STELLAR ASTROPHYSICS CENTRE

Science cases: The Aarhus Red Giants Challenge

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This document describes the activities carried out in the framework of the Aarhus red giants challenge workshop. It include all relevant science cases, details on the input physics utilised, the models compared, the file format used, and the naming convention of the files. New participants interested in taking part in the workshops and comparisons should compute the required models and send them to the organisers.

1 Definition of fundamental constants

The following constants are defined and used throughout the science cases:

Solar mass $\mathrm{M}_{\odot} = 1.9890 \times 10^{33}$ (gr).

Solar radius $\mathrm{R}_{\odot} = 6.95508 \times 10^{10}$ (cm).

Solar luminosity $\mathrm{L}_{\odot} = 3.846 \times 10^{33}$ (erg/s).

Gravitational constant G= 6.67232×10^{-8} ($\mathrm{cm}^3/\mathrm{gs}^2$).

Models requested at a fixed radius throughout the exercises should comply with a minimum acceptable convergence, defined with the following example for a model of $a\times\mathrm{M}_{\odot}$ at $b\times\mathrm{R}_{\odot}$:

$$\Delta_{\mathrm{convergence}} = \left| 1.0 - \frac{G_{\mathtt{model}}\, M_{\mathtt{model}}/R^3_{\mathtt{model}}}{G\,(\alpha\times\mathrm{M}_{\odot})/(\beta\times\mathrm{R}_{\odot})^3} \right| \leq 2\times10^{-4}\,, \tag{1}$$

2 Solar radius calibrated models

The first set of tracks and models are computed with a fixed and predefined combination of input physics, and performing a solar radius calibration as described below.

2.1 Input Physics

Nuclear reactions NACRE nuclear reactions in its original version (Angulo et al. 1999), no update to any rate.

Equation of state OPAL EOS in its 2005 version (Rogers et al. 1996).

Opacities OPAL Opacities (Rogers et al. 1996; Iglesias & Rogers 1996) and Potekhin conductive opacities (Cassisi et al. 2007).

Solar abundances the Grevesse & Noels (1993) solar mixture.

Convective efficiency Mixing length theory for convective treatment.

Macrophysics No overshooting, no mass loss, no microscopic diffusion.

Boundary condition Eddington T-tau relation for the atmosphere.

Evolution the evolutionary sequence should be started from the Zero Age Main Sequence (ZAMS).

2.2 Solar Radius Calibration

Using the above set of input physics, perform a solar calibration fixing the initial hydrogen abundance to X=0.7. The parameters to match are solar radius of 6.95508×10^{10} cm at 4.57 Gyr. This will result in a calibrated mixing length parameter for this solar model, to be used in the calculation of the models for this science case. Note that since there is not diffusion and the quantity Z/X is fixed by the solar mixture to $Z/X = 0.0245$, the composition is completely defined ($X = 0.7$, $Y = 0.28285$, $Z = 0.01715$) and therefore the current solar luminosity is NOT a quantity to be matched in the calibration. Provide the solar interior model and the corresponding evolutionary sequence in the format described in appendix A, named `'code_100_R01_suncal.fgong'` and `'code_suncal.track'`. Use the folder named `'suncal'` in the dropbox.

2.3 Science cases

Using the input physics defined in section 2.1, the obtained solar calibrated $\alpha_{\rm MLT}$ as described in section 2.2, **and changing the initial composition (see below)**, provide results for the following models.

Initial abundances $Y = 0.28$, $Z = 0.02$.

1.0 $\mathbf{M}_\odot$ model in the main-sequence phase at 1 $R_\odot$.

1.0 $\mathbf{M}_\odot$ models in the turn-off phase defined as $X_{\rm c} = 10^{-5}$.

1.0 $\mathbf{M}_\odot$ models in the RGB phase at 7 $R_\odot$ and 12 $R_\odot$.

1.0 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{\rm He}) = 2.0$.

1.5 $M_\odot$ models in the RGB phase at 7 $R_\odot$ and 12 $R_\odot$.

1.5 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{\rm He}) = 2.0$.

2.0 $M_\odot$ model in the RGB phase at 10 $R_\odot$.

2.0 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{\rm He}) = 2.0$.

2.5 $M_\odot$ model in the RGB phase at 10 $R_\odot$.

2.5 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{\rm He}) = 0.0$.

All tracks evolution until core-helium exhaustion, defined as $Y_{\rm c} = 10^{-5}$.

The tracks should be provided in one file for each mass value that includes the evolution from the ZAMS to the end of the clump phase, while the individual models requested should be delivered in `fgong` format. If the code cannot evolve models through the helium flash, provide the track until the most advanced model computed. Content and description of these file formats are given in appendix A. The naming convection for labelling files should follow the description given in section 6: '`Cod_Mas_Sto_Pha_solarcal.fgong`' and '`Cod_Mas_solarcal.track`'. Use the folder named '`solar_calibrated`' in the dropbox.

3 Red giant branch calibrated models

The aim of this exercise is to match a given $T_{\rm eff}$ value on the RGB for each of the masses given in section 2.3 by modifying the value of the mixing-length parameter. The input physics remains the same as defined in section 2.1.

3.1 Science cases

Adjusting the value of the mixing-length parameter, math the required effective temperature and provide results for the following models.

Initial abundances $Y = 0.28$, $Z = 0.02$.

1.0 $M_\odot$ model at 7 $R_\odot$ in the RGB: $T_{\rm eff} = 4722$ K.

1.5 $M_\odot$ model at 7 $R_\odot$ in the RGB: $T_{\rm eff} = 4914$ K.

2.0 $M_\odot$ model at 10 $R_\odot$ in the RGB: $T_{\rm eff} = 4908$ K.

2.5 $M_\odot$ model at 10 $R_\odot$ in the RGB: $T_{\rm eff} = 5041$ K.

With the $\alpha_{\rm MLT}$ value found for each mass, continue the evolution and provide the corresponding evolutionary tracks and interior models:

1.0 $M_\odot$ model in the RGB phase at 12 $R_\odot$.

1.0 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{He}) = 2.0$.

1.5 $M_\odot$ model in the RGB phase at 12 $R_\odot$.

1.5 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{He}) = 2.0$.

2.0 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{He}) = 2.0$.

2.5 $M_\odot$ model at the tip of the RGB phase, defined as $\log(L_{He}) = 0.0$.

All tracks evolution until core-helium exhaustion, defined as $Y_c = 10^{-5}$.

The naming convection for labelling files should follow the description given in section 6: `'Cod_Mas_Sto_Pha_teffcal.fgong'` and `'Cod_Mas_teffcal.track'`. Use the folder named `'RGB_calibrated'` in the dropbox.

4 Core helium-burning models

For the individual models in the core-helium burning phase, the modellers must provide the following information:

- Details of the neutrino emission used (whose neutrino emission).
- Details of the adopted screening formulae
- Details of the adopted Equation of State for the High-temperature and He- C- and O-rich regions of the He-burning stage.
- Details of the opacities (implementation) for C/O-rich material
- Ensure that the NACRE rates for 12C+alpha & triple alpha reactions are being used.

4.1 Evolution from the same initial model

In this exercise, modellers must read in an `fgong` file of a clump model selected from the `MESA` track of 1 $M_\odot$ solar radius calibrated science case. This model is chosen at the beginning of the clump phase where the thermal energy emission is at the minimum. Each modeller should read in the provided structure and relax it without any chemical evolution, changing the mixing-length parameter until the effective temperature of the input model is reproduced. Once this is achieved, the model is evolved throughout the core helium-burning phase with the following models requested:

- 1 $M_\odot$ model obtained after the relaxation process.
- Evolutionary track until central helium exhaustion ($Y_c = 10^{-5}$).

Provide the interior model and the corresponding evolutionary sequence in the format described in appendix A, named `'code_100_RC_mesainput.fgong'` and `'code_100_RC_mesainput.track'`. Use the folder named `'Core_helium'` in the dropbox.

5 Free physics models

The model comparisons thus far have been systematic and controlled in order to isolate numerical differences between the codes. All participants have adopted the same microphysics and for each science case calibrated their mixing parameter to match a reference model. Our methodology has ensured that the models are both seismically and structurally comparable arriving at the required temperature for a chosen large frequency separation (cf., Eq. 1).

We now wish investigate 'the role of the modeller in our experiment'. Participants are encouraged to adopt their preferred set of stellar physics with a few caveats. First that they retain the GN93 solar mixture and secondly that their mixing length parameter remains fixed within a stellar track. This will of course require a slight change in strategy as can no longer enforce the codes to produce the same radii AND temperatures at our comparison points. We will therefore calculate a set of models for structure comparisons (chosen by luminosity and hence core-mass) and set for seismic comparisons (chosen by radii). **Note: all models at a fixed radius must comply with the convergence criterion defined in Eq. 1**.

5.1 Solar Calibration

We ask that you perform a complete solar calibration using the input physics of your choice. The only restrictions are the following:

Solar abundances the Grevesse & Noels (1993) solar mixture.

Solar age 4.57 Gyr.

Macrophysics include microscopic diffusion.

Please provide us with your model `'cod_100_R01_fphys_solarcal.fgong'` and a text file outlining the choices in physics that differ from the homogeneous comparisons and the associated mixing length parameter. **Please specify clearly how you treat material at convective boundaries**. Use the folder named `'Free_physics'` in the dropbox.

5.2 Science cases

For the actual science cases use the Grevesse & Noels (1993) solar mixture, the **initial** composition and mixing length parameter derived from the solar calibration described above, and the same nuclear reactions and T-τ relation used in the solar

calibration. You are free to modify the rest of the input physics if you think it is appropriate for the rest of the computations. The following models should be provided in this exercise:

1 $M_\odot$ Track

- A model, `'code_100_CHE_MSP_fphys.fgong'`, at the end of core hydrogen abundance which we define as the star having a central hydrogen abundance of $X_c = 10^{-5}$.
- A model, `'code_100_R0123_MSP_fphys.fgong'`, close to CHE with radius $R = 1.23\ R_\odot$
- A model, `'code_100_R07_RGB_fphys.fgong'`, with radius $R = 7\ R_\odot$ on the RGB
- A model, `'code_100_L133_RGB_fphys.fgong'`, with luminosity $\log_{10}(L/L_\odot) = 1.33$ on the RGB
- A model, `'code_100_R12_RGB_fphys.fgong'`, with radius $R = 12\ R_\odot$ on the RGB
- A model, `'code_100_L172_RGB_fphys.fgong'`, with luminosity $\log_{10}(L/L_\odot) = 1.72$ on the RGB
- The full track `'code_100_fphys.track'`, computed to the end of core helium exhaustion which we define as the star having a central helium abundance of $Y_c = 10^{-5}$.

1.5 $M_\odot$ Track

- A model, `'code_150_CHE_MSP_fphys.fgong'`, at the end of core hydrogen abundance which we define as the star having a central hydrogen abundance of $X_c = 10^{-5}$.
- A model, `'code_150_R0212_MSP_fphys.fgong'`, close to CHE with radius $R = 2.12\ R_\odot$
- A model, `'code_150_R07_RGB_fphys.fgong'`, with radius $R = 7\ R_\odot$ on the RGB
- A model, `'code_150_L140_RGB_fphys.fgong'`, with luminosity $\log_{10}(L/L_\odot) = 1.40$ on the RGB
- A model, `'code_150_R12_RGB_fphys.fgong'`, with radius $R = 12\ R_\odot$ on the RGB
- A model, `'code_150_L180_RGB_fphys.fgong'`, with luminosity $\log_{10}(L/L_\odot) = 1.80$ on the RGB

- The full track `'code_150_fphys.track'`, computed to the end of core helium exhaustion which we define as the star having a central helium abundance of $Y_c = 10^{-5}$.

2.0 $M_\odot$ Track

- A model, `'code_200_CHE_MSP_fphys.fgong'`, at the end of core hydrogen abundance which we define as the star having a central hydrogen abundance of $X_c = 10^{-5}$.
- A model, `'code_200_R0283_MSP_fphys.fgong'`, close to CHE with radius $R = 2.83\ R_\odot$
- A model, `'code_200_R10_RGB_fphys.fgong'`, with radius $R = 10\ R_\odot$ on the RGB
- A model, `'code_200_L186_RGB_fphys.fgong'`, with luminosity $\log_{10}(L/L_\odot) = 1.86$ on the RGB
- The full track `'code_200_fphys.track'`, computed to the end of core helium exhaustion which we define as the star having a central helium abundance of $Y_c = 10^{-5}$.

2.5 $M_\odot$ Track

- A model, `'code_250_CHE_MSP_fphys.fgong'`, at the end of core hydrogen abundance which we define as the star having a central hydrogen abundance of $X_c = 10^{-5}$.
- A model, `'code_250_R0320_MSP_fphys.fgong'`, close to CHE with radius $R = 3.20\ R_\odot$
- A model, `'code_250_R10_RGB_fphys.fgong'`, with radius $R = 10\ R_\odot$ on the RGB
- A model, `'code_250_L176_RGB_fphys.fgong'`, with luminosity $\log_{10}(L/L_\odot) = 1.76$ on the RGB
- The full track `'code_250_fphys.track'`, computed to the end of core helium exhaustion which we define as the star having a central helium abundance of $Y_c = 10^{-5}$.

5.3 Fundamental constants

For this exercise, each code used a its own preferred set of fundamental constants in its calculations which are listed in Table 1. We note that the gravitational constant G must also be included in the `fgong` file as part of the standard output (see section A).

Table 1: Fundamental constants used in each evolutionary code for the free physics science case (see Section 5).

	G (cm^3/gs^2)	$R_\odot$ (cm)	$L_\odot$ (erg/s)	$M_\odot$ (gr)
ASTEC	6.67232×10^{-8}	6.9599×10^{10}	3.846×10^{33}	1.9890×10^{33}
BaSTI	6.672320×10^{-8}	6.9599×10^{10}	3.842×10^{33}	1.9891×10^{33}
CESAM2k	6.67168×10^{-8}			
GARSTEC	6.67384×10^{-8}	6.95508×10^{10}	3.846×10^{33}	1.9891×10^{33}
Geneva				
LPCODE	6.67384×10^{-8}	6.95508×10^{10}	3.846×10^{33}	
MESA	6.67428×10^{-8}	6.9598×10^{10}	3.8418×10^{33}	1.9892×10^{33}
MONSTAR	6.668×10^{-8}			
YREC				
YaPSI	6.6725×10^{-8}	6.957×10^{10}	3.828×10^{33}	1.988×10^{33}
MESA-10k	6.67408×10^{-8}	6.957×10^{10}	3.828×10^{33}	1.988475×10^{33}

6 Naming convention

Files of the individual models for each of the science cases have been uniformly named using the following syntax: `Cod_Mas_Sto_Pha_Cal.fgong`, where the characters refer to

Cod evolutionary code. The following ones are so far included: ASTEC (AST), BaSTI (BAS), CESAM2k (CES), GARSTEC (GAR), Geneva (GEN), LPCODE (LPC), MESA (MES), MONSTAR (MON), YREC (YRE).

Mas mass of the model

Sto place where the code was stopped to do the comparisons. At a certain radius (RXX), logarithm of helium luminosity (LHeXX), point of highest luminosity in the RGB (Tip), central helium abundance (YXX, etc), main-sequence turn-off (CHE) defined as the point where $X_c = 10^{-5}$, central helium exhaustion (CHeE) defined as the point where $Y_c = 10^{-5}$.

Pha evolutionary phase, either MSP (main-sequence phase), RGB or HeB (Helium-burning phase).

Cal calibration used in the science case, such as matching the solar radius (solarcal), an RGB temperature (teffcal), or the free physics case (fphys).

Example: `'AST_100_R12_RGB_teffcal.fgong'` for the ASTEC model of 1.0 $M_\odot$ at 12 $R_\odot$ in the red giant branch phase, calibrated to match an effective temperature value.

In the case of the evolutionary tracks, the naming convention should follow the same logic: we label the tracks with the syntax `'Cod_Mas_Cal.track'`. For example, the 1 $M_\odot$ track calibrated to the solar radius for the GARSTEC code should be called `'GAR_100_solarcal.track'`.

References

Angulo, C., Arnould, M., Rayet, M., et al. 1999, Nuclear Physics A, 656, 3

Cassisi, S., Potekhin, A. Y., Pietrinferni, A., Catelan, M., & Salaris, M. 2007, ApJ, 661, 1094

Christensen-Dalsgaard, J., Dappen, W., Ajukov, S. V., et al. 1996, Science, 272, 1286

Grevesse, N. & Noels, A. 1993, in 35ème cours de perfectionnement de l'Association Vandoise des Chercheurs en Physique, eds. B. Hauck, S. Paltani, & D. Raboud (AVCP, Lausanne), 205

Iglesias, C. A. & Rogers, F. J. 1996, ApJ, 464, 943

Osaki, J. 1975, Astronomical Society of Japan, 27, 237

Rogers, F. J., Swenson, F. J., & Iglesias, C. A. 1996, ApJ, 456, 902

Scuflaire, R. 1974, Astronomy and Astrophysics, 36, 107

Takata, M. 2006, Publications of the Astronomical Society of Japan, 58, 893

A File format

The following sections describe the file formats of the outputs required for the comparisons, most of which have been compiled by J. Christensen-Dalsgaard.

A.1 Format for evolutionary tracks

Evolutionary tracks should be provided in `ASCII` files including the information described in Table 2, with particular attention to the units required in each case. Missing quantities should be included and labelled by -99.999, and any additional line included in the file (such as header) should start with the character #.

A.2 Format for interior model comparison

The present notes provide a definition of the data format used for comparison of solar models within the GONG models team. They essentially correspond to Section 9 of *Computational procedures for GONG model project* (in the following **CP**) and have been extracted here for convenience of reference. For details on the GONG model comparisons project, involving simplified physics, **CP** should be consulted. The focus here is on the exchange of realistic solar models, involving detailed physics.

Table 2: Format for the `ASCII` files containing evolutionary sequences. Note: if the model has more than one outer convective region (i.e., in 2.5 $M_\odot$ cases), provide the mass and distance to the innermost one in columns 12 and 14.

Column	Quantity	Unit
1	Model number	integer
2	Mass	gr
3	Radius	cm
4	Effective temperature	K
5	Luminosity	erg/s
6	Age	yr
7	Central hydrogen content	mass fraction
8	Central helium content	mass fraction
9	Surface hydrogen content	mass fraction
10	Surface helium content	mass fraction
11	Mass contained within the edge of the conv. core	gr
12	Mass contained within the inner edge of the conv. envelope	gr
13	Distance from the centre to the edge of the conv. core	cm
14	Distance from the centre to the inner edge of the conv. envelope	cm
15	Central temperature	log(K)
16	Central density	log(gr/cm^3)

The use of the format has been extended to more general exchange of stellar models, including in model comparisons related to the *Kepler* project. The main update in the present version is the introduction of a new data format, corresponding essentially to full binary double precision. This is discussed in Section A.2.5.

A.2.1 File structure, and variables

The exchange of models will take place by means of ASCII files, to avoid problems with different binary formats. This makes it possible to send the data electronically without problems (other than caused by the amount).

For each model the file consists of a header, with descriptive information, a set of global variables, and a set of variables given at each mesh point. The set of variables is likely to develop, particularly in the context of the use of the format for transferring realistic solar and stellar models. The actual set used is defined by the variable `ivers` included in the header to the file; this includes `ivers0` depending on the variable set, and a flag for the output format (see also Section A.2.5). In addition, the total number of global parameters, meshpoints and variables at each meshpoint are given in the header, so that the information required to read the file is available.

The variables should be given in *cgs* units, unless otherwise stated.

A.2.2 Text header

The first record should contain the name of the model, its date, and an identification of its origin (such as name or institute). In addition the header may contain text further describing the calculation, and information about the remaining data.

An example of a header may be the following (describing Model S of the *Science* series of articles, see Christensen-Dalsgaard et al. (1996)):

```
   L5BI.D.15.PRES.950912.AARHUS
   Level 5 physics, present Sun.  (OPAL opacity, OPAL EOS). He,
Z diffusion.
   Age of present Sun:  4.6 Gyr.
```

A.2.3 Global parameters

These are set up in the array `glob(i)`, `i = 1, ..., iconst`, with the following definition:

1: M (total mass).
2: R (photospheric radius).
3: L_s (surface luminosity).
4: Z_0 (initial heavy-element abundance).[a)]
5: X_0 (initial hydrogen abundance).
6: $\alpha = \ell/H_p$ (mixing-length parameter; H_p is pressure scale height)
7: ϕ (another convection theory parameter)[b)]
8: ξ (yet another convection-theory parameter)[b)]
9: β (parameter in surface-pressure condition)[c)]
10: λ (parameter in surface-luminosity condition)[c)]
11: $\dfrac{R^2}{p_c}\dfrac{\mathrm{d}^2 p_c}{\mathrm{d}r^2}$ (at centre)[d)]
12: $\dfrac{R^2}{\rho_d}\dfrac{\mathrm{d}^2 \rho_c}{\mathrm{d}r^2}$ (at centre)[d)]
13: Model age (in years)[e)]
14: $T_{\rm eff}$ (effective temperature, in K)[f)]
15: Gravitational constant G, in cgs units[g)]

Notes:

a) Text changed 18/2/13 to emphasize that this should be the *initial* heavy-element abundance.

b) The parameters ϕ and ξ are defined in **CP**. For the Böhm-Vitense formulation, $\phi = 9/4$ and $\xi = 1/162$.

c) These parameters are used in boundary conditions for models with simplified physics (*cf.* **CP**). For realistic models, $\beta = \lambda = 1$.

d) These second derivatives may be needed in the central boundary conditions for oscillation calculations.

e) Added 30/7/96. If the calculation includes pre-main-sequence evolution, the definition of zero age must be specified, in the character header or in accompanying notes. It would be useful to arrive at a common definition; suggestions are welcome.
It appears that this was not properly included (at least in some models) before 9/1/13.

f) Added 9/1/13.

g) Added 30/4/14.

A.2.4 Model variables at each mesh point

These are set up in the array `var(i, n)`, `i = 1, ..., ivar`, `n = 1, ..., nn`. The current set of variables, as defined in October 2005 is characterized by having `ivers0` = 300.

1: r (distance to centre)
2: $\ln q, \quad q = m/M$ (m is mass interior to r and M is total mass)
3: T (temperature)
4: p (pressure)
5: ρ (density)
6: X (hydrogen abundance by mass)
7: $L(r)$ (luminosity at distance r from centre)
8: κ (opacity)
9: ϵ (energy generation rate per unit mass)
10: $\Gamma_1 = \left(\dfrac{\partial \ln p}{\partial \ln \rho}\right)_{\rm ad}$
11: $\nabla_{\rm ad} = \left(\dfrac{\partial \ln T}{\partial \ln p}\right)_{\rm ad}$
12: $\delta = -\left(\dfrac{\partial \log \rho}{\partial \log T}\right)_p$
13: c_p (specific heat at constant pressure)
14: $\mu_{\rm e}^{-1}$ [see note ii) below]
15: $\dfrac{1}{\Gamma_1}\dfrac{{\rm d}\log p}{{\rm d}\log r} - \dfrac{{\rm d}\log \rho}{{\rm d}\log r}$
16: r_X (rate of change in X from nuclear reactions)
17: Z (heavy-element abundance per unit mass)
18: $R - r$
19: $\epsilon_{\rm g}$ (rate of gravitational energy release)

20: L_g (local gravitational luminosity; this has only been included in J. Reiter's models so far)

21: $X(^3\mathrm{He})$ (^{3}He abundance by mass)

22: $X(^{12}\mathrm{C})$ (^{12}C abundance by mass)

23: $X(^{13}\mathrm{C})$ (^{13}C abundance by mass)

24: $X(^{14}\mathrm{N})$ (^{14}N abundance by mass)

25: $X(^{16}\mathrm{O})$ (^{16}O abundance by mass)

26: $\left(\frac{\partial \ln \Gamma_1}{\partial \ln \rho}\right)_{p,Y}$

27: $\left(\frac{\partial \ln \Gamma_1}{\partial \ln p}\right)_{\rho,Y}$

28: $\left(\frac{\partial \ln \Gamma_1}{\partial Y}\right)_{p,\rho}$

29: $X(^2\mathrm{H})$ (^{2}H abundance by mass)

30: $X(^4\mathrm{He})$ (^{4}He abundance by mass)

31: $X(^7\mathrm{Li})$ (^{7}Li abundance by mass)

32: $X(^7\mathrm{Be})$ (^{7}Be abundance by mass)

33: $X(^{15}\mathrm{N})$ (^{15}N abundance by mass)

34: $X(^{17}\mathrm{O})$ (^{17}O abundance by mass)

35: $X(^{18}\mathrm{O})$ (^{18}O abundance by mass)

36: $X(^{20}\mathrm{Ne})$ (^{20}Ne abundance by mass)

37 – 40: Currently not used.

Comments:

i) In variable 2, $\ln q$ is used instead of m to give a better indication of variation close to the surface. Here "ln" is natural logarithm.

ii) $\mu_e^{-1} = N_e m_u$, where N_e is the number of free electrons per unit mass and m_u is the atomic mass unit; thus μ_e is the mean molecular weight per electron. It has been included to give some indication of the ionization state.

iii) Variable 18 was introduced Aug. 6 1993, to avoid problems with interpolation in r near surface. (For versions before `ivers0` = 210, but after that date, $R - r$ was set in variable 17.)

iv) The derivatives of Γ_1 were introduced principally to allow calculation of kernels involving Y, in realistic models; they are less essential for the comparison of models.

v) Calculations that do not follow all details of the CNO cycle may not have available some of the CNO abundances. In that case, the corresponding abundances can be set to zero. (An example might be a calculation following

the conversion of ^{16}O into ^{14}N but ignoring the details of carbon burning into ^{14}N.)

vi) More generally, for a given code there might be variables that are difficult to obtain. In that case, the above numbering should be maintained, but the missing variables may be set identically to zero.

The present set of variables has been chosen to give a reasonably comprehensive basis for comparing evolution models, and to be adequate for the computation of adiabatic oscillations. In particular these variables should define completely the stellar structure equations, and so permit a check of the accuracy to which the equations are satisfied. For a more detailed comparison even more variables may be needed (such as ionization levels), but I suggest that that be arranged separately.

For non-adiabatic calculations, which we should eventually get to, more variables are certainly needed. Typical examples are the derivatives of κ and ϵ with respect to p and T, and possibly variables relating to the perturbation of the convective flux. They can be included later, by extending the basic set given above.

A.2.5 Format for data transfer

The data exchange should be carried out by means of formatted ASCII files, using the following structure:

Record 1: Name of model (as a character string)
Record 2 – 4: Explanatory text, in free format
Record 5: `nn, iconst, ivar, ivers`
Record 6 – 8: `glob(i), i = 1, ..., iconst`
Record 9 – : `var(i,n), i = 1, ..., ivar, n = 1, ... nn`.

Here `nn` is the number of mesh points in the model, `iconst` is the number of global variables (given in the array `glob`), and `ivar` is the number of variables at each mesh point (given in the array `var`). Thus with the present set `iconst` = 15 and `ivar` = 40, but there is room for expansion. The version number `ivers` is written as `ivers = ivers1 + ivers0`. Here `ivers0` was defined above; as discussed below `ivers1 = 0` corresponds to the old (before 8 October 2015) data format, while `ivers1 = 1000` should be used for the new more precise data format.
The integer variable Record 5 should be written with the format

```
4i10
```

There are two options for the format for the real-variable Records 6 and up. Before 8 October 2015 the only format was

```
1p5e16.9
```

The revision has implemented the format

```
1p,5(x,e26.18e3)
```

The use of this format must be flagged by adding 1000 in `ivers`.

A.2.6 Summary of earlier versions

The format has been in used for more than a decade, with various extensions or modifications; these are characterized by the version number, as summarized below:

– **Version 100**: `var(1) - var(16)` defined as above. In some models `var(17)` was set to $R - r$, and `var(19)` and `var(20)` were set as above; otherwise these variables were set to zero; `ivar` = 20.

– **Version 200**: `var(1) - var(16)` and `var(19) - var(25)` defined as above, `var(17)` set to $R - r$; `ivar` = 25.

– **Version 210**: `var(1) - var(25)` defined as above; `ivar` = 25.

– **Version 250**: `var(1) - var(28)` defined as above; `ivar` = 30.

A.3 Format for frequency comparison

For the comparison of oscillation calculations it is convenient to have a standard format of results of adiabatic calculations, in parallel with the `fgong` format used for model comparison. I propose to base this on the ADIPLS `obs` format, but with higher precision. Specifically, the file should provide

$$l, n, \nu, E\ [, n_{\rm p}, n_{\rm g}]$$

Here l is the degree, n is the radial order, ν is the frequency in μHz and E is the mode inertia (see below). Also, the optional quantities $n_{\rm p}$ and $n_{\rm g}$ are the number of p- and g-nodes in the classification scheme, including the Takata (2006) scheme for dipolar modes (see below for details on the definition of the mode order).

For the output I suggest the fixed format (given the very high order in the g-mode behaviour we need quite a long format for the order)

```
2i8, 1p2e16.8, 2i8
```

The file should be ordered with the degree increasing most slowly, i.e.,

```
0 1 xxx eee ii jj
0 2 xxx eee ii jj
.....................
1 -50 xxx eee ii jj
1 -49 xxx eee ii jj
```

etc.

The mode inertia should be defined as

$$E = \frac{\int_0^{R_{\rm s}} [\xi_r^2 + l(l+1)\xi_{\rm h}^2] \rho r^2 {\rm d}r}{M[\xi_r(R_{\rm phot})^2 + l(l+1)\xi_{\rm h}(R_{\rm phot})^2]} = \frac{\int_0^{x_{\rm s}} \left[y_1^2 + y_2^2/l(l+1)\right] qU{\rm d}x/x}{4\pi[y_1(x_{\rm phot})^2 + y_2(x_{\rm phot})^2/l(l+1)]} \ .$$

Note in particular the normalization by the total displacement at the photosphere, $r = R_{\rm phot}$.

The second equality is in terms of the `ADIPLS` solutions,

$$\begin{aligned} y_1 &= \frac{\xi_r}{R} \,, \\ y_2 &= x\left(\frac{p'}{\rho} + \Phi'\right)\frac{l(l+1)}{\omega^2 r^2} = \frac{l(l+1)}{R}\xi_{\rm h} \,, \\ y_3 &= -x\frac{\Phi'}{gr} \,, \\ y_4 &= x^2\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y_3}{x}\right) \,, \end{aligned}$$

where p' and Φ' are Eulerian perturbations to pressure and gravitational potential and g is the local gravity. Also, $q = m/M$ and $U = \mathrm{d}\ln m/\mathrm{d}\ln r = 4\pi r^3\rho/m$, where m and M are the local interior and the total mass, respectively, and ρ is density; $x = r/R_{\rm phot}$ and hence $x_{\rm phot}$ should be 1, and $R_{\rm s}$ is the radius of the outermost point in the model, with $x_{\rm s} = R_{\rm s}/R_{\rm phot}$. For $l = 0$ the terms in $\xi_{\rm h}$ and y_2 should be excluded.

I name the resulting file `fobs......`, as an extension of my normal `obs....` files.

Definition of the mode order

The 'classical' definition of the mode order is based on the so-called Eckart scheme introduced by Scuflaire (1974) and Osaki (1975). This is based on counting the number of nodes of ξ_r with a sign depending on the variation with r of the solution in a $(\xi_r, \xi_{\rm h})$ phase diagram. We let $n_{\rm p}$ be the number of zero-crossings of ξ_r in the counter-clockwise direction and $n_{\rm g}$ the number of crossings in the clockwise direction, with increasing r; then the mode order n is obtained as

$$n = n_{\rm p} - n_{\rm g} \,.$$

It can be proved that this defines a unique and invariant labelling of the modes in the Cowling approximation, where the perturbation to the gravitational potential is neglected, or for radial modes. For nonradial modes with $l \geq 2$ it also seems to be generally valid, although no complete proof of this has been made, as far as I am aware. In these cases I suggest simply to let $n_{\rm p}$ and $n_{\rm g}$ be defined as above.

For dipolar modes when the Cowling approximation is not made the mode labelling becomes problematic for even moderately centrally condensed models, including models of the present Sun, and very much so for more evolved models. Here Takata (2006) demonstrated that a unique and invariant labelling can be

defined based on the phase diagram $(\mathcal{Y}_1, \mathcal{Y}_2)$, where

$$\begin{aligned} \mathcal{Y}_1 &= \frac{1}{g}\left[\frac{\delta\Phi}{r} - \delta\left(\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right)\right] \\ \mathcal{Y}_2 &= \frac{\delta p}{p} \, . \end{aligned}$$

Defining n_p and n_g in terms of zero-crossings of $\mathcal{Y}_1$, as above, the order is determined as

$$n = \begin{cases} n_\mathrm{p} - n_\mathrm{g} + 1 & \text{for } n_\mathrm{p} \geq n_\mathrm{g} \\ n_\mathrm{p} - n_\mathrm{g} & \text{for } n_\mathrm{p} < n_\mathrm{g} \, . \end{cases}$$

I recommend following this definition for $l = 1$ in the full case, despite the more complication relation between n_p, n_g and n. This is also the way that the Takata (2006) labelling is included in `ADIPLS`.