Dynamic Distributed Optimization of Energy Consumption: Real-Time Pricing Scenario

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***Abstract*— This study presents a dynamic distributed control algorithm with a consumption strategy for consumers in a smart grid based on real-time pricing (RTP) scenario. The proposed strategy seek to optimize the current curtailment cost, curtailment cost over a time-horizon, and the cost paid by each consumer, simultaneously. To this end, a dynamic convex optimization is solved at each time step. The algorithm improves the previous results in this area from two main perspectives. It converges to the Nash equilibrium by only one iteration based on utilizing the Newton-based approach. Furthermore, the proposed algorithm is dynamic, i.e., the forecast of future consumption is utilized to dynamically prevent overloading during peak times and reschedule the consumption for future time slots. Numerical results illustrate that the proposed distributed algorithm provides an efficient and optimal energy consumption solution by simply one iteration in each time slot.**

***Index Terms*—Real time pricing, consumption control, Newton's method, optimization**

I. Introduction

Nowadays, one of the most challenges in the energy market is to meet the peak demand. Increasing the capacity of the supply system is not an efficient solution anymore. Instead, the researchers have focused on demand-side management in smart grids. Specifically, implement ion of optimization-based control strategies for consumers would help to recommend (sub)optimal energy consumption at fixed time slots [1]-[4].

Real-time pricing mechanism firstly introduced in [5], modifies the behaviour of consumers by varying price signals. The interaction of consumers becomes more challenging introducing the users' comfort to the energy consumption mechanism. Utility maximization and load consumption minimization yield an optimization problem in which the consumption of each consumer is calculated in each time slot. The duration of the time slots depends on the system decision; it can be every quarter, hourly, or daily [6]-[8].

The primary challenge of solving the underlying optimization problem is the necessity of consumers' privacy. Private information of each consumer is not available for the other consumers. Accordingly, [9]-[10] have utilized the distributed dual sub-gradient approach to determine sub-optimal solutions in a distributed manner. However, convergence to the sub-optimal solution appeared to be slow and highly sensitive to the step size [13].

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As an alternative approach, the interactions among the users are modeled as a non-cooperative game. The utility of players is divided into two parts including the consumption price during the time slot and their welfare. The consumption price is affected by the real-time pricing method. The welfare of the players is estimated as a quadratic term by incorporation of the square error of the consumption level with respect to the desired consumption [6], [11], [12].

In this paper, the energy consumption problem is modeled as a non-cooperative game. We draw upon the main benefits of [6] from two main perspectives. In contrast to the previously developed methods, the proposed scheme is dynamic, i.e., the forecast of future consumption would help to dynamically prevent overloading during peak times and reschedule the consumption (if possible) for future time slots. The total desired load consumption plays a key role in dynamic form as well. Moreover, by utilizing Newton-based methods, the convergence in each time slot is achieved by only one single fixed iteration.

The rest of the paper is organized as follows. In Section II, the energy consumption problem is modeled as a non-cooperating game. The utility function and the feedback mechanism based on the real-time pricing is presented, as well. A dynamic approach for distributed energy management is presented in Section III, Section IV illustrates the simulation results and Section V concludes the paper.

II. Problem Statement

Consider an energy market comprising an energy provider and several consumers as shown in Figure 1. The energy provider sells electricity to the consumers based on their request in each time slot. The information exchange is permitted only between each consumer and the provider. Let N and T be the number of consumers, and the number of time slots in a day, respectively. The consumption of consumer i at time slot k is denoted by l_i^k. To model the Real-Time Pricing scheme, assume that at time slot k, the provider announces the linear pricing strategy as,

$$p(\boldsymbol{l}^k) = \lambda \sum_{i \in N} l_i^k + p_0 \tag{1}$$

Where $p(\boldsymbol{l}^k)$ denotes the energy price which is set by the provider in the time-slot k. p_0 is the base price and λ is an elastic parameter in the pricing function regulated by the provider in order to match supply with demand. (1) reveals that the price at each time slot is determined by the total consumption. Accordingly, excessive load consumption by a consumer results in higher costs paid by all the consumers in

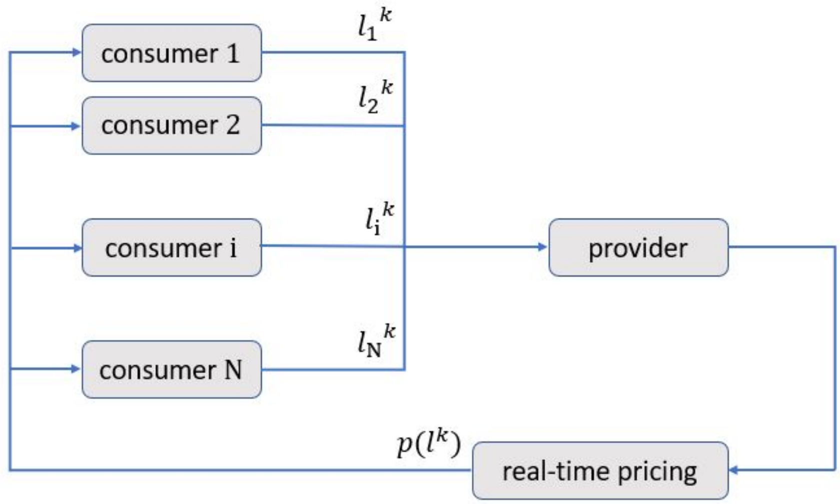

Fig. 1. Energy consumption system loop based on real-time pricing model for time slot k

the network. However, each consumer should decide about its consumption based on its own welfare and consumption information and a feedback signal from the provider regarding the total price $p(\boldsymbol{l}^k)$ at time slot k. In this paper, the problem is formulated as a non-cooperative game where each consumer is regarded as an active consumer.

The utility function of consumer i at time slot k denoted by U_i^k is introduced as,

$$U_i^k = -\sum_{j=k}^{T} \theta \left(l_i^j - \hat{l}_i^j \right)^2 - \sum_{j=k}^{T} \left(\lambda \sum_{i=1}^{N} l_i^j + p_0 \right) l_i^j - \alpha \left(\sum_{j=k}^{T} l_i^j - \sum_{j=k}^{T} \hat{l}_i^j - \sum_{j=1}^{k-1} \hat{l}_{di}^j \right)^2 \tag{2}$$

Based on (2), the utility function comprises three terms:

1) The first term denotes the load curtailment cost along the determined time horizon. The consumer i tries to keep the desired consumption schedule denoted by $\hat{l}_i^j$ for $j = k, k+1, \dots T$.
2) The second term represents the electricity cost paid by the consumer i during the time slots $j = k, k+1, \dots T$.
3) The third term tries to compensate the load curtailment at the past time slots $j = 1, \dots k-1$ by introducing a quadratic term to match the total desired consumption and total consumption. The term $\hat{l}_{di}^j$ denotes the consumption mismatch with the desired values at time slots $j = 0, 1, \dots k-1$ which can be reprogrammed for the time slots $j = k, k+1, \dots T$.

Moreover, the regularization parameter θ regulates the level of emphasis on the desired load consumption in each time slot, while parameter α demonstrates the importance of consuming the total required load over a time horizon. The utility function in (2) is concave and quadratic. It will be shown in the next Section that under some mild conditions, this non-cooperative game has a unique Nash equilibrium. Consequently, an efficient distributed optimization algorithm would be proposed to solve the game with the utility function given in (2).

III. Demand-Side Distributed Energy Management: A Newton-based Approach

The first step is to investigate the existence and uniqueness of the Nash equilibrium in the non-cooperative game discussed in Section II. The following Theorem shows that under some mild conditions, the Nash equilibrium of the non-cooperative game is unique.

Theorem 1: The non-cooperative game of dynamic energy consumption with real-time pricing scheme of (1) and utility function (2) has a unique Nash equilibrium at time-slot k if,

$$\lambda \leq \frac{2\theta + 2\alpha - 2\alpha T + 2\alpha k}{N-3}.$$

Proof: To show the existence of the Nash equilibrium, it will be shown that the utility function U_j^k is concave with respect to the variable,

$$l_i = \begin{bmatrix} l_i^k & l_i^{k+1} & \dots & l_i^T \end{bmatrix}.$$

The second-derivative of U_j^k with respect to the vector l_i results in the Hessian matrix given by,

$$H_{l_i} = \begin{bmatrix} -2\theta - 2\lambda - 2\alpha & \cdots & -2\alpha \\ \vdots & \ddots & \vdots \\ -2\alpha & \cdots & -2\theta - 2\lambda - 2\alpha \end{bmatrix}_{(T-k+1)\times(T-k+1)}$$

$$= -2\alpha \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix} - 2(\theta + \lambda) I$$

Thus, the Hessian matrix can be expressed as sum of two symmetric matrices. The first matrix is negative semi-definite since it has $T-k$ eigenvalues equal to zero (note that it is a rank one matrix), and one eigenvalue equal to $\alpha(T-k+1)$. The second matrix is also a diagonal negative semi-definite matrix. Accordingly, the Hessian matrix with respect to the variable l_i is negative semi-definite and the utility function is concave with respect to the l_i. As a consequence, the Nash equilibrium always exists.

To investigate the uniqueness of Nash equilibrium in dynamic case, an upper-bound condition on the price factor λ is obtained under which the utility function U_j^k is concave with respect to the overall vector,

$$l=\begin{bmatrix}l_1^k & l_1^{k+1} & \dots & l_1^T & \dots & l_N^k & l_N^{k+1} & \dots & l_N^T\end{bmatrix}.$$

The second-derivative of U_j^k with respect to the new vector l results in the Hessian matrix given by,

$$H_l=\begin{bmatrix}H_{l_1} & \cdots & -2\lambda I\\ \vdots & \ddots & \vdots\\ -2\lambda I & \cdots & H_{l_N}\end{bmatrix}.$$

The Hessian matrix is a $(T-k+1)N\times(T-k+1)N$ with diagonal blocks equal to H_{l_1} obtained before. The off-diagonal blocks are all equal to $-2\lambda I_{(T-k+1)}$.. Each row (and each column) of H_l comprises i) a diagonal term $-2\theta-2\lambda-2\alpha$, ii) $T-k$ off-diagonal elements of H_{l_i} equal to -2α, and iii) $N-1$ elements equal to -2λ which corresponds to $N-1$ blocks of $-2\lambda I$. Since the diagonal elements of H_l is negative, its diagonal dominance can provide a sufficient condition for its negative definiteness [17]. Accordingly, by considering an upper-bound for λ given as,

$$\lambda\le\frac{2\theta+2\alpha-2\alpha T+2\alpha k}{N-3},$$

at time slot k, the uniqueness of Nash equilibrium is guaranteed [18]. ∎

A. Distributed Static Optimization or Energy Consumption Based on Newton's Method

For simplicity, first assume that we neglect the dynamic interactions and accordingly, the coefficient α in (2) is zero. This scenario is previously studied in [6] and an iterative algorithm is proposed whose performance is highly dependent on the selection of step-size and number of iterations. In this section, we show that Newton-based algorithms can be utilized to solve the problem and obtain the Nash equilibrium in a single iteration.

Assuming $\alpha=0$, the utility function of consumer i at time slot k takes the form,

$$U_i^k=-\sum_{j=k}^{T}\theta\left(l_i^j-\hat{l}_i^j\right)^2-\sum_{j=k}^{T}\left(\lambda\sum_{i=1}^{N}l_i^j+p_0\right)l_i^j. \quad (3)$$

To maximize the utility function (3) a Newton-based algorithm is applicable. Implementation of each iteration of a distributed optimization algorithm in a real-time pricing scheme would require communication of the consumers with the provider. Since the number of iterations in Newton-based algorithm can be much less than first-order methods, it would help to reduce communication and computation burden and enhance the reliability of the dynamics.

Each step of the Newton algorithm is supposed to find the stationary points of the function's second-order Taylor series approximations [15], [16]. For an arbitrary smooth convex function $f(x)$, every step of the Newton algorithm uses the Hessian matrix associated with $f(x)$ to update the decision variable x as follows,

$$x^{(n+1)}=x^{(n)}-\left(\nabla^2 f\left(x^{(n)}\right)\right)^{-1}.\nabla f\left(x^{(n)}\right). \quad (4)$$

Unlike the gradient descent optimization, when the second-order Taylor series approximations is the same as the total cost function (which is the case for general utility function (2) and special static case (3) as well), the Newton-based algorithm convergences to the extremum by only a single iteration. Accordingly, to obtain the Nash equilibrium, consumer i may apply the Newton algorithm to the objective function (3) with respect to its own set of variables l_i^k as follows,

$$l_i^{(n+1)}=l_i^{(n)}-\left(\nabla^2U_i(l_i^{(n)})\right)^{-1}.\nabla U_i(l_i^{(n)}) \quad (5)$$

in which the variable vector l_i, the gradient vector $\nabla U_i(l_i)$, and the Hessian matrix $\nabla^2U_i(l_i)$ are given by,

$$l_i=\begin{bmatrix}l^1\\ \vdots\\ l^T\end{bmatrix},\quad \nabla U_i(l_i)=\begin{bmatrix}\frac{\partial U_i}{\partial l^1}\\ \vdots\\ \frac{\partial U_i}{\partial l^T}\end{bmatrix},$$

$$\nabla^2U_i(l_i)=\begin{bmatrix}\frac{\partial^2U_i}{\partial (l^1)^2} & \cdots & \frac{\partial^2U_i}{\partial l^1\partial l^T}\\ \vdots & \ddots & \vdots\\ \frac{\partial^2U_i}{\partial l^T\partial l^1} & \cdots & \frac{\partial^2U_i}{\partial (l^T)^2}\end{bmatrix}.$$

Subsequently, the one step update from any arbitrary initial point $l_i^{(0)}$ for consumer i is given by,

$$\begin{bmatrix}l_i^1\\ \vdots\\ l_i^T\end{bmatrix}=\begin{bmatrix}l_i^1\\ \vdots\\ l_i^T\end{bmatrix}^{(0)}-\begin{bmatrix}-2\theta-2\lambda & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & -2\theta-2\lambda\end{bmatrix}^{-1}\times\begin{bmatrix}-2\theta\left(l_i^1-\hat{l}_i^1\right)-\lambda l_i^1-p(l^1)\\ \vdots\\ -2\theta\left(l_i^T-\hat{l}_i^T\right)-\lambda l_i^T-p(l^T)\end{bmatrix} \quad (6)$$

The Hessian matrix in (6) is positive definite and thus the optimal point is achievable by only a single iteration through Newton algorithm. Note that in the static case, i.e., the utility function given by (3), the objective function at time slot k is independent of the utility at time-slot $k+1$. Accordingly, the Hessian matrix with respect to the variable l_i takes a diagonal form as shown in (6).

The equation reveals a couple of important points from implementation point of view, as well. First, note that the per-iteration cost in the Newton-based algorithms are substantially significant. However, in this setting the Hessian matrix is diagonal and thus the consumption of consumer i can be easily updated without necessity of computing the inverse of a $T \times T$ matrix, as follows,

$$l_i^k = {l_i^k}^0 + \frac{1}{2\theta + 2\lambda} \left(-2\theta \left({l_i^k}^0 - \hat{l}_i^k \right) - \lambda {l_i^k}^0 - p(l^k) \right) \quad (7)$$

Furthermore, in (6), all of the required information from the other consumers can be stacked as the price $p(l^k)$. Accordingly, the consumer i only requires a feedback signal from the provider to get access to the real-time price, without any level of communication with the other consumers. Only one iteration is enough to achieve the optimal value of consumption for the demand-side. The result remains independent of the initial point $l_i^{(0)}$ according to the optimally of the step-size in Newton algorithm for convex quadratic programs [19]. Consequently, Newton's method has superiority over other optimization approaches such as the gradient descent method [6].

B. Distributed Dynamic Optimization or Energy Consumption Based on Newton's Method

The procedure in subsection III-A can effectively reduce the peak of demand. However, an important challenge from the demand-side point of view is to shift the consumption of the loads for the future as well. Accordingly, in this Subsection, the proposed framework in Subsection III-A is extended to the case where the decision about consumption in time slot k is dependent on what is planned for the next time slots and how much consumption is sacrificed from the beginning of the duration up to time k. Accordingly, the time shiftable loads can be rescheduled for the next time slots. To this end, consider the utility function in (2) for the general case $\alpha > 0$. In this case, the utility for consumer i depends on the total consumption as well. Accordingly, the consumer has the opportunity to postpone its consumption to some other time slots.

The following steps are performed at time slot k to determine the consumption schedule dynamically for $k,\ k+1,\ \dots T$.

1) Solve an optimization problem with the utility function (2) and $\alpha > 0$ over the time horizon $k,\ k+1 \dots T$ and obtain the optimal solution ${l_i^k}^*,\ {l_i^{k+1}}^*,\ \dots\ {l_i^T}^*$.
2) Follow the first step of the schedule ${l_i^k}^*$.

Similar to the static case, the objective is to solve (2) and achieve the Nash equilibrium by only a single iteration.

Application of the Newton algorithm to the dynamic case results in the following set of updates for consumer i at time slot k, as illustrated in (8).

Note that based on the proof of Theorem 1, the Hessian matrix used in (8) is negative definite, and accordingly, the optimal point is achievable by only a single iteration through Newton algorithm. Again, note that all of the required information from the other consumers can be stacked as the price vector with the elements $p(l^k),\ p(l^{k+1}), \dots p(l^T)$. Accordingly, the consumer i only requires a feedback signal from the provider to get access to the real-time price, without any level of communication with the other consumers. Only one iteration is enough to achieve the optimal value of consumption for the demand-side. The result remain independent of the initial point $l_i^{(0)}$ according to the optimally of the step-size in Newton algorithm for convex quadratic programs.

IV. Simulation Studies

In this section, the proposed algorithm for distributed consumption management is evaluated in the static and dynamic cases. The results of the static case are also compared with the previously developed method in [6]. To this end, PJM Hourly Energy Consumption Data, available at www.Kaggle.com, is utilized. The energy market is simulated with a provider and $N = 30$ consumers. Each consumer acts as a single player in a non-cooperative game and maximizes its utility based on the static and dynamic distributed optimizations discussed in Section III.

The results of the non-iterative algorithm, in terms of the suggested total consumption schedule in a day for the static and dynamic cases are reported in Figure 2. The desired level of consumption is also illustrated in Figure 2. The dynamic Newton-based algorithm tries to postpone some parts of the consumption for future time-slots. The results for consumer 1 is also reported in Figure 3.

Moreover, the total cost versus time is plotted for the first consumer in Figure 4. Note that the total cost corresponds to the utility with a minus sign. Accordingly, minimization of the total costs plotted in Figure 4, corresponds to the utility maximization. Let I represent the percent of improvement in the total utility function over a day by the dynamic model. By evaluating I as,

$$I = \frac{U_{\text{dynamic}} - U_{\text{static}}}{|U_{\text{dynamic}}|} \times 100 \quad (9)$$

Where U shows the total utility in each case calculated over the 24 hours. In this experiment, an improvement of 10% is achieved.

Finally, Fig. 5 provides a comparison of the total utility of all the consumers in the static mode, achieved by the Newton's method and gradient descent method in [6]. Improvement of the results is achieved from two main perspectives. Firstly, the total cost of the Newton method is always smaller than the total cost achieved by the gradient descent. Secondly, the results of the Newton-based method requires only one iteration while the results of the gradient descent method are obtained after nine iterations.

V. Conclusions

Demand-side optimization in an energy market with a real-time pricing scenario is studied. The proposed algorithm improves the previous results in this area from two main perspectives. It converges to the Nash equilibrium by only one iteration based on utilizing the Newton-based approach. Furthermore, the proposed algorithm is dynamic, i.e., the

$$
\begin{bmatrix} l_i^k \\ \vdots \\ l_i^T \end{bmatrix} = \begin{bmatrix} l_i^k \\ \vdots \\ l_i^T \end{bmatrix}^{(0)} - \begin{bmatrix} -2\theta - 2\lambda - 2\alpha & \cdots & -2\alpha \\ \vdots & \ddots & \vdots \\ -2\alpha & \cdots & -2\theta - 2\lambda - 2\alpha \end{bmatrix}^{-1}
$$
$$
\times \begin{bmatrix} -2\theta\left(l_i^k - \hat{l}_i^k\right) - \lambda l_i^k - p(l^k) - 2\alpha\left(\sum_{j=k}^{T} l_i^j - \sum_{j=k}^{T} \hat{l}_i^j - \sum_{j=1}^{k-1} \hat{l}_{di}^j\right) \\ \vdots \\ -2\theta\left(l_i^T - \hat{l}_i^T\right) - \lambda l_i^T - p(l^T) - 2\alpha\left(\sum_{j=T}^{T} l_i^j - \sum_{j=T}^{T} \hat{l}_i^j - \sum_{j=1}^{T-1} \hat{l}_{di}^j\right) \end{bmatrix}. \tag{8}
$$

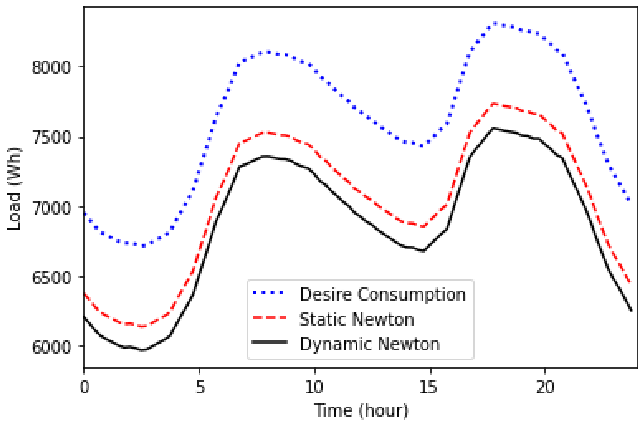

Fig. 2. Total load versus time (hour)

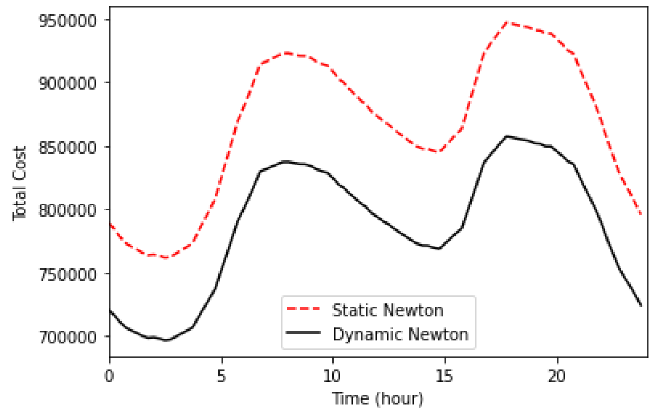

Fig. 4. Total Cost versus time (hour)

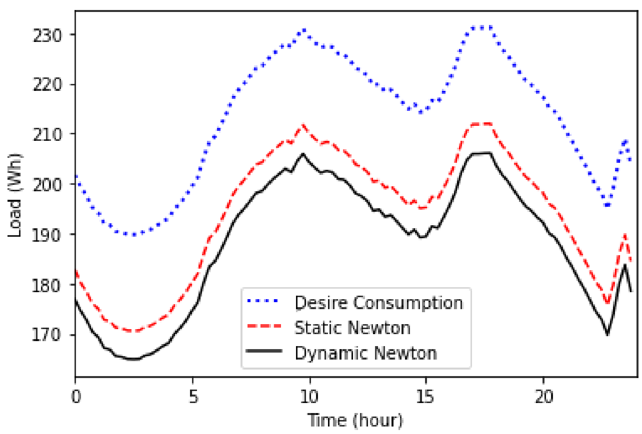

Fig. 3. Load versus time (hour) for the first consumer

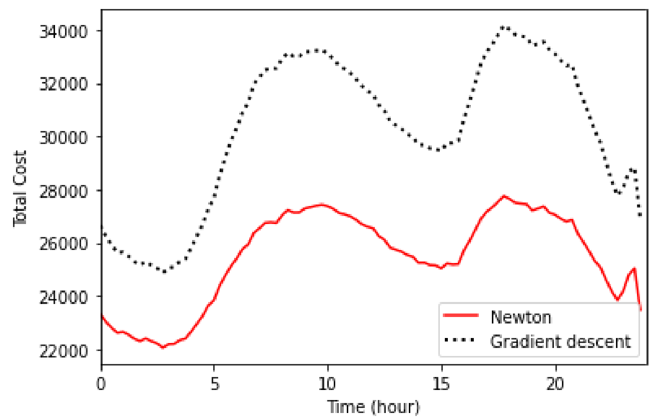

Fig. 5. Total Cost versus time (hour)

forecast of future consumption would help to dynamically prevent overloading during peak times and reschedule the consumption (if possible) for future time slots. Future research directions include investigations on a non-cooperative game with multiple providers and extending the algorithm to the case where the real-time pricing scenario is not linear.

References

[1] H. Wang and Y. Gao, Distributed real-time pricing method incorporating load uncertainty based on nonsmooth equations for smart grid. Mathematical Problems in Engineering, 2019.

[2] J. Morikawa, T. Yamaguchi, and C. Ninagawa, Smart grid real-time pricing optimization management on power consumption of building multi-type air-conditioners. IEEJ Transactions on Electrical and Electronic Engineering, vol. 11, no.6, pp. 823-825, 2016.

[3] Ma. Kai, Hu. Guoqiang, and Spanos. Costas "Distributed Energy Consumption Control via Real-Time Pricing Feedback in Smart Grid," in: IEEE Transactions on Control Systems Technology, Volume: 22, Issue: 5, Sept. 2014.

[4] N. Antonin and P. Energetika, "Ripple control in the Czech Republic and demand side management," in Proc. 20th Int. Conf. Exhibit. Electr. Distrib., Jun. 2009, pp. 1–5.

[5] M. C. Caramanis, R. E. Bohn, and F. C. Schweppe. "Optimal spot pricing: Practice and theory." IEEE Transactions on Power Apparatus and Systems, vol. 9, pp. 3234-3245, 1982

[6] J. Kondoh, N. Lu, and D. J. Hammerstrom, "An evaluation of the water heater load potential for providing regulation service," IEEE Trans. Power Syst., vol. 26, no. 3, pp. 1309–1316, Aug. 2011.

[7] N. Lu, "An evaluation of the HVAC load potential for providing load balancing service," IEEE Trans. Smart Grid, vol. 3, no. 3, pp.

1263–1270, Sep. 2012.

[8] M. H. Albadi and E. F. El-Saadany, “A summary of demand response in electricity markets,” Electron. Power Syst. Res., vol. 78, no. 11, pp. 1989–1996, 2008.

[9] Y. Wang, S. Mao, and R.M., Nelms, “Distributed online algorithm for optimal real-time energy distribution in the smart grid,” IEEE Internet of Things Journal, vol. 1, no. 1 pp.70-80, 2014.

[10] P. Samadi, H. Mohsenian-Rad, R. Schober, and V. W. S. Wong, “Advanced demand side management for the future smart grid usingmechanism design,” IEEE Transactions on SmartGrid, vol. 3, no. 3, pp. 1170–1180, 2012

[11] A.-H. M. Rad, V. W. S. Wong, J. Jatskevich, R. Schober, and A. L. Garcia, “Autonomous demand-Side management based on gametheoretic energy consumption scheduling for the future smart grid,” IEEE Trans. Smart Grid, vol. 1, no. 3, pp. 320–331, Dec. 2010.

[12] Z. Baharlouei, M. Hashemi, H. Narimani, and A.-H. M. Rad, “Achieving optimality and fairness in autonomous demand response: Benchmarks and billing mechanisms,” IEEE Trans. Smart Grid, vol. 4, no. 2, pp. 968–975, May 2013.

[13] N. Gatsis and G. B. Giannakis, “Residential load control: Distributed scheduling and convergence with lost AMI messages,” IEEE Trans. Smart Grid, vol. 3, no. 2, pp. 770–786, Jun. 2012.

[14] Deming, W. Edwards. The New Economics: For Industry, Government, Education. MIT Press. 1993

[15] A.Galántai, “The theory of Newton’s method,” Journal of Computational and Applied Mathematics Volume 124, Issues 1–2, 1 December 2000, Pages 25-44

[16] Yamamoto, Tetsuro ”Historical Developments in Convergence Analysis for Newton’s and Newton-like Methods”. In Brezinski, C.; Wuytack, L. (eds.). Numerical Analysis : Historical Developments in the 20th Century. North-Holland. 2010. pp. 241–263

[17] R. Horn and C. Johnson, Matrix Analysis. Cambridge, U.K.: Cambridge, Univ. Press, 1999

[18] J. Rosen, “Existence and uniqueness of equilibrium points for concave n-person games,” Econometrica, vol. 33, no. 3, pp. 520–534, 1965.

[19] S. Boyd, and L. Vandenberghe, “Convex optimization”. Cambridge university press, 2004.