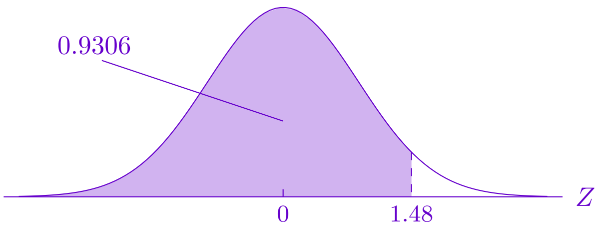
For each problem, you should draw and label a sketch, include probability notation when appropriate, and show any work used to calculate probabilities and z-scores.

1. The height of hobbits, X, is normally distributed with a mean of 42 in. and a standard deviation of 2.7 in.

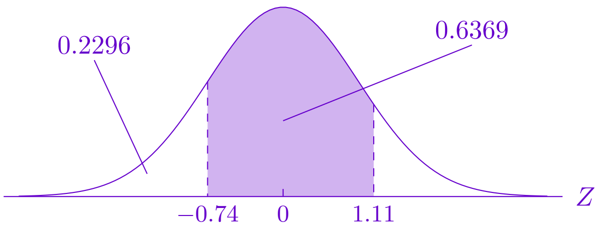
   (a) Frodo is 46 inches tall. What proportion of hobbits is shorter than Frodo?



$$z = \frac{46-42}{2.7} = 1.48$$

$$P(X < 46) = P(Z < 1.48) = 0.9306$$

   (b) What is the probability that a randomly selected hobbit is between 40 and 45 inches tall?



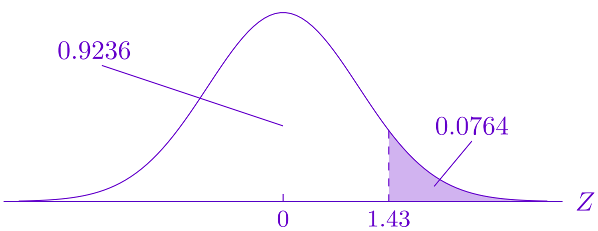
$$z_1 = \frac{40-42}{2.7} = -0.74$$

$$z_2 = \frac{45-42}{2.7} = 1.11$$

$$P(40 < X < 45) = P(-0.74 < Z < 1.11) = P(Z < 1.11) - P(Z < -0.74) = 0.8665 - 0.2296 = 0.6369$$

2. The height of elves, Y, is normally distributed with a mean of 75 in. and a standard deviation of 3.5 in.

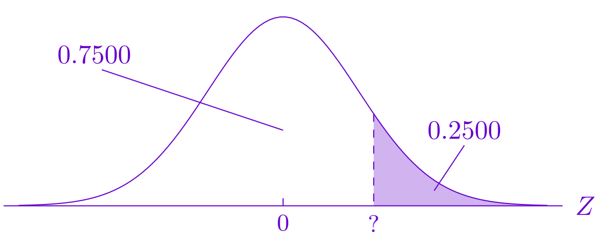
   (a) Legolas is 80 inches tall. What is the probability that a randomly selected elf is taller than Legolas?



$$z = \frac{80-75}{3.5} = 1.43$$

$$P(X > 80) = P(Z > 1.43) = 1 - P(Z < 1.43) = 1 - 0.9236 = 0.0764$$

   (b) Find the minimum height of an elf who falls in the tallest 25% of elves.



From the table: $z \approx 0.67$

$$0.67 = \frac{x-75}{3.5}$$

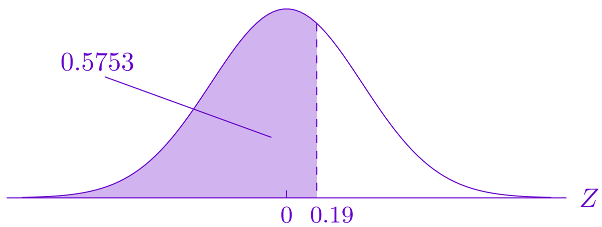
$$x = 0.67(3.5) + 75 = 77.345 \text{ in.}$$

Use the standard normal distribution table to answer the following questions. Draw a sketch with the correct shaded area and show any work used to compute z-scores.

3. Find the following probabilities for a standard normal random variable Z.

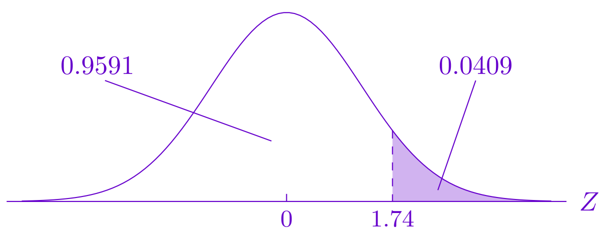
   (a) $P(Z < 0.19)$

   $P(Z < 0.19) = 0.5753$

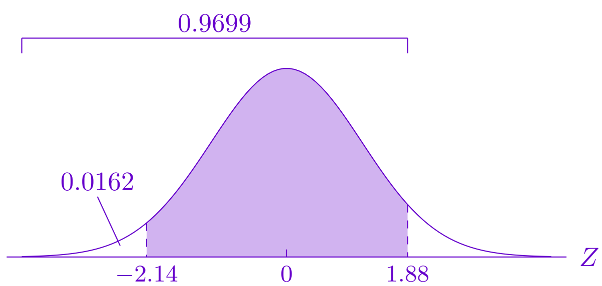
   (b) $P(Z > 1.74)$

   $P(Z > 1.74) = 1 - P(Z < 1.74) = 1 - 0.9591 = 0.0409$

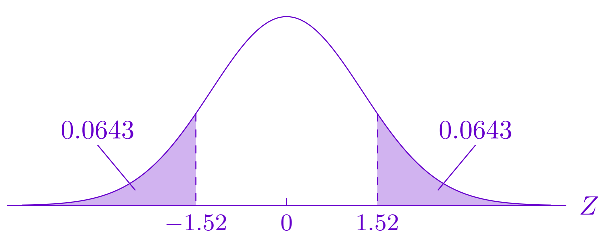
   (c) $P(-2.14 \leq Z \leq 1.88)$

   $P(-2.14 \leq Z \leq 1.88) = P(Z < 1.88) - P(Z < -2.14) = 0.9699 - 0.0162 = 0.9537$

   (d) $P(|Z| > 1.52)$

   $P(|Z| > 1.52) = P(Z > 1.52) + P(Z < -1.52) = 2(0.0643) = 0.1286$

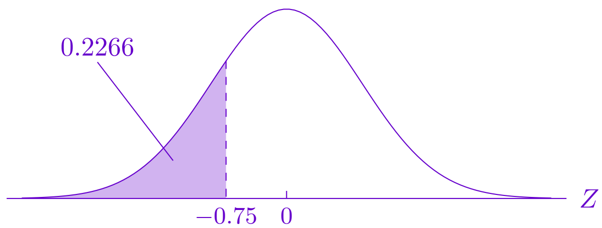
   

4. The annual salaries of employees in a large company are approximately normally distributed with a mean of \$50,000 and a standard deviation of \$20,000.

   (a) What proportion of employees at the company earn less than \$35,000?

   $z = \frac{35,000-50,000}{20,000} = -0.75$

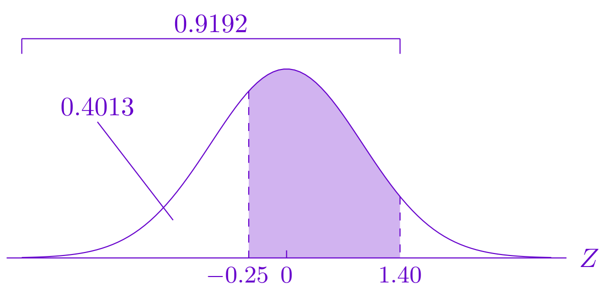
   $P(Z < -0.75) = 0.2266$



   (b) What is the probability that a randomly selected employee at the company earn between \$45,000 and \$78,000?

   $z = \frac{45,000-50,000}{20,000} = -0.25$ $\qquad$ $z = \frac{78,000-50,000}{20,000} = 1.40$

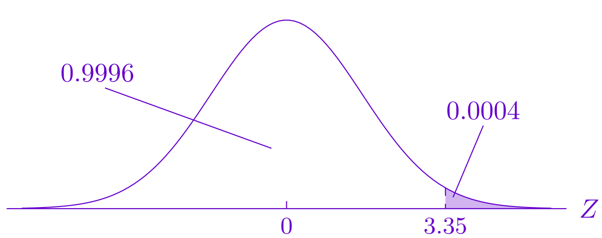
   $P(-0.25 < Z < 1.40) = P(Z < 1.40) - P(Z < -0.25) = 0.9192 - 0.4013 = 0.5179$



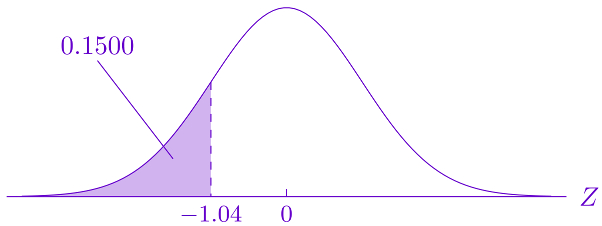
   (c) What proportion of people earn more than \$117,000?

   $z = \frac{117,000-50,000}{20,000} = 3.35$

   $P(Z > 3.35) = 1 - P(Z < 3.35) = 1 - 0.9996 = 0.0004$



(d) When the coronavirus pandemic caused business operations to be suspended and people to be placed on temporary leave from their jobs, the company gave its employees with the lowest 15% of salaries a stipend to make up for lost time on the job. What is the salary cutoff that determines whether someone receives the stipend?

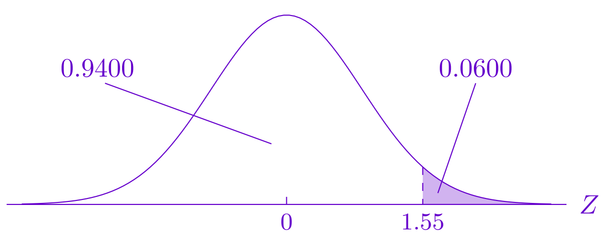


From the table: $z \approx -1.04$
(between $z = -1.03$ and $z = -1.04$, take the one with the probability closer to 0.1500)

$-1.04 = \frac{x-50{,}000}{20{,}000}$

$x = -1.04(20,000) + 50,000 = \$29,200$

(e) To make up for some of the business losses, the company plans to ask employees with the highest 6% of salaries to take one week of unpaid leave. What is the minimum salary that an employee who needs to take a week of unpaid leave will have?



From the table: $z = \frac{1.55+1.56}{2} = 1.555$
(between $z = 1.55$ and $z = 1.56$, probabilities are equally far from 0.9400, so take the average)

$1.555 = \frac{x-50{,}000}{20{,}000}$

$x = 1.555(20,000) + 50,000 = \$81,100$