5.5: The Substitution Method (Review)

Learning Objectives. Upon successful review of Section 5.5, you will be able to...

- Answer conceptual questions involving the Substitution Rule.
- Evaluate indefinite integrals using substitution.
- Evaluate definite integrals using substitution.
- Evaluate integrals with $\sin^2 x$ and $\cos^2 x$.
- Find the area of a region using integration that requires substitution.

A Review of u-Substitution

Recall from Calculus I the **substitution method** for integration, also commonly referred to as **u-substitution**. Applying the substitution method can be thought of as applying the chain rule for differentiation in reverse.

Strategy for Indefinite Integrals. Suppose we have an indefinite integral of the form

$$\int f\big(g(x)\big)g'(x)\,dx.$$

1. Set $u = g(x)$ so that $du = g'(x)\,dx$.
2. The integral may now be expressed as $\displaystyle\int f(u)\,du$.
3. If F is an antiderivative for f, then $\displaystyle\int f(u)\,du = F(u) + C$.
4. We can now substitute $u = g(x)$ into F to obtain the final result.

$$\int f\big(g(x)\big)g'(x)\,dx = F\big(g(x)\big) + C$$

Strategy for Definite Integrals. For definite integrals, the process of u-substitution is nearly the same, but we must modify the limits of integration to correspond with the change of variable.

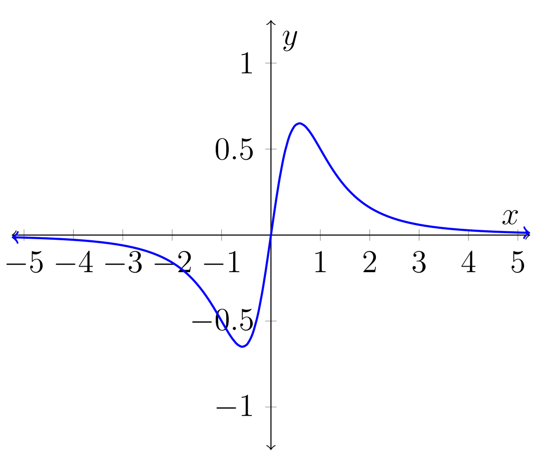
In other words, if $u = g(x)$ and $du = g'(x)\,dx$, then

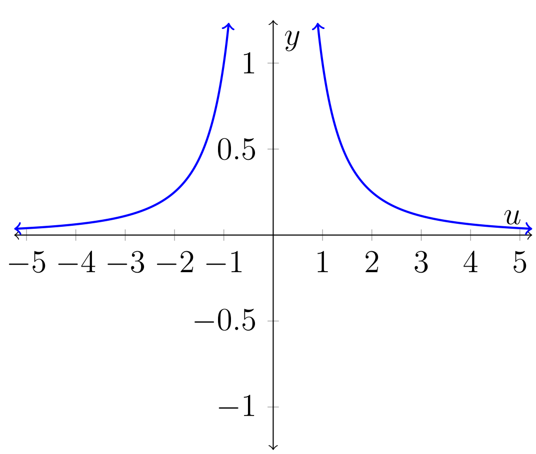
$$\int_a^b f\big(g(x)\big)g'(x)\,dx = \int_{u(a)}^{u(b)} f(u)\,du.$$

✍ **Example.** $\displaystyle\int \tan x\,dx$

✍ **Example.** $\displaystyle\int \frac{x}{1+x^4}\,dx$

✍ **Example.** $\displaystyle\int_0^2 \frac{2x}{(x^2+1)^2}\,dx$





✍ **Example.** $\displaystyle\int_0^2 x^3\sqrt{16-x^4}\,dx$

✍ **Example.** $\displaystyle\int_0^3 \frac{w^2+1}{\sqrt{w^3+3w+4}}\,dw$

✍ **Example.** $\displaystyle\int_0^{\pi/4} \frac{\sin\theta}{\cos^3\theta}\,d\theta$

✍ **Example.** $\displaystyle\int_0^1 xe^{-x^2}\,dx$

✍ **Example.** $\displaystyle\int_0^1 \frac{e^z+1}{e^z+z}\,dz$

✍ **Example.** $\displaystyle\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$