Lab Assignment 4

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Part A

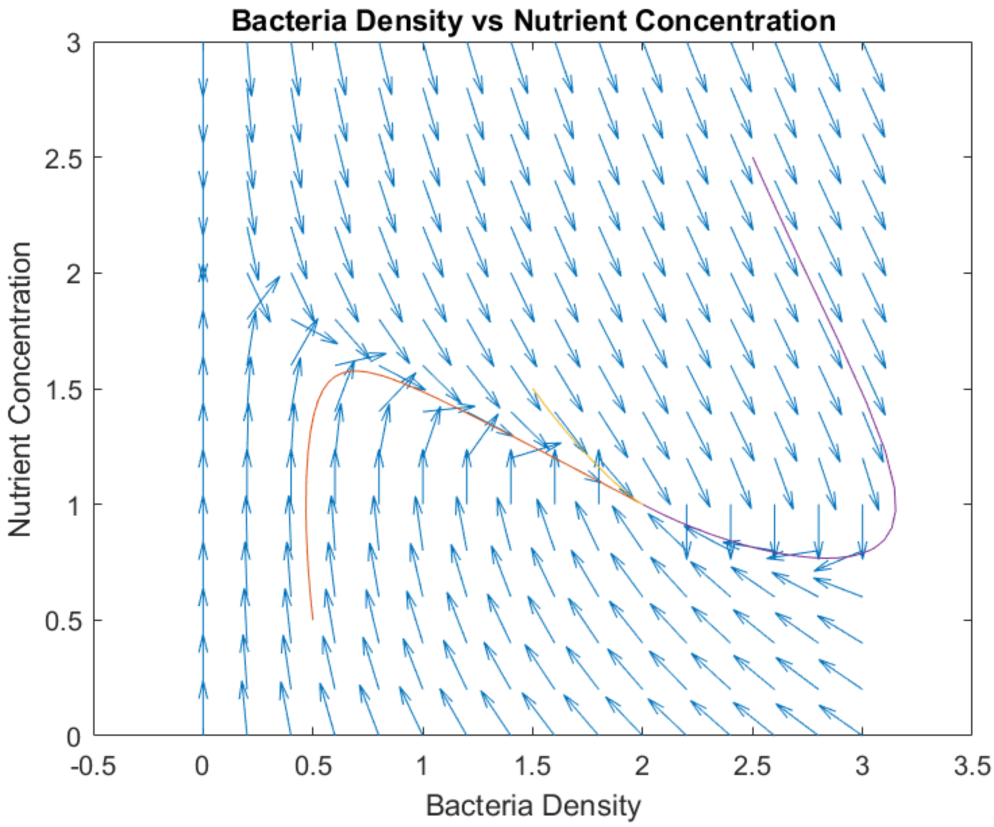

Figure 1: $\alpha_1 = 2$, $\alpha_2 = 2$

When there is low bacteria density and low nutrient concentration, the density will increase rapidly at first, before hitting a limit and decreasing. Conversely, if there is high bacteria density and high nutrient concentration, the density will increase until there is too much bacteria, then the density will reduce until it hit a stable point, at roughly $(2, 1)$. This makes sense for the values of $\alpha_{1,2}$, as both are positive modifiers and with limited nutrients bacteria cannot expand infinitely.

Part B

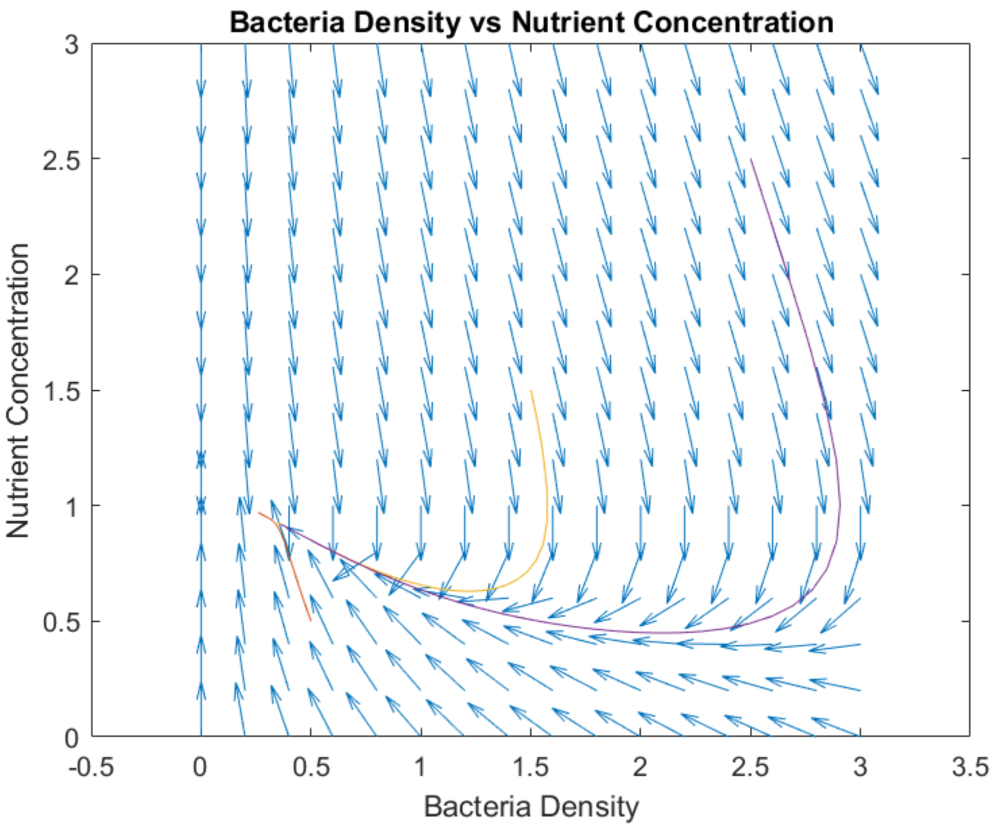

Figure 2: $\alpha_1 = 2$, $\alpha_2 = 1.1$

Similarly to Part A, when there is low bacteria density and low nutrient concentration, the density will increase rapidly at first, before hitting a limit and decreasing and if there is high bacteria density and high nutrient concentration, the density will increase until there is too much bacteria, then the density will reduce until it hit a stable point. Here, the stable point seems to be roughly $(0.25, 1)$. Like in Part A, this behavior makes sense for the values of $\alpha_{1,2}$ for the same reasons. It also makes sense that there is less "growth" periods on the phase plane, as since α_2 is smaller here, there is greater decrease in concentration.

Part C

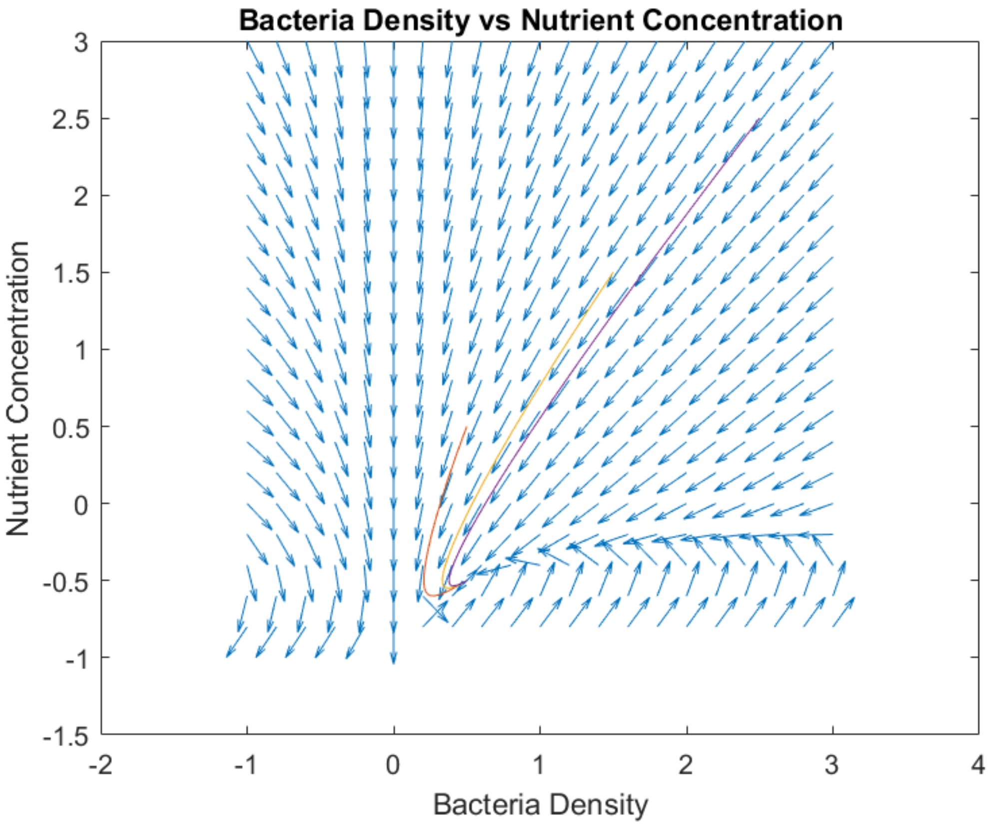

Figure 3: $\alpha_1 = -1$, $\alpha_2 = -1$

Regardless of where the bacteria density and nutrient concentration starts - so long as it is not negative, which isn't biologically feasible anyways - the density and concentration decreases rapidly until nutrient concentration reaches zero. At some negative concentration value (approx. -0.5) the bacteria density will increase, which is not realistic biologically.

In terms of the values of $\alpha_{1,2}$, this behavior makes sense as $\frac{dC}{dt}$ is negative. Biologically, bar all values where concentration and/or density are negative, it makes sense: with decreasing concentration, the bacteria density would decrease rapidly until it reaches 0. Interestingly, the density trajectories do not converge at 0, though upon reflection that makes sense as the bacteria density does not necessarily have to decay at the same rate as the concentration - there could be density left when there is no more concentration left. We just don't have a model for how precisely these stragglers die out.