6.3: Volume by Slicing

Learning Objectives. Upon successful completion of Section 6.3, you will be able to...

- Answer conceptual questions involving the general slicing, disk, and washer methods.
- Use the general slicing method to find volumes of solids.
- Use the disk and washer methods to find the volume of solids of revolution about the x-axis.
- Use the disk and washer methods to find the volume of solids of revolution about the y-axis.
- Use the disk and washer methods to find the volume of solids of revolution about horizontal lines ($y = a$) other than the x-axis and vertical lines ($x = b$) other than the y-axis.
- Describe the solid whose volume is given by an integral.
- Solve applications involving the general slicing, disk, and washer methods.

General Slicing Method

Definition. Let S be a solid that lies between $x = a$ and $x = b$. If the cross sections of S are perpendicular to the x-axis and have area $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A\left(x_i^*\right) \Delta x = \int_a^b A(x)\, dx.$$

Note: If S is a solid lying between $y = c$ and $y = d$ with cross sections perpendicular to the y-axis that have area $A(y)$, where A is a continuous function, then

$$V = \int_c^d A(y)\, dy.$$

✍ **Example.** Find the volume of the solid with circular base of radius 5 whose cross sections perpendicular to the base and parallel to the x-axis are equilateral triangles.

A specific type of solid we will work with is a **solid of revolution**. Suppose f is a continuous function on $[a, b]$ and R is the region bounded by the graph of f, the x-axis, and the lines $x = a$ and $x = b$.

Goal: Find the volume of the solid generated by revolving the region R about the x-axis.

Idea: Think about slices.

✍ **Example.** Let R be the region bounded by $y = \sqrt{x-1}$, $y = 0$, and $x = 5$. Find the volume of the solid generated when R is rotated about the x-axis.

✍ **Example.** Let R be the region bounded by $y = x^3$, $y = x$, and $x \geq 0$. Find the volume of the solid generated when R is rotated about the x-axis.

✍ **Example.** Let R be the region bounded by $y = \sqrt{x-1}$, $y = 0$, and $x = 5$. Set up the integral(s) needed to find the volume of the solid generated when R is rotated about the line $y = 2$.

✍ **Example.** Let R be the region bounded by $y = \ln x$, $y = 1$, $y = 2$, and $x = 0$.

(a) Find the volume of the solid generated when R is rotated about the y-axis.

(b) Set up the integral(s) needed to find the volume of the solid generated when R is rotated about the line $x = -2$.