

## 12.3: Calculus in Polar Coordinates

**Learning Objectives.** Upon successful completion of Section 12.3, you will be able to...

- Answer conceptual questions involving calculus in polar coordinates.
- Find the slope of the line tangent to a polar curve at a given point.
- Find the points at which a polar curve has horizontal or vertical tangent lines.
- Find intersection points for two polar curves.
- Find the area of a region bounded by polar curves.
- Find the lengths of polar curves.

### Tangents to Polar Curves

Given a polar curve  $r = f(\theta)$ , how do we find  $\frac{dy}{dx}$ ?

▣ **Example.** Let's consider the polar curve  $r = 1 + \cos \theta$ . Find the slope of the line tangent to the curve at  $\theta = \frac{\pi}{2}$ .

**Tangents to Polar Curves.** The slope  $\frac{dy}{dx}$  of the line tangent to a polar curve  $r = f(\theta)$  is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}.$$

**Horizontal tangents** occur where \_\_\_\_\_, provided that \_\_\_\_\_.

**Vertical tangents** occur where \_\_\_\_\_, provided that \_\_\_\_\_.

✦ **Example.** Let's again consider the polar curve  $r = 1 + \cos \theta$ . Find the points  $(r, \theta)$  where the graph has horizontal or vertical tangents.