6.2: Regions Between Curves

Learning Objectives. Upon successful completion of Section 6.2, you will be able to...

- Answer conceptual questions about finding areas between curves.
- Find the area of a region using geometry.
- Find the area of a region (either by integrating with respect to x or by integrating with respect to y) when the figure of the region is provided.
- Set up the area of a region in two ways: (1) in terms of one or more integrals with respect to x and (2) in terms of one or more integrals with respect to y.
- Sketch a given region and find its area.

Introduction

In the next few sections, we will look at some geometric applications of integrals. In Calculus I, we looked at the problem of finding the area between a curve $y = f(x)$ and the x-axis. Now, we are interested in finding the area between two or more curves.

Area Between Two Curves

We want to find the area of a region bounded by the curves $y = f(x)$ and $y = g(x)$ on the interval $a \leq x \leq b$.

Definition. If f and g are continuous functions with $f(x) \geq g(x)$ on $[a, b]$, then the **area of the region** bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\, dx.$$

✍ **Example.** Find the area enclosed by the curves $y = e^x$, $y = x^2 - 1$, $x = -1$, and $x = 1$.

✍ **Example.** Find the area between $y = 12 - x^2$ and $y = x^2 - 6$.

✍ **Example.** Find the area of the region enclosed by the curves $y = \cos x$, $y = \sin 2x$, $x = 0$, and $x = \dfrac{\pi}{2}$.

✍ **Example.** Consider the region enclosed by $4x + y^2 = 12$ and $x = y$. Set up the integral(s) for the area...

(a) with respect to x.

(b) with respect to y.