NKG Summer School

## Exercise 3 - Sensor Fusion

Given a 2-dimensional car navigation system that consists of three sensors that provides information about the position. The available measurements are:

- Position (2D) from GNSS
- Heading from digital compass
- Speed from gearbox

The measurements from the available sensors are given in the file: `data.txt`

We assume the car maintains a fairly constant speed, however it will experience random accelerations as it drives along. The random accelerations has the magnitude of $\sqrt{q}$ in both directions.

The state vector elements $x$ are given as:

$$x = \begin{bmatrix} x_n \\ \dot{x}_n \\ x_e \\ \dot{x}_e \end{bmatrix}$$

Initial values:

$$x_0 = \begin{bmatrix} 0 \\ 15\,\mathrm{m\,s^{-1}} \\ 0 \\ 0 \end{bmatrix} \qquad P_0 = \begin{bmatrix} (3\,\mathrm{m})^2 & 0 & 0 & 0 \\ 0 & (1\,\mathrm{m\,s^{-1}})^2 & 0 & 0 \\ 0 & 0 & (3\,\mathrm{m})^2 & 0 \\ 0 & 0 & 0 & (1\,\mathrm{m\,s^{-1}})^2 \end{bmatrix}$$

Measurement noise: $\sigma_n = \sigma_e = 2\,\mathrm{m}$, $\sigma_{vn} = \sigma_{ve} = 1\,\mathrm{m\,s^{-1}}$ and $\sigma_{head} = 0.01\,\mathrm{rad}$.

Process noise: $q = (1\,\mathrm{m^2/s^2})/\mathrm{s}$

## Exercise

1. (a) Derive the dynamic equations for the car navigation system.

   (b) Derive the measurement equations relating the observed position to the elements of the state vector

   (c) Derive the measurement equations relating the observed heading and speed to the elements of the state vector.

2. (a) Design a simple Kalman filter for the car navigation system and estimate the positions and velocities of the car based in the available sensor readings from $t_1$ to $t_{30}$.

   (b) Plot the estimated trajectory of the car together with the estimated standard deviation of the elements of the state vector.

## Discrete Kalman Filter Loop

*Time update*

$$
\begin{aligned}
\tilde{x}_k =& \phi_{k-1}\hat{x}_{k-1} \\
\tilde{P}_k =& \phi_{k-1}\hat{P}_{k-1}\phi_{k-1}^T + Q_{k-1}
\end{aligned}
$$

*Measurement update*

$$
\begin{aligned}
K_k =& \tilde{P}_k H_k^T (H_k \tilde{P}_k H_k^T + R_k)^{-1} \\
\hat{x}_k =& \tilde{x}_k + K_k(z_k - H_k \tilde{x}_k) \\
\hat{P}_k =& (I - K_k H_k)\tilde{P}_k
\end{aligned}
$$