Representation Learning with Fuzzy Sets

Kajal Sharma
Department of Computer Science
University of California, Los Angeles
ksharma29@g.ucla.edu

Muskan Rizwan Shaikh
Department of Computer Science
University of California, Los Angeles
muskanrs@g.ucla.edu

Dhakshina Ilango
Department of Computer Science
University of California, Los Angeles
dhakshina@g.ucla.edu

Vani Agrawal
Department of Computer Science
University of California, Los Angeles
vaniagrawal@g.ucla.edu

Abstract

Knowledge Graph (KG) embedding models are essential for learning the representation of KGs by projecting entities and relations into a low-dimensional vector space. Traditional approaches often focus on instance-view triples and may lose the semantics of entities and relations, leading to the prediction of invalid triples. This research project introduces a novel approach by extending the jointly embedded instance-view and ontology-view KGs into a fuzzy set theory embedding space. The fuzzy set theory allows for the representation of ambiguous information by assigning degrees of membership to entities and modelling relationships with varying degrees of strength or confidence. The proposed extension to JOIE model leverages the two-view KG structure and fuzzy set theory to provide a more flexible and accurate framework for modelling complex and imprecise concepts. We evaluate our model's performance against traditional KG embedding models on tasks such as triple completion.

Link to the code repository: https://github.com/vagrawal22/cs249-project-joie-fuzzysets/tree/main

1 Introduction

Representation of Knowledge Graphs (KG) can be learnt by using KG embedding models. The entities and relations are projected in a low-dimensional vector space to preserve the existing triples present in the KG as well as predict new valid triples. Embedding models might do a good job of learning the representation of the input KG, but due to the nature of machine learning (ML) approaches, they often lose the semantics of entities and relations, which might lead to the prediction of invalid triples. In the realm of KG modelling, we encounter two primary views: the instance view and the ontology view. The instance view focuses on the specific entities and their direct relationships (for example, "UCLA", "is_located_in", "Los Angeles") while the ontology view is a higher-level structure that deals with the abstract concepts and categories, defining the types of entities. It also consists of semantic meta-relations of abstract concepts (for example, "Singer", "is_a", "Artist"). Cross-view links bridge the two views together by connecting the ontological concepts and instances, indicating whether a particular entity is an instantiation of an abstract concept. Traditionally, KG models focus on representing either the instance view or the ontology view, but rarely both at the same time. However, leveraging both views simultaneously can provide a more comprehensive understanding of the data.

Preprint. Under review.

Crisp Set logic is traditionally used to model entities and their relationships in a 2-view KG. In this case, an entity either belongs to a set or it does not and relationships are either True or False without any degree of uncertainty. This type of modelling struggles to represent and reason about incomplete information which is often the case with real-life datasets. The rigid binary nature of relationships does not capture the nuanced and variable nature of real-world data.

In this research project, we propose extending the jointly embedded instance-view and ontology-view knowledge graphs into a fuzzy set theory embedding space. Fuzzy Set Theory overcomes the above issue by assigning degrees of membership to entities and allowing for the representation of ambiguous information of real-world data. Entities can partially belong to multiple sets with varying degrees of membership, providing a more accurate and flexible representation of reality.Relationships can be modelled with varying degrees of strength or confidence, allowing for more nuanced and realistic modelling of associations between entities. Overall, leveraging the two view KG and fuzzy set theory provides a more flexible framework for modelling complex and imprecise concepts, which can lead to more accurate and adaptable models in various domains.

2 Problem Definition and Formalization

In this research paper, our primary contribution to the JOIE model involves addressing the need for greater granularity in the association between head entities and their corresponding tail entities. To achieve this, we focused on the integration of fuzzy sets embeddings as an alternative to the traditional state-of-the-art techniques for embedding retrieval, to enhance the precision and efficiency of these associations within JOIE.

3 Related Work

Much of the work present in the literature revolves around learning instance-view KG embeddings. The input to the models is in the form of triples (h, r, t) where h is the head entity, t is the tail entity and r stands for the relation between the two entities. The objective of these KG embedding models is to optimise a plausibility scoring function which helps to assess the validity of the relationship between the head and tail entities. The KG embeddings are useful for a variety of tasks such as relation extraction [1], question answering [2], dialogue agents [3], knowledge alignment [4] and visual semantic labelling [5]. KG embedding models can be classified into two broad groups - Translational models and Similarity models. Each type of model has a different approach to learning these embeddings and capturing the semantics of the knowledge graph. Translational models, also known as distance-based models, operate on the principle that relationships between entities can be interpreted as translations in the embedding space. A representative example is TransE [6] where the relation embedding acts as a translation vector that moves the head entity embedding close to the tail embedding entity t. Similarity models, also known as semantic matching models, focus on capturing the similarity between entities and relations in the embedding space. These models define a scoring function that measures how similar or compatible the embeddings of the entities and the relation in a triple are. A representative example is DistMult [7] which uses a bilinear scoring function. HolE [8] is another embedding model that extends DistMult by using circular correlation.

The predominant focus of current embedding models lies in capturing deterministic knowledge. Consequently, it becomes crucial to incorporate the representation of ambiguous information within knowledge graph embeddings. UKGE [9], a knowledge graph embedding model, shifts its attention to representing uncertain knowledge. Employing two distinct mapping functions, UKGE transforms the KGE scores function into a confidence score value within the range [0,1]. This approach aligns with the principles of fuzzy set logic, where entities may exhibit relations with other entities characterized by varying degrees of confidence. These models mainly focus on instance-view triple completion and do not use instance-view knowledge to improve the ontology population.

More recent research work involves bridging multiple KGs and leveraging both instance and ontology views. ReasonKGE [10] is a novel approach which integrates ontological reasoning into the embedding learning process. The framework employs a regularization technique that integrates ontological rules into the loss function of the embedding model. This helps in guiding the learning process to produce embeddings that are consistent with the ontological knowledge. After comparing the performance of ReasonKGE with traditional KG embedding models on tasks like link prediction and triple classification, the authors concluded that incorporating ontological reasoning results in more accurate and semantically meaningful embeddings. This, in turn, enhances the overall performance of downstream tasks. Another example of a two-view knowledge graph is MTransE [11], a translational model designed for multilingual learning that aligns entities across different languages by learning embeddings within a shared space. This allows the model to capture the semantic similarity between entities that have the same meaning but are represented in different languages. However, this approach relies on the assumption that the KGs have similar structures and fails to capture associations between the two views when the vocabulary is disjoint. In contrast, DGS [12], extends the JOIE [13] model and offers a novel approach to addressing these issues. DGS leverages dual geometric spaces to embed entities and relationships from two distinct views of a knowledge graph. By doing so, it overcomes the limitations posed by structural discrepancies and disjoint vocabularies, enabling more effective alignment and representation of knowledge across multiple views. This transition highlights the need for advanced models like JOIE and DGS, which offer reasoning over KGs of dissimilar topologies.

4 Methodology

Our proposed approach integrates fuzzy set theory logic into the JOIE framework which jointly embeds entities and ontological concepts using two model components: cross-view association model and intra-view model.

4.1 Cross-view Association Model

Cross-view association model aims to model the cross-view links which connect the entity embedding space and the concept embedding space. There are two ways in which this can be done: Cross-view Grouping (CG) and Cross-View Transformations (CT).

4.1.1 Cross-view Grouping (CG)

The cross-view grouping method can be considered as a grouping-based regularization method which assumes that both the instance and ontology views can be embedded into the same embedding space. This requires the embedding dimensionalities to be the same ie. d = d_c = d_e. The categorical association loss shown in Equation 1, for a given pair of entities and concepts connected by cross-view link (e,c), is defined as the distance between the embedding of e and c compared with margin γ^{CG}.

$$J_{\mathrm{Cross}}^{\mathrm{CG}} = \frac{1}{|S|} \sum_{(e,c)\in S} \left[\|c - e\|_2 - \gamma^{\mathrm{CG}}\right]_+ \tag{1}$$

where $[x]_+$ is the positive part of the input x. CG models cross-view links by clustering It makes the entity embedding close to their concept embedding in the end.

4.1.2 Cross-view Transformation (CT)

Another way to model the cross-view links is to perform a non-linear affine transformation to convert the entity embedding space information to concept embedding space. In contrast to CG, this requires the embedding spaces of the two views to be completely different from each other. Once the transformation is completed an instance will be mapped to an embedding in the ontology view space corresponding to the related abstract concept:

$$c \leftarrow f_{\mathrm{CT}}(e), \forall (e,c) \in S \tag{2}$$

In Equation 2, $f_{\mathrm{CT}}(e)$ is defined as follows: $f_{\mathrm{CT}}(e) = \sigma(W_{\mathrm{ct}} \cdot e + b_{\mathrm{ct}})$ where $W_{\mathrm{ct}} \in \mathbb{R}^{d_2 \times d_1}$ is a weight matrix and b_{ct} is a bias vector. $\sigma(\cdot)$ denotes the non-linear activation function, for which we adopt the tanh function.

In Equation 3, The total loss of the cross-view association model is formulated as follows:

$$J_{\text{Cross}}^{\text{CT}} = \frac{1}{|S|} \sum_{(e,c)\in S \wedge (e,c')\notin S} \left[\gamma^{\text{CT}} + \|c - f_{\text{CT}}(e)\|_2 - \|c' - f_{\text{CT}}(e)\|_2\right]_+ \tag{3}$$

4.2 Intra-view Model

The goal of the intra-view model is to preserve the original structural information of the two views in separate embedding spaces. There are two intra-view model techniques for encoding heterogeneous and hierarchical KG structures.

4.2.1 Default Intra-view Model

The triple encoding technique can be used to represent triples in both views of the Knowledge Base. The TransE model is used to encode the triples in both views. The plausibility score function is described in Equation 4 as:

$$f_{\text{TransE}}(h, r, t) = -\|h + r - t\|_2 \tag{4}$$

To effectively learn the embeddings of the nodes in a graph, a hinge loss as specified in Equation 5 is optimized for all the triples in the graph.

$$J_{\text{Intra}}^{\text{G}} = \frac{1}{|G|} \sum_{(h,r,t)\in G \wedge (h',r,t')\notin G} \left[\gamma^{G} + f(h', r, t') - f(h, r, t)\right]_+ \tag{5}$$

The loss function of the instance view and ontology-view graphs can be combined as shown in Equation 6 :

$$J_{\text{Intra}} = J_{\text{Intra}}^{\text{G}_\text{I}} + \alpha_1 \cdot J_{\text{Intra}}^{\text{G}_\text{O}} \tag{6}$$

4.3 Hierarchy-Aware Intra-view Model for the Ontology

In the ontology-view hierarchical relations are present through meta-relations for example "subclass_of" and "is_a". These structures require a different type of modelling compared to other meta-relations. To address this, a hierarchy-aware (HA) intra-view model is used which extends the cross-view transformation (CT) method to account for the non-linear relationships between coarser and finer concepts. This involves introducing an ontology hierarchy loss term to the training process. The total training loss of the HA intra-view model as shown in Equation 7, combines losses from regular semantic relations and hierarchical meta-relations, weighted by hyperparameters.

$$J_{\text{Intra}} = J_{\text{Intra}}^{\text{G}_\text{I}} + \alpha_1 \cdot J_{\text{Intra}}^{\text{G}_\text{O}\setminus\text{T}} + \alpha_2 \cdot J_{\text{Intra}}^{\text{HA}} \tag{7}$$

The JOIE framework assumes that knowledge graphs (KGs) have rich ontology hierarchies and semantic relations and is designed to encode these properties effectively.

4.4 Fuzzy Set Embedding Model (Our Model)

Our extension to JOIE model aimed to retrieve the fuzzy set embeddings instead of the embeddings generated from other traditional approaches. Our motivation behind this approach was to cater to the following limitations of other traditional ways of getting embeddings. The first is handling uncertainty. Oftentimes, the relationships between the entities aren't binary or clear-cut. This can be only modeled through fuzzy logic which is known to handle vagueness or uncertainties. Fuzzy set embeddings can help capture more nuanced and complex relations between the entities that traditional embeddings would miss. The second is scalability and flexibility. Fuzzy set embedding provides greater scalability and flexibility by accommodating diverse datasets and knowledge graphs, whereas, traditional approaches can be rigid and less adaptable to varying data types and structures. Lastly, there is improved generalization and interpretability. Fuzzy set embeddings help enhance interpretability by providing clearer insights into how entities are related, which could pose a more significant challenge in other traditional approaches. Having these clearer insights is also especially valuable for domains such as healthcare, where the reasoning behind relationships is particularly essential. For our extension to JOIE, we decided to take two different approaches. The first approach involved modeling both entities and relations as fuzzy sets instead of taking triples, and the second approach involved assigning a membership function to the entities. We have explained more about these two approaches in the subsections below.

4.4.1 Entities and Relations as Fuzzy Sets (Approach 1)

In this section, we will discuss the first approach we took for our fuzzy set embedding model. This approach involved representing both entities and relations as fuzzy sets, along with subsequently applying fuzzy logic to further enhance knowledge graph representation. The reason behind representing both the entities and relations as fuzzy sets is that this would provide a versatile framework for encoding various types of information, such as hierarchical, relational, and overlapping. They can capture more complex relationships, in scenarios where a category could contain more specific subcategories or belong to multiple categories simultaneously. For example, in a hierarchical relationship, the category "Technology" could contain other subcategories such as "Artificial Intelligence" or "Machine Learning". Another example, in an overlapping relationship, an entity "Dr. John" could belong to the "professor" category and the "researcher" category, simultaneously. Integrating the approach involved modifying the KG and MultiG classes to load triples and alignments where the entities and relations are represented as fuzzy sets. The model was updated to handle fuzzy sets and the trainer class along with the loss calculation was adjusted to incorporate fuzzy sets into the training process. However, integrating this approach into the JOIE framework posed several challenges. Firstly, defining and maintaining entities and relations as fuzzy sets added considerable complexity to the model. Secondly, the JOIE framework was not initially designed to handle the intricacies of fuzzy set theory, so this required extensive modifications and led to compatibility issues between the approach and the framework. The final challenge was computational overhead, resulting in slower processing times. Due to these challenges, we decided to proceed with the second approach.

4.4.2 Fuzzy Membership Function (Approach 2)

In this section, we will discuss the second approach we took for our fuzzy set embedding model to address the limitations and challenges identified in the first approach. This approach involved assigning a fuzzy membership function to the entities to help capture the degree to which the entity belongs to a particular concept. This approach provided several advantages such as degree of belonging, semantic similarity, and enhanced reasoning and inference. Assigning fuzzy membership functions to the entities enabled us to capture the degree to which the entity belonged to a particular concept. Unlike binary classification, fuzzy membership enables a more flexible and accurate representation of an entity belonging to multiple categories. For the fuzzy set embedding score calculation for this approach, we decided to use element-wise multiplication of all three components (entity, relation, target) followed by a summation. This interprets the score as a fuzzy membership value.

4.4.3 Cross-view model

Instance view- Entities: John('e1'), Siya('e2'), Peter('e3'); Relations- 'r1': is-a Ontology-view: Concepts: 'c1'(persons), 'c2'(artists) Relations - 'r2': subclass-of

So, in our fuzzy model, the membership of an entity 'e' in concept 'c' can be denoted as

$$\mu(e_i, c_j) = \sigma(e_i.c_j) \tag{8}$$

Here '.' denotes the dot product.

For our cross view model, we want to bridge the embeddings of instance-view entities with ontology-view concepts.

Cross view Association model: We can model the cross-view association in our fuzzy model using fuzzy intersection theory. The model can be defined as follows:

$$\mu_{cv}(e_i, c_j) = min(\mu(e_i, c_j), \mu(r_k, c_j)) \tag{9}$$

where r_k is the relation linking e_i to c_j

Cross-view loss function: To train our model, we can use a loss function that minimizes the loss between the predicted memberships and the actual memberships.

$$L = \sum_{e_i, c_j} (\mu_{cv}(e_i, c_j) - y_{ij})^2 \tag{10}$$

y_{ij} represents the ground truth membership value that is, if e_i belongs to c_j then set $y_{ij} = 1$ otherwise set it to 0.

This approach however assumes that if we want to correctly model the interactions between the entities and the concepts, it's pivotal that their embeddings exist in a common or compatible space. If the embeddings are not in the common space then we need to perform extra steps like transformation or projection, which we are not covering here.

4.4.4 Intra-View model

In the instance-view, we model the relationships between the entities. Let's denote the embeddings of the entities e_i and e_j as e_i and e_j and r_k represents the embedding of the intra-view relation.

Now, the degree to which the relation r_k is held between e_i and e_j can be computed by the fuzzy membership function that is,

$$\mu(e_i, e_j, r_k) = \sigma(e_i.r_k.e_j) \tag{11}$$

Again, for the ontology view, we consider associative or hierarchical relationships between the concepts. (same as in instance-view)

$$\mu(c_i, c_j, r_m) = \sigma(c_i.r_m.c_j) \tag{12}$$

and the loss can be obtained using via modelling combined loss from both the views.

$$L = \sum_{e_i, e_j, r_k} (\mu(e_i, e_j, r_k) - y_{ijk})^2 + \sum_{c_i, c_j, r_m} (\mu(c_i, c_j, r_m) - y_{ijm})^2 \tag{13}$$

5 Experiments

In this section, we perform evaluation of the proposed version of the JOIE model to the fuzzy set embedding space and compare it with the variants of the JOIE model. We perform the evaluation of the triple completion task.

5.1 Datasets

Most of the available datasets have a one view setting, either the instance view or the ontology view settings. In order to utilize the two-view setting we utilize the dataset DB111K-174. It has been extracted from [14], and was prepared from [13]. Table 1 provides the dataset splits for both the views. This dataset incorporates both factual triples and hierarchical relationships. The dataset is already split into both instances and concepts that are used to test JOIE's ability to jointly embed these instances and concepts. The dataset is split into training, validation and test sets with approximately 85%, 5% and 10%.

5.2 Models

For the project, we chose to train four models: two utilizing the JOIE framework with TransE embeddings, and two incorporating the fuzzy set embeddings with the JOIE framework. The details of each model is mentioned below.

- JOIE-TransE-CT: This model uses the TransE embedding method with the cross-view transformation as the bridge between the instance view and ontology. Here entity embeddings are of the dimension 300 and concept embeddings are of the dimension 100.
- JOIE-FuzzySet-CT: This model uses the proposed fuzzy set method to form the embeddings along with the cross-view transformation as the bridge between the instance view and ontology view. The entities have the dimension 300 and concepts have the dimension 100.

Table 1: Dataset details. Dataset from [13]

DB111K-174		
Instance Graph $\mathcal{G}_I$		
Number of Entities	Number of Relations	Number of Triples
111,762	305	863,643
Ontology Graph $\mathcal{G}_O$		
Number of Concepts	Number of Meta-relations	Number of Triples
174	20	763
Type Links $\mathcal{S}$		
99,748		

- JOIE-TransE-CG: This model uses the TransE embedding method with the cross-view grouping as the bridge between the instance view and ontology view. Here entity and concept embeddings are of the same dimensions 200.
- JOIE-FuzzySet-CG: This model uses the proposed fuzzy set method to form the embeddings as the bridge between the instance view and ontology view. Here entity and concept embeddings are of the same dimensions 100.

5.3 Knowledge Graph Triple Completion

In a triple completion task we aim to construct a missing fact in a knowledge graph structure (**h, r, ?t**). Given a triple $(h^{(O)}, r^{(O)}, t^{(O)}) \in \mathcal{G}_O$, we calculate the plausibility score for every tail entity $t^{'(O)}$ in the test candidate set. Then we rank all the scores and calculate the mean reciprocal rank (MRR).To indicate triple completion performance, the higher the MRR value the better. For the scope of this project we carry out this task on the ontology-view knowledge graph.

For the hyperparameter setting for this task, the margin γ^{G_I} and γ^{G_O} was set to 0.5 and the hyperparameter α values were as follows, α_1 =2.5, α_2 =1.0. All the models were trained for 120 epochs each.

We compare the JOIE fuzzy set model variants with the JOIE TransE model variants. The results of the triple completion task are shown in Table 2.

Results As shown in Table 2, we can observe that the proposed JOIE fuzzy set embedding models underperform as compared to the two JOIE TransE embedding models.

As we can see from the results that the fuzzy model didn't do well as compared to TransE, leading to contradicting our assumption that fuzzy modelling will be able to capture more nuanced relationships. However, this is possible because of different reasons which are the steps for future exploration. Though, we trained both the models i.e., JOIE transE and our fuzzy model for same number of epochs but it is very likely that fuzzy model hasn't converged yet. The reason being, introducing fuzzy concept will in return introduce more complexity compared to TransE, potentially making it harder to train with the same amount of data or train time. Furthermore, fuzzy set models are often super-sensitive to hyperparameters like membership function, which require careful fine-tuning. We can experiment with different ways of assigning memberships like triangular memberships, Gaussian memberships and the like to contrast the two models. Additionally, it is also very likely that the chosen dimensions are not optimal for fuzzy model in contrast to transE and hence our model didn't perform well. Having stated all these reasons, we aim to investigate more into our results by incorporating the stated enhancements as our future direction to the project.

5.4 Additional Experiments

In an attempt to extend the evaluation section we also carried out experiments for the entity typing task. The entity typing task involves predicting the association of concepts to entities. In case of using a cross-view transformation bridge, we first take the projection of the entity embedding in the

Table 2: Results on Triple Completion Task

Graph	$\mathcal{G}_O$ KG Completion
Metric	MRR
JOIE-TransE-CG	0.3326
JOIE-TransE-CT	0.3493
JOIE-FuzzySet-CG	0.0494
JOIE-FuzzySet-CT	0.0721

Table 3: Results on Entity Typing Task

Metric	MRR
JOIE-TransE-CG	0.0303
JOIE-TransE-CT	0.8091

concept embedding space and then rank all the candidate concepts for that particular entity in order to calculate the MRR. The cross-view links of the dataset are separated into training and testing sets with the ratio of 60% to 40%.

We were able to perform tests on only two models, the JOIE TransE models. Each model was trained for 120 epochs and the results are shown in Table 3.

Results As we can see from Table 3, we were able to obtain results that are close to those shown in [13], for the JOIE-TransE-CT model. For the JOIE TransE CG model, we were expecting the model to perform better, as we were aiming to place the entities and concepts in a higher dimensional space to get a better representation but the results were not up to the mark. We carried out this experiment with the dimensions of both KG views set to 200 here. Whereas, in the paper [13] it was set to 100.

6 Conclusion and Future Scope

The future scope of this research project includes several key areas for further exploration and enhancement. First, we can incorporate a hybrid model approach by combining the concepts of a fuzzy membership function and representing entities and relations as fuzzy sets. This approach will help leverage the strengths of both methodologies by enhancing the precision and granularity of associations within the framework. Second, we can develop an advanced loss computation technique tailored for fuzzy set embeddings. This approach will help optimize the learning process and improve the performance of the embeddings. Third, another potential direction is to investigate more sophisticated methods for combining fuzzy set representations with embedding models, such as Relational Graph Convolutional Networks (R-GCNs). Exploring how R-GCNs can effectively capture the complex relational structures within knowledge graphs while accommodating fuzzy set semantics could lead to more robust and interpretable embeddings. Lastly, as a future extension, we plan to integrate this model approach with the FUSE model, which is a related project of a general representation learning framework that was designed to generate fuzzy set embeddings. Integrating our model with FUSE will enable the application of proper loss function computation specifically designed for fuzzy set embeddings.

In conclusion, this report introduces a novel approach to knowledge graph representation learning by integrating fuzzy set theory into the JOIE framework. By extending jointly embedded instance-view and ontology-view knowledge graphs into a fuzzy set theory embedding space. Through experiments comparing fuzzy set embedding models with traditional KG embedding models, valuable insights into the performance and challenges of the proposed approach were gained. Despite encountering obstacles such as model complexity and computational overhead, the results provide a foundation for future exploration.

References

[1] Wang, Zhen, et al. "Knowledge graph embedding by translating on hyperplanes." Proceedings of the AAAI conference on artificial intelligence. Vol. 28. No. 1. 2014.

[2] Bordes, Antoine, Jason Weston, and Nicolas Usunier. "Open question answering with weakly supervised embedding models." Machine Learning and Knowledge Discovery in Databases: European Conference, ECML PKDD 2014, Nancy, France, September 15-19, 2014. Proceedings, Part I 14. Springer Berlin Heidelberg, 2014.

[3] He, He, et al. "Learning symmetric collaborative dialogue agents with dynamic knowledge graph embeddings." arXiv preprint arXiv:1704.07130 (2017).

[4] Chen, Muhao, et al. "Multilingual knowledge graph embeddings for cross-lingual knowledge alignment." arXiv preprint arXiv:1611.03954 (2016).

[5] Fang, Yuan, et al. "Object detection meets knowledge graphs." International Joint Conferences on Artificial Intelligence, 2017.

[6] Bordes, Antoine, et al. "Translating embeddings for modelling multi-relational data." Advances in neural information processing systems 26 (2013).

[7] Yang, Bishan, et al. "Embedding entities and relations for learning and inference in knowledge bases." arXiv preprint arXiv:1412.6575 (2014).

[8] Nickel, Maximilian, Lorenzo Rosasco, and Tomaso Poggio. "Holographic embeddings of knowledge graphs." Proceedings of the AAAI conference on artificial intelligence. Vol. 30. No. 1. 2016.

[9] Chen, Xuelu, et al. "Embedding uncertain knowledge graphs." Proceedings of the AAAI conference on artificial intelligence. Vol. 33. No. 01. 2019.

[10] Jain, Nitisha, et al. "Improving knowledge graph embeddings with ontological reasoning." International Semantic Web Conference. Cham: Springer International Publishing, 2021.

[11] Chen, Muhao, et al. "Multilingual knowledge graph embeddings for cross-lingual knowledge alignment." arXiv preprint arXiv:1611.03954 (2016).

[12] Iyer, Roshni G., et al. "Dual-geometric space embedding model for two-view knowledge graphs." Proceedings of the 28th ACM SIGKDD Conference on Knowledge Discovery and Data Mining. 2022.

[13] Hao, Junheng, et al. "Universal representation learning of knowledge bases by jointly embedding instances and ontological concepts." Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining. 2019.

[14] Lehmann, Jens et al. “DBpedia - A large-scale, multilingual knowledge base extracted from Wikipedia.” Semantic Web 6 (2015): 167-195.

Task Distribution

Please refer to Table 4 below for the group task distribution.

Table 4: Task Distribution

Task	People
1. Fuzzy Model Implementation	Kajal and Vani
2. Testing and Evaluation	Muskan and Dhakshina
3. Report	Dhakshina, Muskan, Vani and Kajal
4. Model Training	Dhakshina, Muskan, Vani and Kajal