6.4: Volume by Shells

Learning Objectives. Upon successful completion of Section 6.4, you will be able to...

- Answer conceptual questions involving the Shell Method.
- Use the shell method to find the volume of the solid of revolution about the y-axis.
- Use the shell method to find the volume of the solid of revolution about the x-axis.
- Use the shell method to find the volume of the solid of revolution about other horizontal and vertical lines (other than the x-axis and y-axis).
- Use both the shell method and washer method to find the volume of the solid of revolution about an indicated axis or line.
- Find the volume of a solid of revolution using any method.
- Solve applications involving the shell method.

Volume by Cylindrical Shells

Motivation. Suppose we want to find the volume of a solid generated when a region is rotated about the y-axis. If using the Disk or Washer Method, we would need to make slices *perpendicular* to the axis of rotation (the y-axis) and integrate with respect to y. To do so, we would need to write $y = f(x)$ in terms of y, but this may be difficult for some functions.

Shell Method

Instead, we can create slices that are **parallel** to the axis of rotation. Rather than generating disk or washer cross sections, this will generate a series of **cylindrical shells**.

Shell Method about a Vertical Line. Let f be a continuous function $f(x) \geq 0$ on the interval $[a, b]$. If the region R bounded by the graph of f, the x-axis, and the lines $x = a$ and $x = b$ is revolved about a vertical line (such as the y-axis), the volume of the resulting solid of revolution is

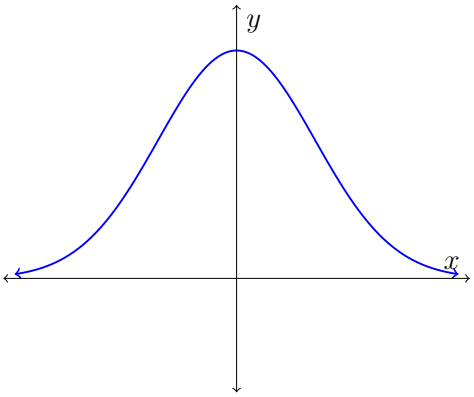
$$V = \int_a^b 2\pi r(x)h(x)\,dx,$$

where $r(x)$ is the radius of the cylindrical shell and $h(x)$ is the height of the shell.

Note: If a region between $y = c$ and $y = d$ is instead rotated about a horizontal line (such as the x-axis), the Shell Method can be used by creating horizontal slices and integrating with respect to y.

$$V = \int_c^d 2\pi r(y)h(y)\,dy$$

✍ **Example.** Let R be the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Use the Shell Method to find the volume of the solid generated when R is rotated about the y-axis.



✍ **Example.** Let R be the region bounded by $y = x^3$, $y = 8$, and $x = 0$. Use the Shell Method to find the volume of the solid generated when R is rotated about the x-axis.

✍ **Example.** Let R be the region bounded by $y = x^3$, $x = 2$, and $y = 0$. Use the Shell Method to set up the integral(s) needed to find the volume of the solid generated when R is rotated about...

(a) the line $x = 3$.

(b) the line $y = -1$.