Dynamic and Stochastic Rational Behavior

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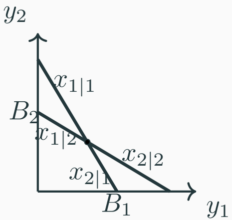
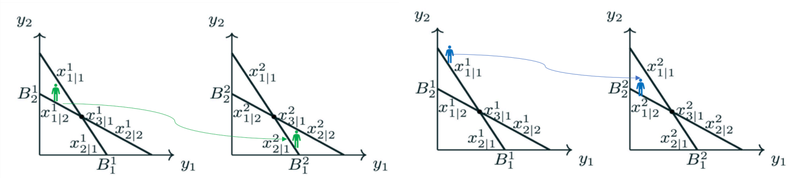
Dynamic Random Utility Model (DRUM)

- Dynamic Random Utility Model–DRUM:
  - Each decision maker (DM) randomly draws a utility function from a stochastic utility process $(u^t)_{t\in\mathcal{T}}$ and maximizes it subject to a menu $(B^t)_{t\in\mathcal{T}}$ in each period.
  - Unrestricted time correlation and cross-section heterogeneity in preferences.
- Contribution: RP characterization of DRUM for a panel of choices and menus. New tech: converts static into dynamic models & obtains dynamic axiomatizations from their static counterparts.
- Static utility maximization is under scrutiny [Choi et al., 2007, Echenique et al., 2011, Ahn et al., 2014, Dean and Martin, 2016].
  - Evidence of changing taste [Cherchye et al., 2017, Adams et al., 2015], evolving errors in valuation [Kurtz-David et al., 2019], time varying risk aversion [Guiso et al., 2018].
- Our results unify static utility maximization–time series–[Afriat, 1967] and random utility–cross-section or pool–[McFadden, 2005, McFadden and Richter, 1990]. In experimental data, we show DRUM succeeds while static rationality fails.

Today Focus on Demand

- **Simple-setup:** 2 goods, $T = 2$, and 2 budgets $B_j^{*t} = B_j^{*s} = B_j^*$, and $B_j^* \cap B_{j'}^* \neq \emptyset$.
- Patches are the **coarsest partition** of intersecting budgets in a period. WLG, dicretization.





Budget path B_2^1, B_1^2. **Choice path** $x^1_{1|2}, x^2_{2|1}$. $\rho(x^1_{1|2}, x^2_{1|1}) + \rho(x^1_{1|2}, x^2_{2|1}) + \rho(x^1_{2|2}, x^2_{2|1}) + \rho(x^1_{2|2}, x^2_{1|1}) = 1.$

Setup

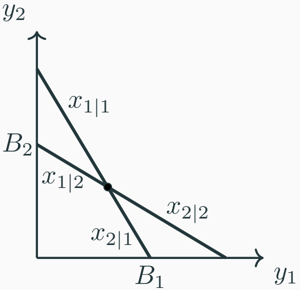
- Time: $\mathcal{T} = \{1, \cdots, T\}$, $T \geq 1$. Finite choice set: X^t, endowed with $>^t$.
- At $t \in \mathcal{T}$, DM faces menu $B^t_{j_t} \in 2^{X^t} \setminus \emptyset$, with $j_t \in \mathcal{J}^t = \{1, \cdots, J^t\}$ ($\mathbf{j} = \{j_t\}_{t \in \mathcal{T}}$). $\mathbf{J}$: **menu paths**.
- The i_t-th element of $j_t \in \mathcal{J}^t$ is $x^t_{i_t|j_t}$ with $i_t \in \mathcal{I}^t_j = \{1, \cdots, I^t_j\}$ ($\mathbf{i} = (i_t)_{t \in \mathcal{T}}$). $\mathbf{I_j}$: **choice paths.**
- Primitive: $\rho_j \in \Delta(\prod_{t \in \mathcal{T}} B^t_{j_t})$. $\rho_{\mathbf{j}}(x_{\mathbf{i}|\mathbf{j}}) \geq 0 \ \forall \ \mathbf{i} \in \mathbf{I_j}$, $\sum_{\mathbf{i} \in \mathbf{I_j}} \rho_{\mathbf{j}}(x_{\mathbf{i}|\mathbf{j}}) = 1$. $B^t_j = \{x^t_{i|j}\}_{i \in \mathcal{I}^t_j}$.
- U^t is the set of all injective, monotone on $>^t$ utility functions s.t. $u^t : X^t \to \mathbb{R}$ ($\mathcal{U} = \prod_{t \in \mathcal{T}} U^t$).
- Dynamic stochastic choice function $\rho = (\rho_{\mathbf{j}})_{\mathbf{j} \in \mathbf{J}}$. ρ, is consistent with DRUM if $\exists \ \mu \in \Delta(\mathcal{U})$:

$$\rho_{\mathbf{j}}(x_{\mathbf{i}|\mathbf{j}}) = \int \prod_{t \in \mathcal{T}} 1(argmax_{y \in B^t_{j_t}} u^t(y) = x^t_{i_t|j_t}) d\mu(u), \forall \mathbf{i} \in \mathbf{I_j}, \mathbf{j} \in \mathbf{J}.$$

- E.g., (i) CD: $u^t(y_1, y_2) = y_1^{\alpha_t} y_2^{(1-\alpha_t)}$ with $\alpha_t = \alpha_{t-1} + \epsilon_t$. (ii) ERA $u^t(y_1, y_2) = \pi_1 \frac{y_1^{1-\sigma_t}}{1-\sigma_t} + (1 - \pi_1) \frac{y_2^{1-\sigma_t}}{1-\sigma_t}$ with $\sigma_t \leq \sigma_{t-1}$. (iii) $u^t(x) = v(x) + \alpha'_t x$.

A Stochastic Revealed Preference Characterization of DRUM.

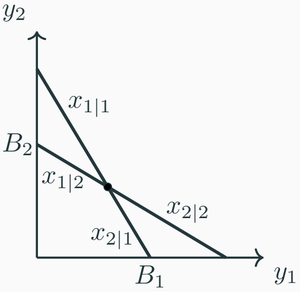
- DRUM is associated with a finite mixture over a finite number of preference profiles: $\mathbf{r} = \{r^1, \cdots, r^T\}$, r^t is a linear order on patches.
- Assume away intersection patches. Static behavior summarized in A^t:



A^t	$r^t_{1,1}$	$r^t_{1,2}$	$r^t_{2,2}$	$r^t_{2,1}$
Rational	yes	yes	yes	**no**
$x^t_{1\|1}$	1	1	-	-
$x^t_{2\|1}$	-	-	1	1
$x^t_{1\|2}$	1	-	-	1
$x^t_{2\|2}$	-	1	1	-

A Stochastic Revealed Preference Characterization of DRUM.

- Encode $\mathbf{r}$, as $a_{\mathbf{r}} = (\prod_{t\in\mathcal{T}} a_{r^t,i_t,j_t})_{\mathbf{i}\in\mathbf{I_j},\mathbf{j}\in\mathbf{J}}$. $a_{r^t,i_t,j_t} = 1 \iff x^{t*}_{i_t|j_t} = argmax_{y\in B^t_j} u^{r^t}(y)$. Matrix $A_{\mathcal{T}} = \otimes_{t\in\mathcal{T}} A^t$.
- Recall $\rho(x_{\mathbf{i}|\mathbf{j}}) = \rho(\{x^t_{i_t|j_t}\}_{t\in\mathcal{T}})$, let $\rho = (\rho_{\mathbf{i}|\mathbf{j}})_{\mathbf{i}\in\mathbf{I_j},\mathbf{j}\in\mathbf{J}}$.
- $A_{\mathcal{T}}\nu = (A^1 \otimes A^2)\nu = \rho$, $\nu \in \Delta^8$ (i.e., 9 demand profiles types, A has dim. 16×9).



	$\mathbf{r}_1 : r^1_{1,1}; r^2_{1,1}$	$\mathbf{r}_2 : r^1_{1,1}; r^2_{1,2}$	$\cdots$
$x_{1\|1}; x_{1\|1}$	1	1	
$x_{1\|1}; x_{2\|1}$	-	-	
$x_{1\|1}; x_{1\|2}$	1	-	
$x_{1\|1}; x_{2\|2}$	-	1	
$\vdots$			

$$\begin{pmatrix} 1 & 1 & \\ 0 & 0 & \\ 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} \rho(x^1_{1|1}; x^2_{1|1}) \\ \rho(x^1_{1|1}; x^2_{2|1}) \\ \rho(x^1_{1|1}; x^2_{1|2}) \\ \rho(x^1_{1|1}; x^2_{2|2}) \end{pmatrix}$$

First Result: Axiom of Dynamic Stochastic Revealed Preference.

Definition

(Axiom of Dynamic Stochastic Revealed Preference, ADSRP) ρ satisfies ADSRP if for every finite sequence of pairs of budget and choice paths (including repetitions), k, $\{(i_k, j_k)\}$ such that $j_k \in J$ and $i_k \in I_{j_k}$

$$\sum_k \rho(x_{i_k|j_k}) \leq \max_{r \in \mathcal{R}} \sum_k a_{r,i_k,j_k}.$$

Theorem

The following are equivalent:

1. *ρ is consistent with DRUM.*
2. *There exists $\nu \in \Delta^{|\mathcal{R}|-1}$ such that $\rho = A_T \nu$.*
3. *There exists $\nu \in \mathbb{R}_+^{|\mathcal{R}|}$ such that $\rho = A_T \nu$.*
4. *ρ satisfies the ADSRP.*

Computational Aspects of Testing

- Theorem 1.(3) $A_T\nu = \rho$ for $v \in \mathbb{R}_+^{|\mathcal{R}|}$ is straightforward to test using tools in [Kitamura and Stoye, 2018].
- **Computational bottleneck:** Computing A_T is costly [Smeulders et al., 2021].

Lemma

Let A^t be the matrix with entries $a^t_{r_t,i_t,j_t} = 1 \iff argmax_{y \in B^t_j} u^{r_t}(y) = x^t_{i_t|j_t}$, then $A_T = A^1 \otimes \cdots \otimes A^T$.

- If we have the same number of budgets, all intersecting $A_t = A_s$, then $A_T = A_1^{\otimes_{t=1}^{\mathcal{T}}}$ (Kronecker power).
- Modular structure: computational savings. Kronecker product structure allows a linear inequality characterization.

Simple-setup necessary conditions: Stability

- (**Stability**) In the simple setup ρ satisfies stability if (i) $\rho(x^1_{l|k}; B^2_j) = \sum_{i\in\{1,2\}} \rho_{l|k;i|j}$ does not depend on B^2_j, (ii) $\rho(x^2_{i|j}; B^1_k) = \sum_{l\in\{1,2\}} \rho_{l|k;i|j}$ does not depend on B^1_k.
- If ρ is DRUM $\exists\ \nu \in \Delta^9$ s.t. $A_T\nu = \rho$. Verify:

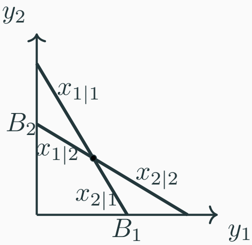
$t=1; t=2$	$x_{1\|1}$	$x_{2\|1}$	$x_{1\|2}$	$x_{2\|2}$
$x_{1\|1}$	$\rho_{1\|1;1\|1} = \nu_1+\nu_2+\nu_4+\nu_5$	$\rho_{1\|1;2\|1} = \nu_3+\nu_6$	$\rho_{1\|1;1\|2} = \nu_1+\nu_4$	$\rho_{1\|1;2\|2} = \nu_2+\nu_3+\nu_5+\nu_6$
$x_{2\|1}$	$\rho_{2\|1;1\|1} = \nu_7+\nu_8$	$\rho_{1\|1;2\|1} = \nu_9$	$\rho_{2\|1;1\|2} = \nu_7$	$\rho_{1\|1;2\|2} = \nu_8+\nu_9$
$x_{1\|2}$	$\rho_{1\|2;1\|1} = \nu_1+\nu_2$	$\rho_{1\|2;2\|1} = \nu_3$	$\rho_{1\|2;1\|2} = \nu_1$	$\rho_{1\|2;2\|2} = \nu_2+\nu_3$
$x_{2\|2}$	$\rho_{2\|2;1\|1} = \nu_4+\nu_5+\nu_7+\nu_8$	$\rho_{2\|2;2\|1} = \nu_6+\nu_9$	$\rho_{2\|2;1\|2} = \nu_4+\nu_7$	$\rho_{2\|2;2\|2} = \nu_5+\nu_6+\nu_8+\nu_9$

Simple-setup necessary conditions: Monotonicity

- Let $x^t_{i_t|j_t} >^t x^t_{l_t|k_t}$ if $x^{t*}_{i_t|j_t} > x^{t*}_{l_t|k_t}$.
- (**Monotonicity**) ρ satisfies monotonicity,

$$\begin{aligned} &\textbf{(i)} \text{ if } x^1_{i'|j'} >^1 x^1_{i|j} \text{ then } \rho(x^1_{i|j}, x^2_{l|k}) \leq \rho(x^1_{i'|j'}, x^2_{l|k}), \\ &\textbf{(ii)} \text{ if } x^2_{l'|k'} >^2 x^2_{l|k} \text{ then } \rho(x^1_{i|j}, x^2_{l|k}) \leq \rho(x^1_{i|j}, x^2_{l'|k'}). \end{aligned}$$

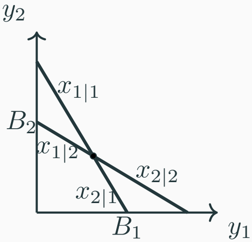
- For $\mathcal{T} = \{t\}$, Monotonicity is captured by a matrix H^t, s.t. $H^t\rho \geq 0$.



$t=1; t=2$	$x_{1\|1}$	$x_{2\|1}$	$x_{1\|2}$	$x_{2\|2}$
$x_{1\|1}$	$\underline{\rho_{1\|1;1\|1} = \nu_1 + \nu_2 + \nu_4 + \nu_5}$	$\rho_{1\|1;2\|1} = \nu_3 + \nu_6$	$\underline{\rho_{1\|1;1\|2} = \nu_1 + \nu_4}$	$\rho_{1\|1;2\|2} = \nu_2 + \nu_3 + \nu_5 + \nu_6$
$x_{2\|1}$	$\rho_{2\|1;1\|1} = \nu_7 + \nu_8$	$\rho_{2\|1;2\|1} = \nu_9$	$\rho_{2\|1;1\|2} = \nu_7$	$\rho_{1\|1;2\|2} = \nu_8 + \nu_9$
$x_{1\|2}$	$\rho_{1\|2;1\|1} = \nu_1 + \nu_2$	$\rho_{1\|2;2\|1} = \nu_3$	$\rho_{1\|2;1\|2} = \nu_1$	$\rho_{1\|2;2\|2} = \nu_2 + \nu_3$
$x_{2\|2}$	$\rho_{2\|2;1\|1} = \nu_4 + \nu_5 + \nu_7 + \nu_8$	$\rho_{2\|2;2\|1} = \nu_6 + \nu_9$	$\rho_{2\|2;1\|2} = \nu_4 + \nu_7$	$\rho_{2\|2;2\|2} = \nu_5 + \nu_6 + \nu_8 + \nu_9$

Simple-setup: D-Monotonicity

- D-monotonicity :Improving a dominant choice path has a higher impact on the probability of choosing a choice path than improving a dominated path.
  - $D^1(x^1_{i'|j'})[\rho(x^1_{i|j}, x^2_{l|k})] = \rho(x^1_{i'|j'}, x^2_{l|k}) - \rho(x^1_{i|j}, x^2_{l|k}) \geq 0$ if $x^1_{i'|j'} >^1 x^1_{i|j}$.
  - $D^2(x_{l'|k'})D^1(x^1_{i'|j'})[\rho(x^1_{i|j}, x^2_{l|k})] =$ $[\rho(x^1_{i'|j'}, x^2_{l'|k'}) - \rho(x^1_{i|j}, x^2_{l'|k'})] - [\rho(x^1_{i'|j'}, x^2_{l|k}) - \rho(x^1_{i|j}, x^2_{l|k})] \geq 0$ if $x^2_{l'|k'} >^2 x^2_{l|k}$ and $x^1_{i'|j'} >^1 x^1_{i|j}$.
- D-monotonicity iff $\otimes_{t\in\mathcal{T}} H^t \rho \geq 0$ (Kronecker product of Static Conditions!).
- E.g., (i) $\rho_{1|1;1|2} - \rho_{1|1;1|1} \leq \rho_{1|2;1|2} - \rho_{1|2;1|1}$; (ii) $\rho_{2|2;1|2} - \rho_{2|2;1|1} \leq \rho_{2|1;1|2} - \rho_{2|1;1|1}$



$t=1; t=2$	$x_{1\|1}$	$x_{2\|1}$	$x_{1\|2}$	$x_{2\|2}$
$x_{1\|1}$	$\rho_{1\|1;1\|1} = \nu_1+\nu_2+\nu_4+\nu_5$	$\rho_{1\|1;2\|1} = \nu_3+\nu_6$	$\rho_{1\|1;1\|2} = \nu_1+\nu_4$	$\rho_{1\|1;2\|2} = \nu_2+\nu_3+\nu_5+\nu_6$
$x_{2\|1}$	$\rho_{2\|1;1\|1} = \nu_7+\nu_8$	$\rho_{1\|1;2\|1} = \nu_9$	$\rho_{2\|1;1\|2} = \nu_7$	$\rho_{1\|1;2\|2} = \nu_8+\nu_9$
$x_{1\|2}$	$\rho_{1\|2;1\|1} = \nu_1+\nu_2$	$\rho_{1\|2;2\|1} = \nu_3$	$\rho_{1\|2;1\|2} = \nu_1$	$\rho_{1\|2;2\|2} = \nu_2+\nu_3$
$x_{2\|2}$	$\rho_{2\|2;1\|1} = \nu_4+\nu_5+\nu_7+\nu_8$	$\rho_{2\|2;2\|1} = \nu_6+\nu_9$	$\rho_{2\|2;1\|2} = \nu_4+\nu_7$	$\rho_{2\|2;2\|2} = \nu_5+\nu_6+\nu_8+\nu_9$

Simple-setup: A simpler characterization.

Theorem

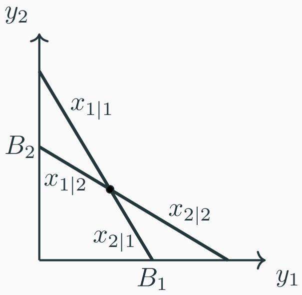
For the simple-setup, the following are equivalent:

1. ρ *is consistent with DRUM*
2. ρ *satisfies (i) stability, (ii) D-monotonicity.*
3. ρ *satisfies stability and* $\otimes_{t\in\mathcal{T}} H^t \rho \geq 0$.

- This thm provides an explicit H-representation of DRUM. D-mon is not only monotonicity on each time period, it's more!
- D-mon is a dynamic version of the Weak Axiom of Stochastic Revealed Preference for the static case [Hoderlein and Stoye, 2014] and a stochastic version of the Weak Axiom of Revealed Preference.
- **In this paper, we show the intution of $1 \iff 3$ holds generally in a general abstract domain.**

Simple-setup: Counterexample of DRUM ρ

- $\rho_1^*(x^1_{2|1}, B^2_1) = \frac{1}{2}$, $\rho_1^*(x^1_{1|2}, B^2_1) = \frac{1}{2}$; and $\rho_1^*(x^1_{2|1}, B^2_2) = \frac{1}{3}$ and $\rho_1^*(x^1_{1|1}, B^2_2) = \frac{2}{3}$.
- Static case with 2 goods: RUM rationalizability is equivalent to $\rho_1^*(x^1_{2|1}, B^2_j) \leq \rho_1^*(x^1_{1|1}, B^2_j)$.
- Yet, stability, monotonicity, and IM are violated.



$t=1; t=2$	$x_{1\|1}$	$x_{2\|1}$	$x_{1\|2}$	$x_{2\|2}$
$x_{1\|1}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0
$x_{2\|1}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$x_{1\|2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	0
$x_{2\|2}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Relation with Samuelson-Afriat: Static Utility Maximization.

- We say DRUM, is **constant** when $u_s = u_t$ almost surely $\forall\ t, s \in \mathcal{T}$.
- (Strong Axiom of Revealed Patch Dominance, SARPD) For $\mathbf{j} \in \mathbf{J}$, $p_{\mathbf{j}}$ satisfies SARPD if $\rho_{\mathbf{j}}(\{x^t_{i_t|j_t}\}_{t\in\mathcal{T}}) = 0$, when $\{x^t_{i_t|j_t}\}_{t\in\mathcal{T}}$is such s.t. $x^{t*}_{i_t|j_t} \in x^t_{i_t|j_t}$,

$$p^{t\prime}_{j_t} x^{t*}_{i_t|j_t} \geq p^{t\prime}_{j_t} x^{s*}_{i_s|j_s}, p^{s\prime}_{j} x^{s*}_{i_s|j_s} \geq p^{s\prime}_{j_s} x^{r*}_{i_r|j_r}, \cdots, p^{r\prime}_{j_r} x^{r*}_{i_r|j_r} \geq p^{r\prime}_{j_r} x^{k*}_{i_k|j_k}, \text{and } p^{k\prime}_{j_k} x^{k*}_{i_k|j_k} \geq p^{k\prime}_{j_k} x^{t*}_{i_t|j_t}.$$

- **Proposition:** If ρ is consistent with a constant DRUM, then $p_{\mathbf{j}}$ satisfies SARPD for all $\mathbf{j} \in \mathbf{J}$.
- DRUM does not impose SARPD, but **it bounds** cycles: $\rho(x^1_{1|2}, x^1_{2|1}) \leq \rho(x^1_{1|1}, x^2_{2|2})$.

Relation with McFadden-Richter: Random Utility Model.

- Define: (marginal) $\rho^m_{t,\mathbf{j}}(x_{i_t|j_t}) = \sum_{\tau\in\mathcal{T}\setminus\{t\}} \sum_{i\in\mathcal{I}^\tau_{j_\tau}} \rho(x_{\mathbf{i}|\mathbf{j}})$, (conditional)$\rho^c_{t,\mathbf{j}}(x_{i_t|j_t}) = \frac{\rho(x_{\mathbf{i}|\mathbf{j}})}{\sum_{i\in\mathcal{I}^t_{j_t}} \rho(x_{\mathbf{i}|\mathbf{j}})}$, (slice) $\rho^s_t(x_{i_t,j_t}) = \sum_{\mathbf{j}\in\mathbf{J}} \rho^m_{t,\mathbf{j}}(x_{i_t|j_t})F(\mathbf{j}|j_t)$, where $F(\mathbf{j}|j_t)$ is the conditional probabiilty of budget path.
- **Proposition:** If ρ is consistent with DRUM, then $\rho^m_{t,\mathbf{j}}$, $\rho^c_{t,\mathbf{j}}$ and ρ^s_t are rationalized by RUM for any t and any $\mathbf{j}\in\mathbf{J}$.
- RUM has "less empirical content" than DRUM by ignoring the time dimension.

Relation with the Literature.

- DRUM was introduced by [Strzalecki, 2021], but the characterization was an open question. The problem is hard! We solved it with a new mathematical technique. Also, we study more domains such as demand.
- [Chambers et al., 2021, Li, 2021] characterize models of "correlated choice" in a discrete choice set environment with menu variation (i.e., no monotonicity, and choice sets are assumed to be nested). This does not work in demand analysis. We provide a characterization for their setup in full generality. Chambers et. al assume $T = 2$ and assume additional structure in one period, and Li assumes $|X^t| \leq 3$.

Conclusions.

- DRUM is a powerful framework for choice analysis: modular/compositional, more informative than McFadden-Richter, yet more flexible than Samuelson-Afriat.
- DRUM synthesizes the static utility maximization model and the random utility framework.
- DRUM, as opposed to the static utility maximization, does not require acyclicity. Yet it bounds revealed demand cycles. It needs panel datasets.
- RUM can be satisfied in “pooled” data, but DRUM can fail. This means RUM is missing some empirical bite from stochastic rationality.

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Simple-setup: A simpler characterization (proof (i)).

- $\mathcal{T} = \{t\}$. $\rho = A^t \nu = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} (\nu_1 + v_2) \\ \nu_3 \\ \nu_1 \\ (\nu_2 + \nu_3) \end{pmatrix} \implies \rho(x^t_{1|1}) \geq \rho(x^t_{1|2})$.
- $\rho = A_T \nu = A^1 \otimes A^2 \nu = \begin{pmatrix} A^1 & A^1 & 0 \\ 0 & 0 & A^1 \\ A^1 & 0 & 0 \\ 0 & A^1 & A^1 \end{pmatrix} \begin{pmatrix} \nu^1_1 \\ \nu^1_2 \\ \nu^1_3 \end{pmatrix} = \begin{pmatrix} A^1(\nu^1_1 + v^1_2) \\ A^1 \nu^1_3 \\ A^1 \nu^1_1 \\ A^1(\nu^1_2 + \nu^1_3) \end{pmatrix} \implies$
- $\implies [\rho(x^1_{1|1}, x^2_{1|1}) - \rho(x^1_{1|2}, x^2_{1|1})] - [\rho(x^1_{1|1}, x^2_{1|2}) - \rho(x^1_{1|2}, x^2_{1|2})] \geq 0.$

Simple-setup: A simpler characterization (proof (ii)).

- For the simple setup $\mathcal{T} = \{t\}$, ρ is consistent with RUM iff $H^t\rho \geq 0$ iff ρ satisfies monotonicity.
- $H^t = \begin{array}{c} \begin{array}{cccc} x^t_{1|1} & x^t_{2|1} & x^t_{1|2} & x^t_{2|2} \end{array} \\ \begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$; $H^t\rho \geq 0; \rho(x^t_{1|1}) \geq \rho(x^t_{1|2})$ + nonnegativity of ρ.
- We apply our general mathematical result that says that in this case, ρ is consistent with DRUM iff $(\otimes_{t\in\mathcal{T}} H^t)\rho \geq 0$.
- ρ is consistent with $D-$monotonicity iff $(\otimes_{t\in\mathcal{T}} H^t)\rho \geq 0$.

Dynamic Random Augmented Utility Model, DRAUM

- Consider $X^* = X \times \mathbb{R}_-$ corresponding to consumption and the expenditure on it. Let F_{X^*} be the set of measurable sets on X^* ($F^T_{X^*} = \prod_{t\in\mathcal{T}} F_X$). Let $\mathbf{j}$ index a price path $(p^1_j, p^2_j, \cdots p^t_j)$. $\rho_{X^*,\mathbf{j}}$is a measure on $F_{X^{*T}}$
- $\rho_{X^*,j}$, is rationalized by a probability measure η over $\mathcal{V}$ ($v = (v^t)_{t\in\mathcal{T}} \in \mathcal{V}$,) if

$$\rho_{X^*,j}(\{O^t\}_{t\in\mathcal{T}}) = \int \prod_{t\in\mathcal{T}} 1(argmax_{y\in\mathbb{R}^K_+} v^t(y, -p^{t\prime}_j y) \in O^t) d\eta(v), \forall \mathbf{j}, \{O^t\}_{t\in\mathcal{T}}.$$

- Consider the random augmented utility:
  $\mathbf{v}^t(x, -p^{t\prime}x) = u(x) + \delta E[V(\mathbf{y}_t + (1 + r_t)s_{t-1}(\mathbf{y}) - p^{t\prime}x)|I_t]$, randomness comes from the information I_t.
- If we assume this information is i.i.d. and independent of prices then this is a special case of DRAUM.
- All our characterization works for DRAUM for transformed patches. See [Deb et al., 2021].

General Axiomatization of DRUM: given RUM

- Define A^{t*} as the submatrix of A^t. Take A^t and delete the last row of each menu.
- We say A^t generates a **unique RUM** when the system $\rho = A^t \nu$ has a unique solution. Unique DRUM defnd. analogously.
- $C^{\otimes k} = \otimes_{j=1}^{k} C$. $\phi^t = \frac{1}{|rows|} \sum_{row} H^t_{row}$, $\gamma_k^{\phi^t} = \frac{1}{k} \sum_{j=1}^{k} \phi^{t,\otimes(j-1)} \otimes I^t \otimes \phi^{t,\otimes(k-j)}$, $\Gamma_k^{\phi} = I^1 \otimes (\otimes_{t \in \mathcal{T} \setminus \{1\}} \gamma_{k_t}^{\phi^t})$.

Theorem

Assume that A^{t*} is full row rank for all $t \in \mathcal{T}$. Then ρ is consistent with DRUM if and only if ρ is stable and

$$\rho \in \bigcap_{k_1, \cdots, k_T} \left\{ \Gamma_k^{\phi^{*\prime}} z : \left(\otimes H^{t, \otimes_{k_t}} \right) z \right\}.$$

Moreover, ρ is consistent with unique DRUM if and only if ρ is table and $(\otimes_{t \in \mathcal{H}} H^t) \rho \geq 0$.

- Given, H^t $\forall t$, we get linear and closed-form necessary and sufficient conditions for DRUM with arbitrary precision.