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Abstract

1 Fundamental matrix

Let us have 2 shots of the same scene made with slightly rotated cameras so that the transformation between the camera coordinate system K' and K is given via the rotation matrix R and the translation vector T, i.e. the coordinates X and X' of the point M are related as follows:

$$X = RX' + T \tag{1}$$

The inverse transformation reads

$$X' = R^T X - R^T T \tag{2}$$

A point M is projected on images on points ξ and ξ' respectively as shown in Fig. 1.

The line joining the camera centers C and C' intersects with the image plains in the points e and e' - the *epipoles*.

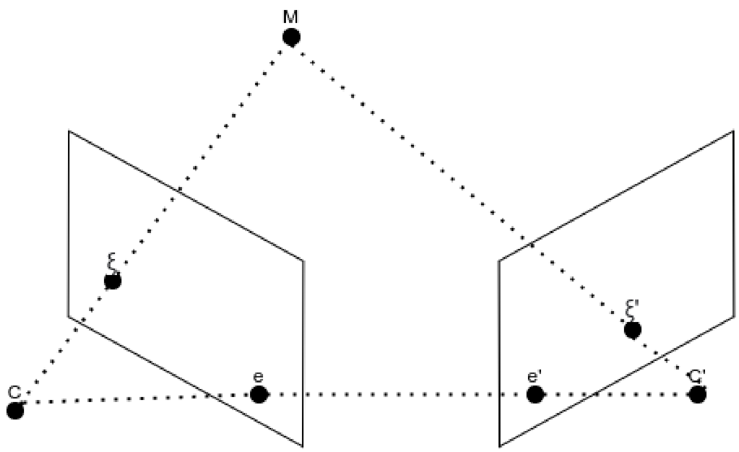

Figure 1: Epipolar geometry

Proposition: For any non-identical euclidean transformation (R, T) there exists a matrix $F \equiv F(R, T)$ such that

$$X^T F X' = 0. \tag{3}$$

Proof: It is easy to derive the coordinates of the epipoles in both coordinate systems. Since the coordinate of each camera center in its own coordinate system is $(0, 0, 0)$, the homogeneous coordinates of e and e' simply read:

$$e = T, \quad e' = -R^T T \tag{4}$$

Each point ξ on the left image corresponds to a ray $\lambda\xi$ in the 3D-space. It is obvious from the construction that this ray corresponds to a line joining ξ' and e' on the second image. On the other hand, a line passing through 2 homogeneous points x and y is given by a cross product

$$l = x \times y. \tag{5}$$

This automatically satisfies the condition

$$l \cdot x = l \cdot y = 0.$$

Applying this to the points e' and X' gives us

$$l' = e' \times X' = -(R^T T) \times (R^T X - R^T T) = -(R^T T) \times (R^T X) \equiv F X \tag{6}$$

By construction $X' \cdot l' = 0$, and this proves the statement.