6.7: Physical Applications

Learning Objectives. Upon successful completion of Section 6.7, you will be able to...

- Find the mass of a thin bar with a given density function.
- Find the work done given constant force.
- Find the work done given a variable force function $f(x)$.
- Solve work problems involving springs and Hooke's law.
- Solve work problems involving lifting ropes/chains/cables.
- Solve work problems involving pumping water.
- Solve applications involving pressure.
- Solve applications involving hydrostatic force.

Density and Mass

Density is the concentration of mass in an object. Usually density indicates mass per volume (e.g., kg/m^3) and an object with uniform density satisfies the equation

$$\text{mass} = \text{density} \cdot \text{volume}.$$

If the density varies with the object, we need to use calculus. The problem of finding the mass of a 2- or 3-dimensional object given a density function that varies by position requires multi-variable calculus. So we will look at the problem of finding the mass of a 1-dimensional object using calculus. For 1-dimensional objects, we use *linear density* (e.g., kg/m).

Goal: Suppose a 1-dimensional object, such as a thin bar or wire, is represented by the interval $a \leq x \leq b$. Find the mass of the object given that the object's linear density $\rho(x)$ varies along its length.

✍ **Example.** A thin bar is represented by the interval $0 \leq x \leq \pi$. Find the mass of this bar if its density is given by $\rho(x) = 1 + \sin x$.

Work

Work is an important concept for determining the amount of energy needed to perform various tasks. Work is done when a force moves an object.

Examples of situations where we want to find the amount of work done.

- the work needed to lift a heavy object
- the work needed to wind up a heavy chain
- the work needed to pump water up and out of a tank
- the work needed to stretch or compress a spring