1. Let X = the number of large bags of popcorn sold by a local movie theater in a day. Suppose that X is normally distributed with a mean of 230 bags and a standard deviation of 29 bags.

   (a) Describe the distribution of $\overline{X}$, the **average** number of large bags of popcorn sold in a random sample of 7 days, by identifying the **mean** $\mu_{\overline{X}}$ and the **standard error** $\sigma_{\overline{X}}$.

   $\mu_{\overline{X}} = \mu_X = 230$ large popcorn bags

   $\sigma_{\overline{X}} = \frac{29}{\sqrt{7}} = 10.96$ large popcorn bags

   (b) Can we use the normal distribution to find probabilities for $\overline{X}$? How do you know?

   Yes. Because the population X is normally distributed, we can conclude that $\overline{X}$ is approximately normally distributed.

   (c) What is the probability that, on a **single** day, the theater will sell more than 250 popcorn bags?

   $z = \frac{250-230}{29} = 0.69$

   $P(X > 250) = P(Z > 0.69) = 1 - P(Z < 0.69) = 1 - 0.7549 = 0.2451$

   (d) If seven days are randomly selected, what is the probability that the **average** number of popcorn bags sold per day will be greater than 250?

   $z = \frac{250-230}{10.96} = 1.82$

   $P(\overline{X} > 250) = P(Z > 1.82) = 1 - P(Z < 1.82) = 1 - 0.9656 = 0.0344$

2. Leslie has been tasked with putting together a report for Ron regarding the use of a park in Pawnee. Previous data show that 72% of the residents in Pawnee visited the park in the last month.

   (a) Describe the distribution of $\hat{p}$, the **proportion** of residents in a random sample of 150 who visit the park in a month, by identifying the **mean** $\mu_{\hat{p}}$ and **standard error** $\sigma_{\hat{p}}$. Round standard error to four decimal places.

   $\mu_{\hat{p}} = p = 0.72$

   $\sigma_{\hat{p}} = \sqrt{\frac{0.72(0.28)}{150}} = 0.0367$

   (b) Is the distribution of $\hat{p}$ approximately normally distributed? How can you tell?

   $np(1-p) = 150(0.72)(0.28) = 30.24 \geq 10 \Rightarrow \hat{p}$ is approximately normally distributed.

   (c) What is the probability that more than 99 individuals in a random sample of 150 residents have visited the park in the last month?

   $\hat{p} = \frac{99}{150} = 0.66$

   $z = \frac{0.66-0.72}{0.0367} = -1.63$

   $P(\hat{p} > 0.66) = P(Z > -1.63) = 1 - P(Z < -1.63) = 1 - 0.0516 = 0.9484$

3. Given that a continuous random variable X is normally distributed with a mean of 40 and a standard deviation of 13, calculate the probability that a sample of size 49 has a mean of...

   (a) Greater than 37

   $z = \frac{37-40}{13/\sqrt{49}} = -1.62$

   $P\left(\overline{X} > 37\right) = P(Z > -1.62) = 1 - 0.0526 = 0.9474$

   (b) At least 42.5

   $z = \frac{42.5-40}{13/\sqrt{49}} = 1.35$

   $P\left(\overline{X} \geq 42.5\right) = P(Z \geq 1.35) = 1 - 0.9115 = 0.0885$

   (c) Between 39 and 43

   Lower $z = \frac{39-40}{13/\sqrt{49}} = -0.54$, Upper $z = \frac{43-40}{13/\sqrt{49}} = 1.62$

   $P\left(39 < \overline{X} < 43\right) = P(-0.54 < Z < 1.62) = 0.9474 - 0.2946 = 0.6528$

   (d) No more than 35

   $z = \frac{35-40}{13/\sqrt{49}} = -2.69$

   $P\left(\overline{X} \leq 35\right) = P(Z < -2.69) = 0.0036$

4. All Clear Windows makes windows for use in homes and commercial buildings. The standards for glass thickness call for the glass to average 0.375 inches with a standard deviation of 0.050 inches. Let $\overline{X}$ represent the mean thickness of 50 randomly selected windows.

   (a) Describe the center, spread, and shape of the distribution of $\overline{X}$.

   Center: $\mu_{\overline{X}} = \mu_X = 0.375$ in.

   Spread: $\sigma_{\overline{X}} = \frac{0.050}{\sqrt{50}} = 0.007$ in.

   Shape: Because $n = 50 \geq 30$, $\overline{X}$ follows an approx. normal distribution by the Central Limit Theorem.

   (b) Suppose a random sample of $n = 50$ windows yields a mean thickness of 0.392 inches. What is the likelihood of observing a sample with a mean thickness at least as thick as ours?

   $z = \frac{0.392-0.375}{0.050/\sqrt{50}} = 2.40$

   $P\left(\overline{X} > 0.392\right) = P(Z > 2.40) = 1 - 0.9918 = 0.0082$

5. A nationwide survey analyzing trends in popular media found that 81% of U.S. college students prefer British baking shows over American baking shows. You are interested to see if this result holds at your university, which has a student population of about 30,000. You take a random sample of 140 students on campus and find that 125 of them prefer watching British baking shows.

   (a) Can you use the normal distribution to find probabilities for the sample proportion $\hat{p}$ of students at your university who prefer British baking shows? Check the appropriate condition to justify your answer.

   Yes. Because $np(1-p) = 140(0.81)(1-0.81) = 21.546 \geq 10$, $\hat{p}$ follows an approximately normal distribution.

   (b) Find the probability of obtaining a sample where $\hat{p}$ is at least as great as your sample.

   $\hat{p} = \frac{125}{140} \approx 0.89$

   $z = \frac{0.89-0.81}{\sqrt{\frac{0.81(0.19)}{140}}} = 2.41$

   $P(\hat{p} > 0.89) = P(Z > 2.41) = 1 - 0.9920 = 0.0080$

   (c) Does your result cause you to suspect that the national result is an over- or an underestimate for your university? Explain your reasoning.

   The probability of obtaining a sample of 140 where 89% or more prefer British baking shows is small (less than 1%), meaning that this is a rare event. This may cause us to suspect that the national result underestimates the true proportion at our university.