

## 8.6: Integration Strategies

**Learning Objectives.** Upon successful completion of Section 8.6, you will be able to...

- Identify integration strategies needed to solve given integrals.
- Answer conceptual questions about integration strategies.
- Evaluate integrals using various integration strategies.
- Find areas, volumes, arc lengths, and surface areas using various integration strategies.

### Summary of Techniques

- Basic integration rules
- Using algebra to simplify
- $U$ -substitution
- Integration by parts
- Trig integrals
- Trig substitution
- Partial fraction decomposition

### Strategy Overview

- ① Simplify the integrand, if possible.
- ② Look for a substitution.
- ③ Classify the integrand type.
  - Is it a **trig integral**?  $\implies$  Think about using trig identities and 8.4 techniques.
  - Is it a **rational function**?  $\implies$  Try a partial fraction decomposition.
  - Is it a **product** of two different types of functions?  $\implies$  Try integration by parts.
  - Does it involve **radicals**?
    - $\implies$  If you have a quadratic under a root, try trig substitution.
    - $\implies$  If not, try to make a substitution to make the integrand a rational function.
- ④ Try again. If the first thing you try doesn't work, go back and try a different approach. For some problems, it may take a few tries to find an approach that works. Often, several integration strategies will be used within one problem.

Identify a technique to evaluate each of the following integrals. What tells you that the technique you identified might be effective? If necessary, explain how to first simplify the integrand before applying the suggested technique. You do not need to evaluate the integrals.

▮ **Example.**  $\int 4xe^{5x} dx$

▮ **Example.**  $\int \frac{\ln x}{x\sqrt{1 + (\ln x)^2}} dx$

▮ **Example.**  $\int \frac{x^3}{\sqrt{64 - x^2}} dx$

▮ **Example.**  $\int \frac{\tan^2 x + 1}{\tan x} dx$

▮ **Example.**  $\int \frac{5x^2 + 18x + 20}{(2x + 3)(x^2 + 4x + 8)} dx$

▮ **Example.**  $\int \frac{\cos^5 x \sin^4 x}{1 - \sin^2 x} dx$