8.3: Trigonometric Integrals

Learning Objectives. Upon successful completion of Section 8.3, you will be able to...

- Answer conceptual questions involving trigonometric integrals.
- Evaluate indefinite integrals involving powers of sine and cosine.
- Evaluate definite integrals involving powers of sine of cosine.
- Evaluate indefinite integrals involving powers of tangent and secant or cotangent and cosecant.
- Evaluate definite integrals involving powers of tangent and secant or cotangent and cosecant.
- Use trigonometric integrals to find areas, volumes, or arc length.

Trig Integrals

Recall the trig antiderivative rules that we know for sine and cosine.

$$\int \sin x \, dx = \qquad\qquad \int \cos x \, dx =$$

We can also identify trig antiderivative rules for the tangent and cotangent functions.

$$\int \tan x \, dx =$$

$$\int \cot x \, dx =$$

What about our other two trig functions?

$\int \sec x \, dx =$

$\int \csc x \, dx =$

A Review of Common Trig Identities

Pythagorean Identities

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ (found by dividing each term of the first identity by $\cos^2 x$)

$\cot^2 x + 1 = \csc^2 x$ (found by dividing each term of the first identity by $\sin^2 x$)

Half-Angle Identities

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

Double-Angle Identities

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \cos^2 x - \sin^2 x$

$\sin 2x = 2\sin x \cos x$

Let's learn some other techniques for integrating trig functions.

Tips for Integrating $\int \sin^m x\,dx$, $\int \cos^n x\,dx$, and $\int \sin^m x \cos^n x\,dx$

If the power of sine m is **odd**, separate out a factor of $\sin x$ and write the remaining $\sin^{m-1} x$ in terms of cosines using the Pythagorean identity. Then use u-substitution with $u = \cos x$.

✍ **Example.** $\displaystyle\int \sin^5 x\,dx$

If the power of cosine n is **odd**, separate out a factor of $\cos x$ and write the remaining $\cos^{n-1} x$ in terms of sines using the Pythagorean identity. Then use u-substitution with $u = \sin x$.

✍ **Example.** $\displaystyle\int \frac{\cos^5 x}{\sin^{3/2} x}\,dx$

If m and n are both **even**, use the half-angle identities.

✍ **Example.** $\displaystyle\int \sin^2 x \cos^4 x \, dx$