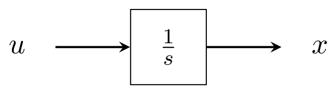
NKG Summer School

Exercise 1 - Tropospheric Delay

Obtaining a realistic model of the tropospheric delay is one of the main challenges when processing raw GNSS observations. One way to handle this delay is to assume it behaves like a stochastic process and use a Kalman filter to provide an estimate of the delay.

Assume that the tropospheric delay can be modeled as random walk, i.e. integrated white noise. The process can then be written as follows:



$$\dot{x} = u$$

The random fluctuations of the process is given as $q = (0.01\,\mathrm{m}^2)/\mathrm{s}$, and the time interval is $\Delta t = 1\,\mathrm{s}$.

The initial values for the process and variance is given as:

$$x_0 = 1\,\mathrm{m} \quad P_0 = 1\,\mathrm{m}^2$$

1. (a) Compute predicted values $\tilde{x}$ and the corresponding variance $\tilde{P}$ from t_1 to t_{100}.

   (b) Plot the predicted values $\tilde{x}$ together with the corresponding variances $\tilde{P}$.

A more realistic model would be to assume that the tropospheric delay can be modeled as a first order Markov process with $\sigma = 0.5\,\mathrm{m}$ and $\beta = 0.1\,\mathrm{rad\,min^{-1}}$, i.e. a time constant of $10\,\mathrm{min}$.

The Power Spectral Density (PSD) of a first order Gauss-Markov process is given by:

$$S_X = \frac{2\sigma^2\beta}{-s^2 + \beta^2} \tag{1}$$

The process can be written as follows:

$$u \longrightarrow \boxed{\frac{\sqrt{2\sigma^2\beta}}{s+\beta}} \longrightarrow x$$

$$\dot{x} = -\beta x + \sqrt{2\sigma^2\beta}\,u \tag{2}$$

2. (a) Compute predicted values $\tilde{x}$ and the corresponding variances $\tilde{P}$ from t_1 to t_{100}.

   (b) Plot the predicted values $\tilde{x}$ together with the corresponding variances $\tilde{P}$.

   (c) How much has the process value decayed after $10\,\mathrm{min}$, i.e. after one time constant?

Discrete Kalman Filter Loop

Time update

$$\begin{aligned}\tilde{x}_k =& \phi_{k-1}\hat{x}_{k-1} \\ \tilde{P}_k =& \phi_{k-1}\hat{P}_{k-1}\phi_{k-1}^T + Q_{k-1}\end{aligned}$$

Measurement update

$$\begin{aligned}K_k =& \tilde{P}_k H_k^T (H_k \tilde{P}_k H_k^T + R_k)^{-1} \\ \hat{x}_k =& \tilde{x}_k + K_k(z_k - H_k\tilde{x}_k) \\ \hat{P}_k =& (I - K_k H_k)\tilde{P}_k\end{aligned}$$