

## 12.2: Polar Coordinates

**Learning Objectives.** Upon successful completion of Section 12.2, you will be able to...

- Answer conceptual questions involving polar coordinates.
- Graph with polar coordinates and give alternate polar representations of polar points.
- Convert between polar and Cartesian coordinates.
- Convert between polar equations and Cartesian equations.
- Graph simple polar curves.
- Identify corresponding points given Cartesian and polar graphs of the same curve.

### Introduction to Polar Coordinates

✚ **Example.** Plot the following polar coordinates.

## Coordinate Conversion

Let the polar axis coincide with the positive  $x$ -axis and the pole with the origin.

▮ **Example.** Find the Cartesian coordinates of the point with polar coordinates  $\left(-4, \frac{3\pi}{4}\right)$ .

▮ **Example.** Find polar coordinates (with  $r > 0$ ) of the point with Cartesian coordinates  $(-4, 4\sqrt{3})$ . Also find polar coordinates with  $r > 0$  for this point.

## Polar Curves

The **graph of a polar equation**  $r = f(\theta)$  consists of all points  $P$  that have at least one polar representation  $(r, \theta)$  whose coordinates satisfy the equation.

### Basic Curves

$r = a$ , Example:  $r = 3$

$\theta = b$ , Example:  $\theta = 5\pi/6$

▮ **Example.** Convert the following equation to Cartesian coordinates. Describe the resulting curve.

$$r = 6 \cos \theta + 8 \sin \theta$$

## Symmetry in Polar Equations

- We have symmetry about the  **$x$ -axis (polar axis)** if  $(r, \theta)$  is on the graph whenever  $(r, -\theta)$  is.
- We have symmetry about the  **$y$ -axis** ( $\theta = \pi/2$ ) if  $(r, \theta)$  is on the graph whenever  $(r, \pi - \theta) = (-r, -\theta)$  is.
- We have symmetry about **the origin (the pole)** if  $(r, \theta)$  is on the graph whenever  $(-r, \theta) = (r, \theta + \pi)$  is.

## Sketching Polar Curves

✎ **Example.** Sketch the graph of the polar curve  $r = 4 + 4\cos\theta$  using a table of values and symmetry.