Build-a-Bear distributors claim that there is only a 2% chance that an unstuffed bear has a sewing defect. Your store received a standard shipment of 200 unstuffed bears. Let X = the number of bears with defects. Round all probabilities in the following problems to **four** decimal places.

1. What is the probability that **exactly** 10 bears have a defect? Show probability notation, the **binomial formula** with values plugged in, and your answer.

$P(X = 10) = {}_{200}C_{10}(0.02)^{10}(0.98)^{190} = 0.0049$

2. What is the probability that **at least two** bears have defects? Show probability notation and your answer.

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 1 - \Big(P(X = 0) + P(X = 1)\Big) \\ &= 1 - \Big({}_{200}C_0(0.02)^0(0.98)^{200} + {}_{200}C_1(0.02)^1(0.98)^{199}\Big) \\ &= 1 - (0.0176 + 0.0718) \\ &= 0.9106 \end{aligned}$$

3. What is the probability that **at most three** bears have a defect? Show probability notation and your answer.

$$\begin{aligned} P(X \leq 3) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\ &= {}_{200}C_0(0.02)^0(0.98)^{200} + {}_{200}C_1(0.02)^1(0.98)^{199} + {}_{200}C_2(0.02)^2(0.98)^{198} + {}_{200}C_3(0.02)^3(0.98)^{197} \\ &= 0.0176 + 0.0718 + 0.1458 + 0.1963 \\ &= 0.4315 \end{aligned}$$

4. What is the probability that **between** three and five bears (inclusive) have defects? Show probability notation and your answer.

$$\begin{aligned} P(3 \leq X \leq 5) &= P(X = 3) + P(X = 4) + P(X = 5) \\ &= {}_{200}C_3(0.02)^3(0.98)^{197} + {}_{200}C_4(0.02)^4(0.98)^{196} + {}_{200}C_5(0.02)^5(0.98)^{195} \\ &= 0.1963 + 0.1973 + 0.1579 \\ &= 0.5515 \end{aligned}$$

5. What is the probability that **more than 10** bears are defective? (Hint: Doing this by hand would be incredibly tedious. Practice answering binomial probability problems using JMP output instead! You can follow the instructions on pages 71–73 of your Lecture Notes to generate the JMP output you would be provided on an exam for this type of problem.)

$P(X > 10) = 1 - P(X \leq 10) = 1 - 0.9975 = 0.0025$

6. What is the **expected number** of defective bears in your shipment? Include units, the appropriate symbol, and your calculations.

   $\mu_X = E(X) = np = 200(0.02) = 4$ bears

7. **Interpret** the expected value you found in Question #5.

   If we observed a large number of 200-bear shipments from Build-a-Bear, we would expect the average number of defective bears to be 4.

8. What is the **standard deviation** of defective bears in your shipment? Include units, the appropriate symbol, and your calculations.

   $\sigma_X = \sqrt{np(1-p)} = \sqrt{200(0.02)(0.98)} = \sqrt{3.92} = 1.98$ bears