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Communication Systems II

Project-Tasks 1,2

Abstract - In this work we analyze and design different constellations considering the trade-offs inherent in simultaneous wireless information and power transfer (SWIPT) systems. More specifically, we compare the behavior of basic modulation schemes (PAM,PSK,QAM) to this of C-QAM [1] and spike-QAM [2] verifying the results from the corresponding papers. Given that energy harvesting is related to the peak-to-average power ratio (PAPR) and the symbol error probability (SER) is a function of the minimum Euclidean distance, this project will focus on this trade-off for SWIPT. Then, a C-s-QAM (Circular-spike-QAM) is designed, investigated and proposed as a new optimal constellation for SWIPT.

I Task 1

In this section we analyse theoretically the PAPR behavior versus $d_{\min}$ for 16-PAM, 16-PSK, 16-QAM and 16-C-QAM, verifying the results in Fig 2 [1] through simulations. For SWIPT systems on the one hand we want high PAPR for better energy harvesting performance, but on the other we want high $d_{\min}$ for better detection and information performance.

16-PAM

Each symbol represents one of 16 different amplitude levels. The symbols are linearly spaced along the amplitude axis. The distance between the symbols is constant equal to $d_{\min}$, which is given from the next formula:

$$d_{\min} = 2\sqrt{Eg}$$

where

$$E_g = \frac{3E_s}{M^2 - 1}$$

If we replace Es=1 and M=16, we will have dmin = 0,2169 Then, the PAPR is calculated from:

$$PAPR = \frac{max(|s_i|^2)}{E_s}$$

The symbol which is the farthest from 0, has a distance of

$$7d_{\min} + \frac{d_{\min}}{2}$$

So, if we do the calculations, we will obtain PAPR approximately equal to 2.65.

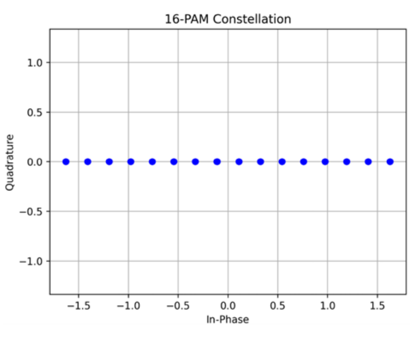

Figure 1: 16-PAM modulation

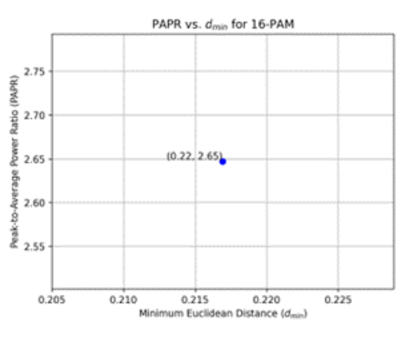

Figure 2: PAPR versus $d_{\min}$ of 16-PAM

16-PSK

In 16-PSK each symbol represents a different phase angle but same amplitude. PAPR is determined by the peak amplitude which is constant for all phase angles, so it will be equal to 1. We will calculate $\mathrm{d}_{\min} from : d_{\min} = 2\sqrt{Es} * sin(\frac{\pi}{M}) = 0.39$

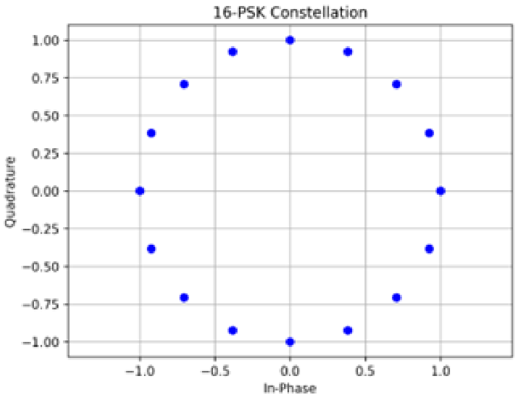

Figure 3: 16-PSK modulation

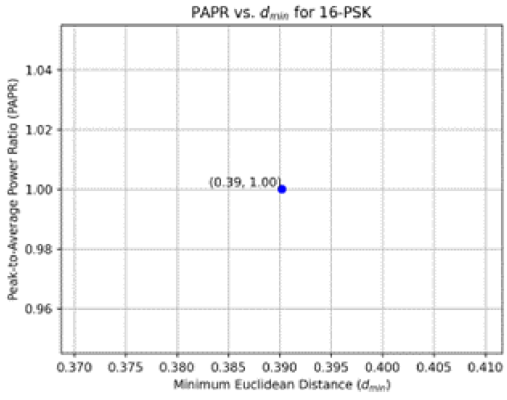

Figure 4: PAPR versus $d_{\min}$ of 16-PSK

16-QAM

In 16-QAM each symbol represents a different combination of amplitude and phase. Because we have

$$M = 4^2$$

so

$$\sqrt{M}$$

is an integer, QAM is created by two $\sqrt{M}$ PAM signals, which each forms the I and Q components. So, we have

$$d_{\min} = \sqrt{\frac{6Es}{(M-1)}}$$

which gives 0.632. The maximum distance of the farthest symbol will be:

$$\frac{\sqrt{2}}{2} d_{\min} + \sqrt{2} d_{\min}$$

So, with the same formula as before we will calculate PAPR = 1.79.

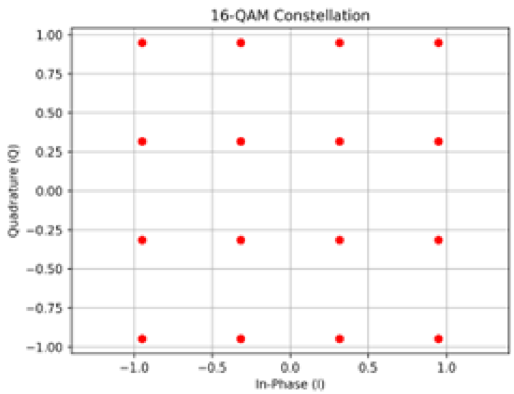

Figure 5: 16-QAM modulation

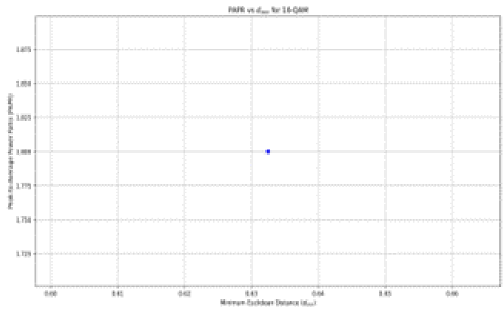

Figure 6: PAPR versus d_{min} of 16-QAM

16-CQAM

The scientists in [1] have proposed CQAM as an appropriate constellation for SWIPT systems, as it provides a controllable trade-off between energy harvesting and information transfer. In this way, CQAM allows to tune the amount of energy to be harvested without affecting the performance.

We constructed the CQAM constellation with the algorithm that it is described in [1] and we obtained the same results that prove its superiority upon the basic modulations, due to its flexibility which comes from allowing to modify the number of energy levels and to trade PAPR with minimum Euclidean distance. Here is an example of CQAM constellation that was designed with a given $d_{\mathrm{min}} = 0.3$ for the best PAPR performance and then the PAPR-d_{min} diagram:

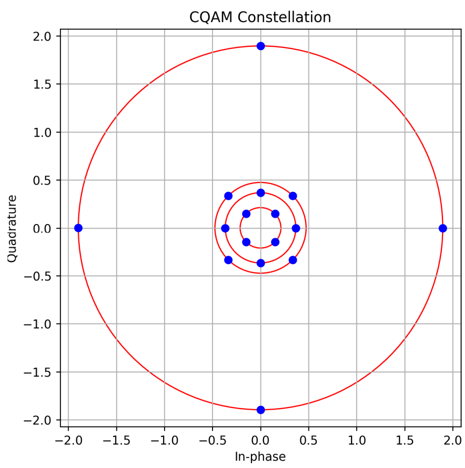

Figure 7: 16-CQAM modulation, N=4

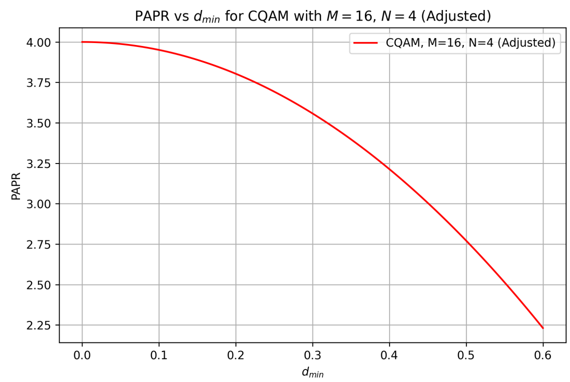

Figure 8: PAPR versus d_{min} of 16-CQAM, N=4

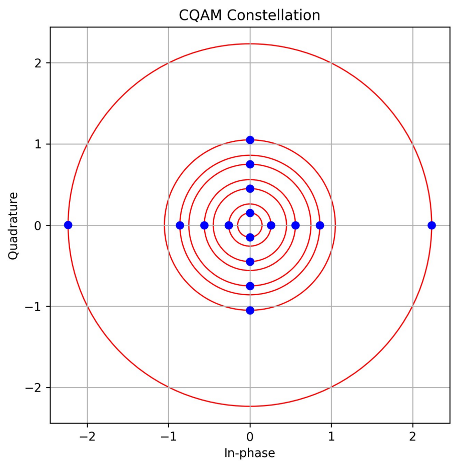

Figure 9: 16-CQAM modulation, N=8

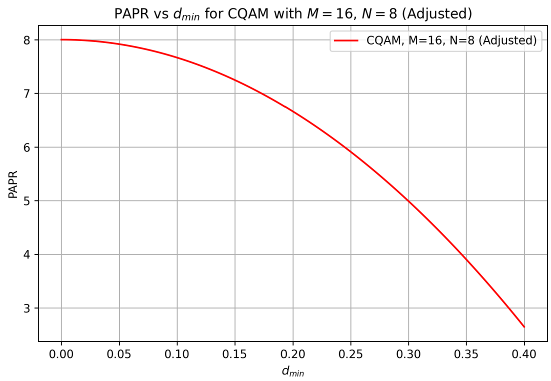

Figure 10: PAPR versus $d_{\min}$ of 16-CQAM, N=8

In the figure below it is presented the behaviour of PAPR versus $d_{\min}$ for every constellation in the same diagram, exactly like the figure 2 [1]:

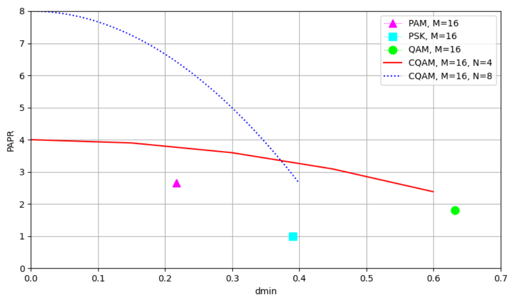

Figure 11: PAPR versus $d_{\min}$ of CQAM and some well-known modulation schemes.

Conclusion

Therefore, from the above diagram the superiority of CQAM over other modulation schemes is clear, as for the same $d_{\min}$ has the highest value of PAPR.

II Task 2

In this section we observe the behavior of SER, firstly versus normalized harvested energy for fixed SNR and secondly versus SNR for fixed normalized energy harvested.

Spike QAM

This modulation,which is presented analytically in [2], is a regular QAM constellation where one to four of the elements with the largest amplitudes are placed further from the center. This constellation is capable of maximizing the SER performance for a given design PAPR, or maximize the PAPR given a SER performance, as it minimizes the SER-PAPR ratio.

Below, there are presented some examples of the constellation with spikes 1,2,3,4 and $d_{\min}$ = 0.4:

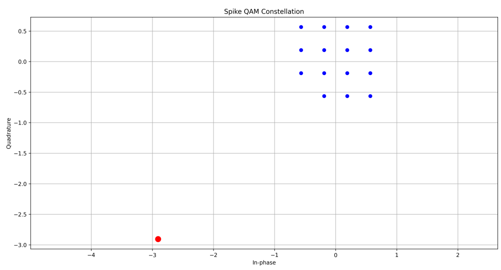

Figure 12: s-QAM number of spikes=1

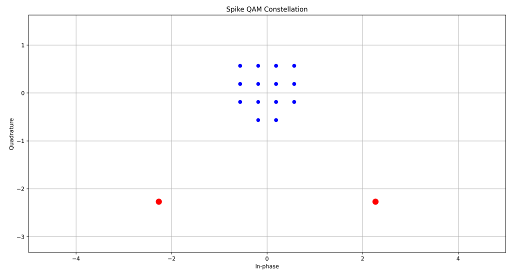

Figure 13: s-QAM number of spikes=2

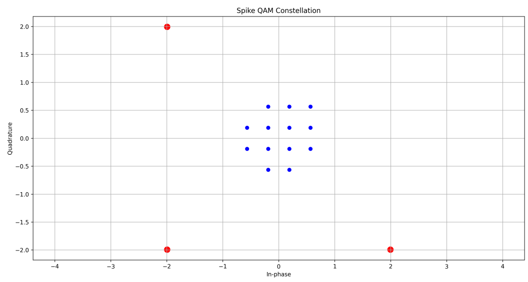

Figure 14: s-QAM number of spikes=3

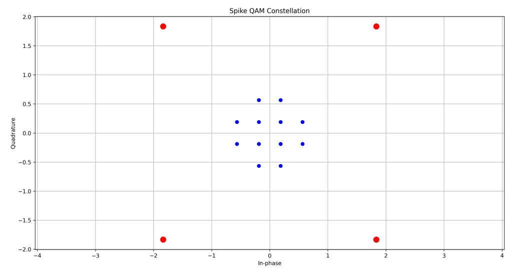

Figure 15: s-QAM number of spikes=4

Fig. 4: SER versus Normalized Harvested Energy [1]

This graph shows the Symbol Error Rate(SER) of PAM, QAM and C-QAM (one optimized for maximum peak-to-average power ratio (PAPR), and one for maximum minimum Euclidean distance ($d_{\min}$)) as a function of the normalized harvested energy. From the graph we understand that the more the harvested energy increases, because more power is diverted from information transfer to energy harvesting, the higher the symbol error rate becomes. This happens because less power is used for info transmission and thus overcoming the channel noise.

At first, we did some analysis in which energy harvested is an overall percentage of Es for the whole constellation, so we used the SER formulas but instead of Es, we put (1-Energy harvested)*Es.

Something that we want to clarify is that we ran these simulations at SNR=20, because in [2] they reported that the simulations are in an AWGN channel with high SNR. In order to obtain the same scale of values 0.1-1, we ran the simulations for C-QAM at SNR=10 too (fig. 20). The range of values that we obtained for each SNR are confirmed by fig 5 too [1], from the curve of CQAM with E[x]=0.2.

Moreover, it is clear that PAM has the best energy harvesting performance from the basic modulations, something that was expected because it has the highest PAPR.

Lastly, spike QAM has almost 0 error for low energy harvested levels and the SER becomes higher only when all the energy is harvested, something that confirms that is is an optimum constellation for SWIPT systems.

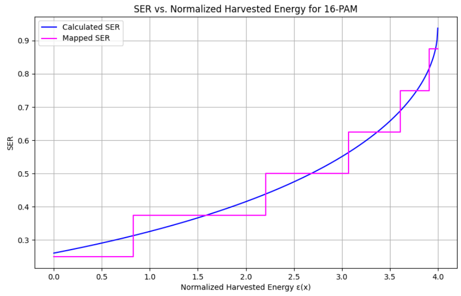

Figure 16: SER versus normalized harvested energy ε(x) of 16-PAM

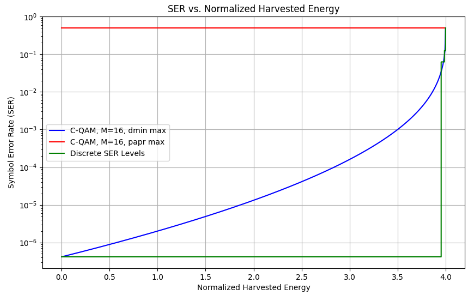

Figure 17: SER versus normalized harvested energy ε(x) of CQAM

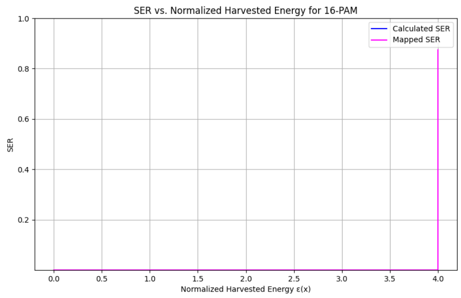

Figure 18: SER versus normalized harvested energy ε(x) of spike-QAM

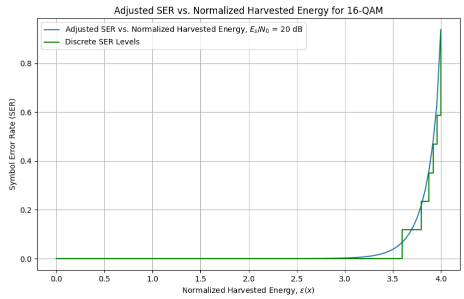

Figure 19: SER versus normalized harvested energy ε(x) of 16-QAM

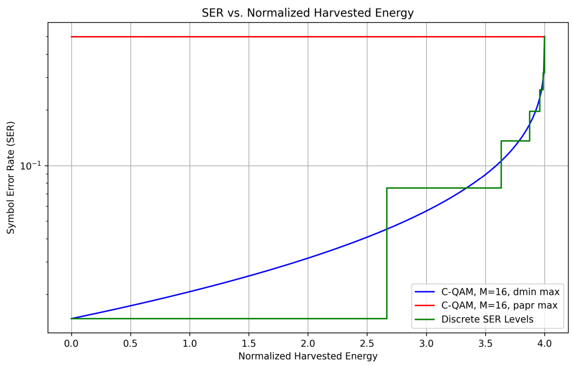

Figure 20: SER versus normalized harvested energy ε(x) of 16-cQAM but with SNR=10

The scientists in [1] did the calculations for energy harvested in another way. Instead of a total removal rate of energy, they subtracted from each symbol the same amount of energy and calculated its new position.

Below, we will follow the same procedure for each constellation.

The energy of each symbol is equal to R_{i}^2, where R_{i} symbolizes its distance from 0. So the new adjusted energy will be

$$E_{\text{adjusted}} = R_{\mathrm{i}}^2 - E_{\text{harvested}}.$$

Consequently, the new magnitude of the distance of the symbol from 0 is equal to $\sqrt{E_{\text{adjusted}}}$.

Something interesting from this diagram that's needed to be said is that each step for each modulation indicates the change-reduction of its N levels. To be more clear, for example, for C-QAM ,when we have maximum $d_{\min}$ equal to 0,617, we initially have 4 levels-circles. When we subtract from each level some energy, at some point the first radius will become zero. At this point all the symbols from this level are located to (0,0) and we have augmentation of SER, as these points give P=1. When we reach the energy of the outermost circle, i.e. $R_{\max}^2$, all the symbols are located to 0 and SER will be equal to 1, something logical, because the possibility of error is 1 , when the decision areas coincide.

In the same way, C-QAM for maximum PAPR ,as it has only one circle and all the other points are initially located in 0, will have only one step in the diagram, which indicates the point where this outermost level with energy reduction becomes equal to zero. We observe that we can design C-QAM either for better information transmission performance (highest $d_{\min}$=0.617, lower SER while energy harvesting but lower PAPR) or for better energy harvesting(highest PAPR-high SER, $d_{\min}$=0).

To summarize, for the same reason PAM has 8 steps-8 different distances-levels from 0- and QAM 2,as it has two levels-squares.

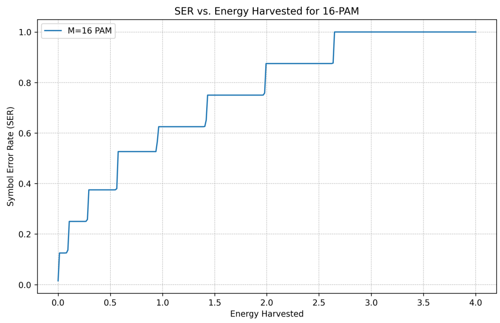

Figure 21: SER versus normalized harvested energy ε(x) of PAM

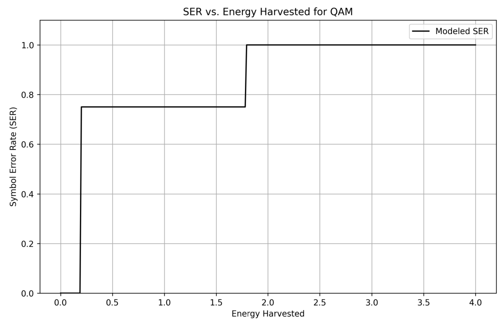

Figure 22: SER versus normalized harvested energy ε(x) of QAM

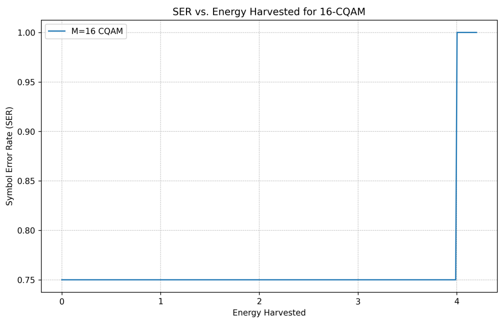

Figure 23: SER versus normalized harvested energy ε(x) of 16-cQAM PAPR max

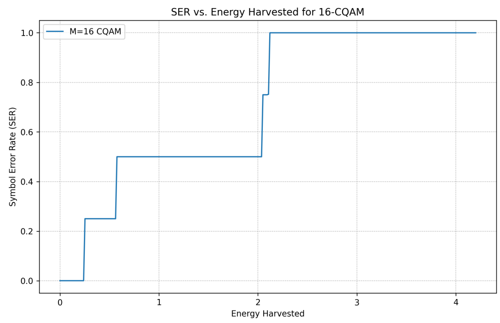

Figure 24: SER versus normalized harvested energy ε(x) of 16-cQAM dmin max

Fig 5: SER versus SNR for Fixed Normalized Energy Harvesting[2]

This graph displays the performance of the same modulation schemes under different SNR conditions, while keeping the normalized energy fixed. As more energy is harvested (comparing E(x)=0 with E(x)=0.2 for each scheme, this time energy harvested is a percentage of the whole energy) we observe once again that SER is increased.

It is evident that at lower SNR, SER is bigger. As the SNR increases, i.e. better channel, the impact of energy harvesting on SER diminishes, which is shown by the flattening of the curves at higher SNR values(really small values of SER). From this we can conclude that the system can compensate for the loss of signal power due to energy harvesting.

Moreover, it is clear that C-QAM has the ability to maintain lower SER at a given SNR when energy is harvested, something that confirms its design for minimization of the loss of signal integrity while maximizing energy harvesting. That's because PAM and QAM have a big change in their performance when energy is harvesting unlike CQAM.

Although in [1] PAM and QAM constellation have a really bad behavior when energy harvested is 0.2 (almost straight line - high values of SER), we didn't get such bad behavior. For PAM, the curve shifted to the right much more than C-QAM and even at a high SNR like 15dB has 0.2 error rate. For QAM we got a better behavior than PAM but worse than C-QAM, as expected.

Last but not least, with this diagram the superiority of spike-QAM is obvious, as when energy harvested is 0.2, SER vs SNR performance has smaller changes than the other constellations. For example, for C-QAM in $d_{\min}$=0.4 case SER reaches the same value in 5dB SNR change, in contrast with the other constellations which need much more better channels. That shows how much suitable is for SWIPT systems, as it has the best energy harvesting and information transfer performance.

Something that we have to note is that for spike-QAM in order to have the same results with [2] we worked with Es/N0 and not Eb/N0, that's why the values of SER are bigger. Below in the 3rd task we analyse its performance versus Eb/N0 too.

The used formulas for SER for each modulation are listed:

PAM

$$P_s = \frac{2(M-1)}{M} Q(\sqrt{\frac{6EsM}{(M^2-1)No}})$$

QAM

$$P_{s,\sqrt{M}} = 2(1 - \frac{1}{\sqrt{M}})Q(\sqrt{\frac{3E_s}{No(M-1)}})$$

$$P_{s,M} = 1 - (1 - P_{s,\sqrt{M}})^2$$

CQAM[1]

$$P_e \approx \frac{1}{N} \sum_{i=1}^{N} \nu(i) Q \left(\sqrt{\frac{E_b}{N_0}} \sin\left(\frac{\pi}{n}\right) \left(\sqrt{R_i^2 - \epsilon(R_i)}\right)_+ \right)$$

s-QAM[2]

$$P_{sQAM} = \frac{M-i}{M} P_{\gamma QAM} + \frac{i}{M} P_{\max QAM}$$

$$\gamma = \frac{(M-i) \cdot PAPR}{M-i} = \frac{(M-i) \cdot |x_M|^2}{M-i}$$

$$P_{\max QAM} = Q\left(\frac{|x_M| - D_{\text{mid}}}{\sqrt{2N_0}}\right)$$

where E_s we used $4E_b$, as M=16

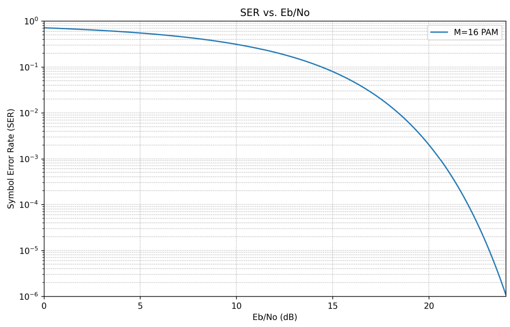

Figure 25: PAM SER vs SNR

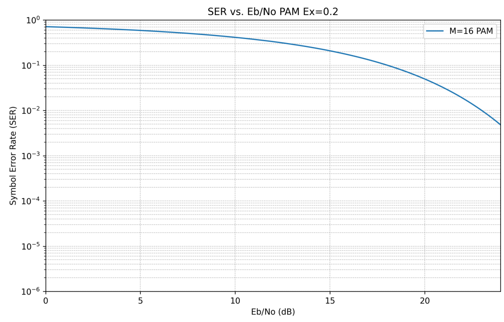

Figure 26: PAM SER vs SNR E(x)=0.2

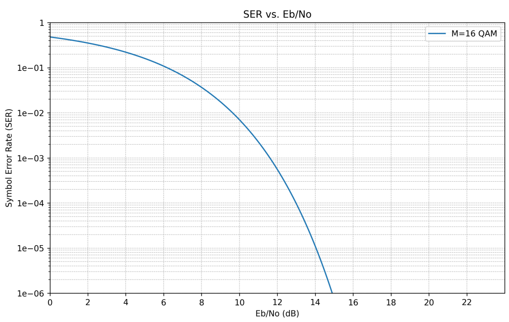

Figure 27: QAM SER vs SNR

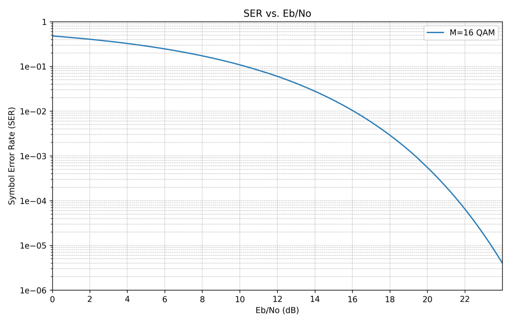

Figure 28: QAM SER vs SNR E(x)=0.2

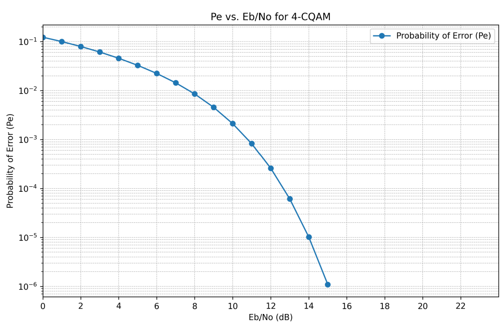

Figure 29: CQAM SER vs SNR

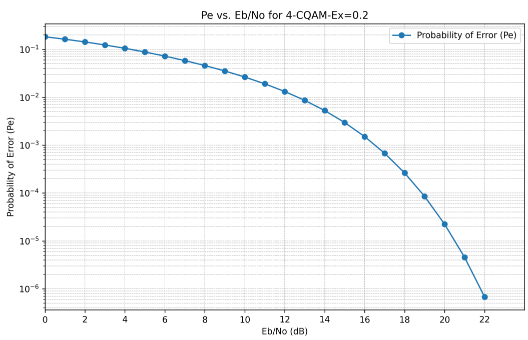

Figure 30: CQAM SER vs SNR E(x)=0.2

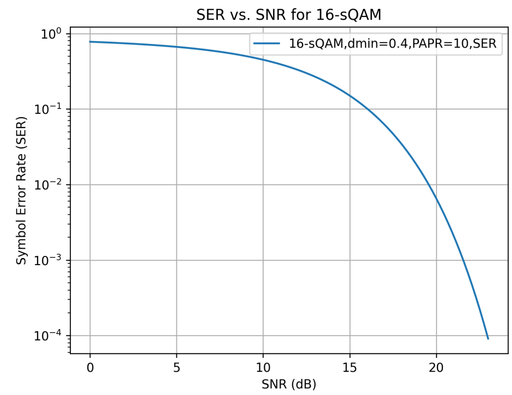

Figure 31: sQAM SER vs SNR M=16 spike=1 $d_{\min}$=0.4 PAPR=10

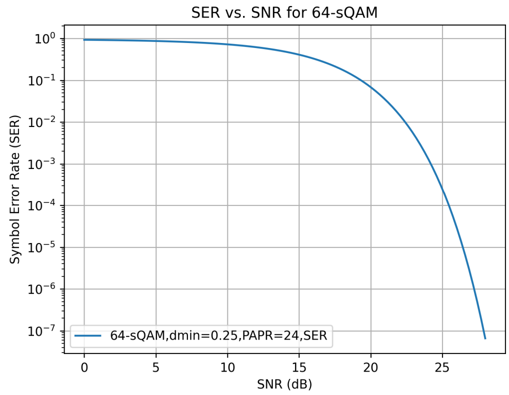

Figure 32: sQAM SER vs SNR M=64 spike=1 $d_{\min}$=0.25 PAPR=24

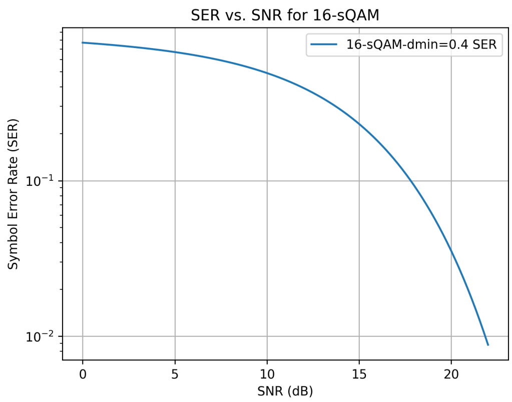

Figure 33: sQAM SER vs SNR E(x)=0.2 M=16 spike=1 $d_{\min}$=0.4 PAPR=10

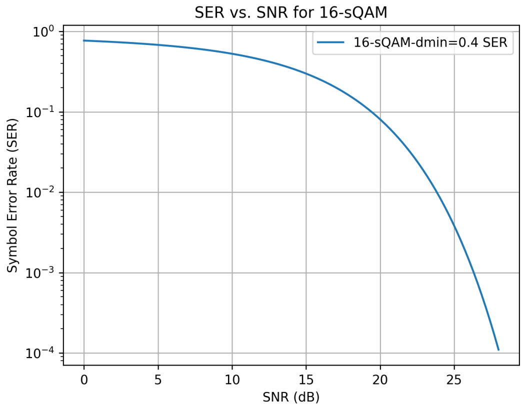

Figure 34: sQAM SER vs SNR E(x)=0.2 M=16 spike=1 $d_{\min}$=0.4 PAPR=10, run with more values of SNR

References

[1] G. M. Kraidy, C. Psomas, and I. Krikidis, "Fundamentals of circular qam for wireless information and power transfer," in *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*. IEEE, 2021, pp. 616–620.

[2] M. J. Lopez Morales, K. Chen-Hu, and A. Garcia Armada, "Optimum constellation for symbol-error-rate to papr ratio minimization in swipt," in *IEEE Vehicular Technology Conference (VTC2022-Spring)*. IEEE, 2022, pp. 1–5.