MATH 1080 Vagnozzi

6.3: Volume by Slicing

Learning Objectives. Upon successful completion of Section 6.3, you will be able to...

- Answer conceptual questions involving the general slicing, disk, and washer methods.
- Use the general slicing method to find volumes of solids.
- Use the disk and washer methods to find the volume of solids of revolution about the x-axis.
- Use the disk and washer methods to find the volume of solids of revolution about the y-axis.
- Use the disk and washer methods to find the volume of solids of revolution about horizontal lines (y = a) other than the x-axis and vertical lines (x = b) other than the y-axis.
- Describe the solid whose volume is given by an integral.
- Solve applications involving the general slicing, disk, and washer methods.

General Slicing Method

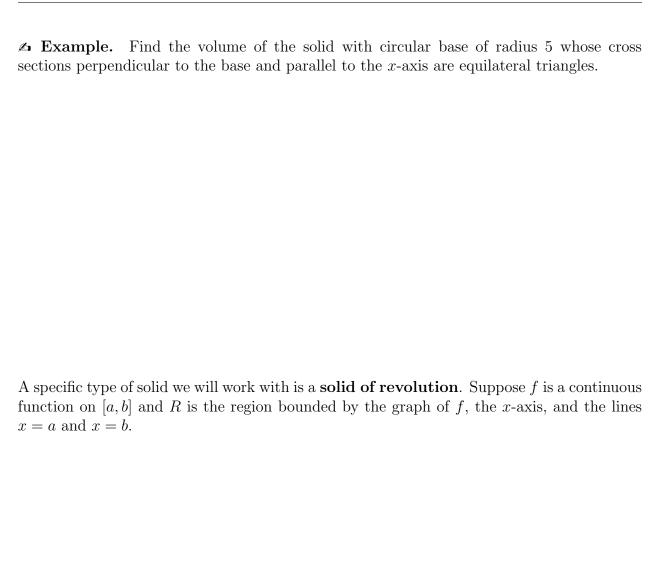
Definition. Let S be a solid that lies between x = a and x = b. If the cross sections of S are perpendicular to the x-axis and have area A(x), where A is a continuous function, then the **volume** of S is

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i^*) \Delta x = \int_a^b A(x) dx.$$

Note: If S is a solid lying between y = c and y = d with cross sections perpendicular to the y-axis that have area A(y), where A is a continuous function, then

$$V = \int_{c}^{d} A(y) \, dy.$$

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Idea: Think about slices.

Example. Let R be the region bounded by $y = \sqrt{x-1}$, y = 0, and x = 5. Find the volume of the solid generated when R is rotated about the x-axis.

Example. Let R be the region bounded by $y = x^3$, y = x, and $x \ge 0$. Find the volume of the solid generated when R is rotated about the x-axis.

Example. Let R be the region bounded by $y = \sqrt{x-1}$, y = 0, and x = 5. Set up the integral(s) needed to find the volume of the solid generated when R is rotated about the line y = 2.

- **Example.** Let R be the region bounded by $y = \ln x$, y = 1, y = 2, and x = 0.
 - (a) Find the volume of the solid generated when R is rotated about the y-axis.

(b) Set up the integral(s) needed to find the volume of the solid generated when R is rotated about the line x=-2.