6.5: Length of Curves

Learning Objectives. Upon successful completion of Section 6.5, you will be able to...

- Answer conceptual questions involving lengths of curves.
- Set up arc length integrals.
- Find the arc length of a curve on a given interval, integrating with respect to x or with respect to y as appropriate.
- Find functions with given arc lengths.
- Find lengths of related curves given the length of a curve f.

Finding a Formula for Arc Length

Goal: Find the length of a curve C defined by $y = f(x)$, $a \leq x \leq b$, where f' is continuous on $[a, b]$.

Arc Length. Consider a curve C defined by $y = f(x)$, where $a \leq x \leq b$ and f' is continuous on $[a, b]$. The length L of the curve is given by

$$L = \int_a^b \sqrt{1 + \big(f'(x)\big)^2}\, dx.$$

✍ **Example.** Find the arc length of the curve $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}$, $1 \leq y \leq 2$.

✍ **Example.** Find the arc length of the curve $y = \frac{1}{2}\left(e^x + e^{-x}\right)$ on $[-\ln 2, \ln 2]$.

✍ **Example.** Set up and simplify the integral that gives the arc length of the curve $y = \ln x$ for $1 \leq x \leq 4$.

Note: Arc length integrals are often difficult to evaluate analytically, even for simple functions. We will be able to evaluate even more types of integrals after learning more integration techniques in Chapter 8.