NKG Summer School 2021

August 31, 2021

1 NKG Summer School 2021

Lecture notes

1.0.1 Your Turn (slide 17)

[1]:
```
from sympy import Matrix, eye, symbols, inverse_laplace_transform

# Define symbols
s, t, phi = symbols('s t phi', positive=True)

# Dynamic matrix
F = Matrix([[0, 1],
            [0, 0]])

# Identity matrix
I = eye(2)

S = (s*I-F).inv()
display(S)

# Transition matrix
phi = inverse_laplace_transform(S, s, t)
display(phi)
```

$$\begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

1.0.2 Your Turn (slide 21)

[2]:
```
from numpy import array, sqrt
from vanloan import numeval

# System parameters
dt = 5    # [seconds]
```

```python
q = 0.3   # [meter^2/second]


# Dynamic matrix
F = array([[0, 1],
           [0, 0]])

# White noise coefficients
G = array([[0],
           [sqrt(q)]])

# Van Loan numerical evaluation
phi, Q = numeval(F, G, dt)
print(phi)
print(Q)
```

```
[[ 1.  5.]
 [-0.  1.]]
[[12.5   3.75]
 [ 3.75  1.5 ]]
```

1.0.3 Your Turn (slide 24)

[3]:
```python
from sympy import Matrix, symbols, atan, diff, simplify, sqrt

x0, y0, x1, y1 = symbols('x0 y0 x1 y1')

alpha = atan((y1 - y0)/(x1 - x0))
s = sqrt((x1 - x0)**2 + (y1 - y0)**2)

# Heading
a1 = diff(alpha, x0)
a2 = diff(alpha, y0)
display(simplify(a1))
display(simplify(a2))
```

$$\frac{-y_0 + y_1}{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$

$$\frac{x_0 - x_1}{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$

[4]:
```python
# Distance
b1 = diff(s, x0)
b2 = diff(s, y0)
display(simplify(b1))
display(simplify(b2))
```

$$\frac{x_0 - x_1}{\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}}$$

$$\frac{y_0 - y_1}{\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}}$$

1.0.4 Your Turn (slide 33)

[5]:

```python
from sympy import symbols, solveset, factor

s = symbols('s')

S = 16/(s**4 + 64)
display(S)

display(solveset(s**4 + 64, s))

Sf = factor(S)
display(Sf)
```

$$\frac{16}{s^4 + 64}$$

$$\{-2 - 2i, -2 + 2i, 2 - 2i, 2 + 2i\}$$

$$\frac{16}{(s^2 - 4s + 8)(s^2 + 4s + 8)}$$

[6]:

```python
solveset(s**2 + 4*s + 8)
```

[6]: $\{-2 - 2i, -2 + 2i\}$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8 & -4 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u$$