books. Visit the page and view the section labeled ***Life expectancy***, which includes the average life-spans of Hobbits, Dwarves, and Men. The dwarf Gimli lived to be 262 years old. Using the information from the table, what is the probability that a randomly-selected dwarf from the books lived longer than Gimli? (Don't forget to define your random variable!)

Use the standard normal distribution table to answer the following questions. Draw a sketch with the correct shaded area and show any work used to compute z-scores.

6. Find the following probabilities for a standard normal random variable Z.

   (a) $P(Z < 0.19)$

   (b) $P(Z > 1.74)$

   (c) $P(-2.14 \leq Z \leq 1.88)$

   (d) $P(|Z| > 1.52)$

7. The annual salaries of employees in a large company are approximately normally distributed with a mean of \$50,000 and a standard deviation of \$20,000.

   (a) What proportion of employees at the company earn less than \$35,000?

   (b) What is the probability that a randomly selected employee at the company earn between \$45,000 and \$78,000?

   (c) What proportion of people earn more than \$117,000?

   (d) When the coronavirus pandemic caused business operations to be suspended and people to be placed on temporary leave from their jobs, the company gave its employees with the lowest 15% of salaries a stipend to make up for lost time on the job. What is the salary cutoff that determines whether someone receives the stipend?

   (e) To make up for some of the business losses, the company plans to ask employees with the highest 6% of salaries to take one week of unpaid leave. What is the minimum salary that an employee who needs to take a week of unpaid leave will have?