Diffuse/Specular Polarization

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1 Introduction

1.1 Contributions

1. We demonstrate a method for separation of Diffuse and Specular reflection components using very few photographs for several viewpoints and polarimetric measurements.
2. We demonstrate that this method allows to estimate simultaneously material properties, including refractive index.
3. We show that compared to other methods ours is based on a fundamental difference between diffuse and specular components and the separation gives us true diffuse and specular component reflection intensities.
4. We also show that this separation allows to enhance even further the depth map quality for a Polarized 3D method.
5. ?POSSIBLY? As a side contribution - we developed a path tracer with light polarization properties, which was used to make a simulations in this paper and will be developed further as well as shared with the community.

2 Related Work

Newcombe, Nayar, Polarized 3D and all the other papers.

3 Theoretical Background

For almost any object the reflected light has two components: specular and diffuse. Each of the components also has two subcomponents: polarized and unpolarized. The proportion of specular polarized (S_p), specular unpolarized (S_{np}), diffuse polarized (D_p) and diffuse unpolarized (D_{np}) depends on the material of the object and the direction of the reflection. Due to the nature of diffuse and specular polarization, these two components have $\pi/2$ shift in polarization angle, and the resulting polarization of the light is a sum of these two components in terms of intensity, since the waves are incoherent.

$$I = S_p + S_{np} + D_p + D_{np} \tag{1}$$

Let's write down all the components in terms of Electric field:

$$\vec{E} = \vec{E}_0 + \vec{E}_{Dp} \sin(\phi_{pol} - \varphi) + \vec{E}_{Sp} \cos(\phi_{pol} - \varphi) \tag{2}$$

and since the waves are incoherent

$$\begin{aligned} |\vec{E}|^2 \propto I = I_0 + D_p \sin^2(\phi_{pol} - \varphi) + S_p \cos^2(\phi_{pol} - \varphi) = \\ = I_0 + \frac{S_p + D_p}{2} + \frac{S_p - D_p}{2} \cos(2(\phi_{pol} - \varphi)) \end{aligned} \tag{3}$$

As one can see from the equation 3, the phase shift doesn't depend on anything except which component - specular polarized or diffuse polarized is bigger. Indeed, if $S_p \gg D_p$ then

$$\begin{aligned} & I = I_0 + \frac{S_p}{2} + \frac{S_p}{2} \cos(2(\phi_{pol} - \varphi)), \\ & \text{and since } I_0 = I_{min}, S_p = I_{max} - I_{min}, \\ & I = \frac{I_{max} + I_{min}}{2} + \frac{I_{max} - I_{min}}{2} \cos(2(\phi_{pol} - \varphi)) \end{aligned} \tag{4}$$

and if $D_p \gg S_p$

$$\begin{aligned} I &= I_0 + \frac{D_p}{2} - \frac{D_p}{2} \cos(2(\phi_{pol} - \varphi)) = \\ &= I_0 + \frac{D_p}{2} + \frac{D_p}{2} \cos(2(\phi_{pol} - \varphi) - \pi) = \\ &= I_0 + \frac{D_p}{2} + \frac{D_p}{2} \cos(2(\phi_{pol} - (\varphi + \pi/2))) \\ & \text{and since } I_0 = I_{min}, D_p = I_{max} - I_{min}, \\ I &= \frac{I_{max} + I_{min}}{2} + \frac{I_{max} - I_{min}}{2} \cos(2(\phi_{pol} - (\varphi + \pi/2))) \end{aligned} \tag{5}$$

If neither of components is much bigger than the other one, then according to 3:

$$\begin{aligned} & I = I_0 + \frac{S_p + D_p}{2} + \frac{S_p - D_p}{2} \cos(2(\phi_{pol} - \varphi)) \\ & I_{min} = I_0 + \frac{S_p + D_p}{2} - \frac{S_p - D_p}{2} = I_0 + D_p; \\ & I_{max} = I_0 + \frac{S_p + D_p}{2} + \frac{S_p - D_p}{2} = I_0 + S_p; \\ & \rho = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{S_p - D_p}{2I_0 + S_p + D_p} \end{aligned} \tag{6}$$

Therefore, the proportion between D_p and S_p affects the degree of polarization and can make rapid $\pi/2$ phase shift. Moreover, if the value of D_p is close to S_p then the light becomes almost unpolarized.

Now, It is true that diffuse polarization usually is much weaker than specular polarization. And we admit that for many objects we measured, the S_p component was dominant, not diffuse one. This does not require changing the equations since projecting the degree of polarization values to zenith angle values is ambiguous anyways (remember, we also manually put some value for refractive index, this is the same thing) and the zenith angle is additionally optimized with coarse depth map. Therefore, choosing the way of initial calculation of zenith angle doesn't affect the result. For all the objects in the paper we had either dominant diffuse polarization measurement or dominant specular (in diffuse polarization dominance case we just manually rotated all the polarization angles in the measurement by $\pi/2$ before optimization process).

It is true that for some objects we can get partly diffuse and partly specular dominant component for different parts of the image and, as long as the change is low-frequency enough, fixing this with coarse depth map is straightforward and will involve one more additional optimization step. We plan to add this in the next revision of the algorithm.

4 Analyzing Model Mismatch

In this section, we quantify the error from model mismatch, as it pertains to estimating azimuth and zenith angles. Previous work has relied mostly on strength of empirical results without full quantification of model error. The error from the mismatch for a zenith angle estimation at each point of the object:

$$err = |\theta_r - \theta_m| \tag{7}$$

where θ_r - the real zenith at the point, θ_m - measured zenith angle with some mixture of diffuse and specular components occuring. Now, if assumed that the refractive index is known and fixed to n,

$$\theta = f(\rho) \tag{8}$$

and moreover, for each ρ there are two possible values for zenith angle. Basically, $\theta_r = f(\rho_r)$ and $\theta_m = f(\rho_m)$, where ρ_r is a degree of polarization in case if only specular reflection would occur and ρ_m is measured degree of polarization. Moreover,

$$\rho_m = \rho_r \frac{I_s}{I} = \rho_r \frac{I_s}{I_s + I_d} = \rho_r \frac{1}{1 + \frac{I_d}{I_s}} = \rho_r \frac{1}{1 + \alpha} \tag{9}$$

where α is a diffuse specular ratio. For each point α lies in the interval $[0, \infty)$, therefore ρ_m lies somewhere between 0 and ρ_r. Hence measured zenith angle is between 0 and θ_r, which means that for each point $err \in [0, \theta_r]$. Here we assume that we can choose the value of zenith angle among two possible options using coarse depth map.

5 Diffuse/Specular Reflection Separation

According to dichromatic reflection model, the light reflected from any point of the object can be represented as a sum of two components: diffuse and specular:

$$I = I_d + I_s \tag{10}$$

Wherein the diffuse component has an equal intensity in all the directions, and specular component is directional, depending on the direction of incoming light. Here we also assume that the diffuse reflection is unpolarized. In general, this is not exact, but it is true that the degree of diffuse polarization is almost an order of magnitude lower than degree of specular polarization. Now, let's write down what will be the degree of polarization of a light that is reflected from some object point:

$$\rho = \frac{I_s^{max} - I_s^{min}}{I_s^{max} + I_s^{min} + I_d} = \rho_0 \frac{I_s}{I} = \rho_0 \frac{I - I_d}{I} \tag{11}$$

where

$$\rho_0 = \frac{2\sin\theta\tan\theta\sqrt{n^2 - \sin^2\theta}}{n^2 - 2\sin^2\theta + \tan^2\theta} \tag{12}$$

and ρ is the measured Degree of Polarization. From Equations 11 and 12 we can easily get the diffuse reflection intensity:

$$I_d = I(1 - \rho\frac{n^2 - 2\sin^2\theta + \tan^2\theta}{2\sin\theta\tan\theta\sqrt{n^2 - \sin^2\theta}}) \tag{13}$$

The unknown parameters in this equation are intensity of diffuse reflection and refractive index. Since the I_d has the same value from all the viewpoints, it is possible to find both diffuse intensity and refractive index having measurement of the degree of polarization, intensity and zenith angle of the point from multiple viewpoints.

$$I_d = f(n, \theta_i, I_i, \rho_i) \tag{14}$$

Therefore several measurements from different viewpoints using depth camera and polarization camera give us a nonlinear system of equations, solving which will allow us to find intensities of both diffuse and specular reflections, as well as the refractive index of the material. Using the information about real refractive index and specular reflection intensity, we can calculate the real zenith angle for the high-frequency details of the object and therefore reconstruct real high-quality normal map and then shape.

5.1 Solving the Nonlinear System

One way to find the values for n and I_d is to write energy function in the following form:

$$E = \sum_{i=1}^{N}(I_d - f(n, \theta_i, I_i, \rho_i))^2 \tag{15}$$

and to minimize it for each point using SQP and varying I_d between 0 and I_i and n between 0 and 2. N here is the number of viewpoints.

6 Results

First, we show how the technique works in the simulations. Secondly, we do realworld measurements and compare with other existing techniques (Newcombe, Riviere, Nayar).

7 Conclusion

8 Implementation Details

Each simulation had three main steps: dataset generation, solving the optimization problem, and normal map estimation.

8.1 Dataset Generation

In order to generate datasets we first used Mitsuba raytracer to generate diffuse and specular reflection components separately from each of 3 viewpoints, as well as depth maps for each viewpoint to be able to reproject the same point on the object on the images from different viewpoints. After images are rendered we calculate the degree of polarization map based on the set refractive index, zenith angle from depth map and proportion between specular and diffuse components (see Equation 6 from the paper). For each point on the object from the viewpoint 0 we create a vector of 3 values of DOP and 3 values of measured intensity (sum of diffuse and specular components).

8.2 Solving the Optimization Problem

We use Matlab implementation of sequential quadratic programming to solve the optimization problem and find simultaneously diffuse component intensity and refractive index. The function is optimized pixel by pixel. For refractive index the bounds were set to $[1, 1.7]$ and for diffuse intensity the bounds were $[0, I]$, where I - is the full intensity at the point. It is worth mentioning that in order to calculate the refractive index only the points with the DOP above average for the object were used (since the value is much more robust for these points). As a possible way to improve the results even further, the estimated refractive index can be used to rerun the optimization and improve the results at points with low DOP. The optimization for 500 by 500 image takes on average about an hour on Core i5-2430 CPU.

8.3 Normal Map Estimation

We first estimate the real DOP for specular reflection using again Equation 6 from the main paper. To clean the noise we set all the values above 2 as NaN, and the values between 1 and 2 as 1 (DOP can't be more than 1). After that we estimate zenith angle point by point. For each point we need to fix the zenith angle ambiguity for specular polarization. We did it using depth data - the angle closer to the zenith angle from depth data was selected for each point (out of two possible values). Also, to make the estimation faster, we use precomputed discrete function of $\rho = f(n, \theta)$. This makes the estimation slightly less precise (1% error) , however, the computations are extremely fast (fraction of a second).