

3.10: Derivatives of Inverse Trig Functions

Learning Objectives. Upon successful completion of Section 3.10, you will be able to...

- Answer conceptual questions involving derivatives of inverse trigonometric functions.
- Find slopes and equations of tangent lines of inverse trigonometric functions.
- Find derivatives of functions involving inverse trigonometric functions.
- Solve applications involving derivatives of inverse functions.

Inverse Trig Derivatives

Derivatives of the inverse trigonometric functions are shown below.

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arccot}(x)) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arcsec}(x)) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\operatorname{arccsc}(x)) = \frac{-1}{|x|\sqrt{x^2-1}}$$

✎ **Example.** Prove that $\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$.

Reminders. Be mindful of recognizing when to apply the *chain rule*.

$$\frac{d}{dx} \left(\arcsin(f(x)) \right) = \frac{1}{\sqrt{1 - (f(x))^2}} \cdot f'(x)$$

Also, be aware of equivalent notations for *inverse trig functions*.

$$\arccos(x) = \cos^{-1}(x)$$

🔗 **Example.** Find the derivative of $y = \arccos(3x - 5)$.

🔗 **Example.** Find the derivative of $y = \frac{\arctan(\pi x)}{\pi x - 2}$.

🔗 **Example.** Find the derivative of $g(\theta) = \arcsin(\sqrt{\theta})$.

▮ **Example.** Find the derivative of $f(x) = \arcsin(\tan x)$.

▮ **Example.** Find the derivative of $f(x) = \tan(x) \arctan(x)$.

▮ **Example.** Find the derivative of $f(x) = \arctan(\arctan x)$.

▮ **Example.** Find the line tangent to $f(x) = \arccos(x^2)$ at $x = \frac{\sqrt{2}}{2}$.

▮ **Example.** Evaluate the limit $\lim_{h \rightarrow 0} \frac{\arctan(1+h) - \arctan(1)}{h}$.