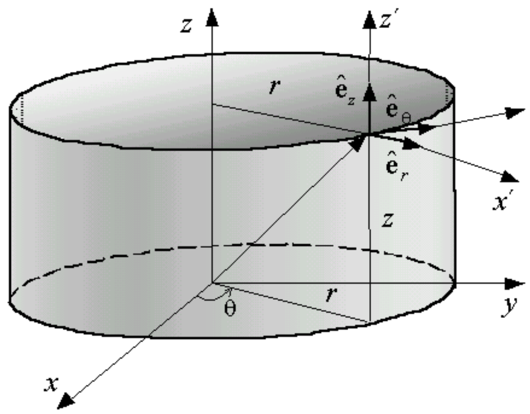
Appendix 1.

Infinitesimal Strain Tensor

Cylindrical Coordinates



$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{rr} & \varepsilon_{r\theta} & \varepsilon_{rz} \\ \varepsilon_{r\theta} & \varepsilon_{\theta\theta} & \varepsilon_{\theta z} \\ \varepsilon_{rz} & \varepsilon_{\theta z} & \varepsilon_{zz} \end{bmatrix}$$

$$\varepsilon_{rr} = \frac{\partial \mathrm{u}_r}{\partial r} \qquad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial \mathrm{u}_\theta}{\partial \theta} + \frac{\mathrm{u}_r}{r} \qquad \varepsilon_{zz} = \frac{\partial \mathrm{u}_z}{\partial z}$$

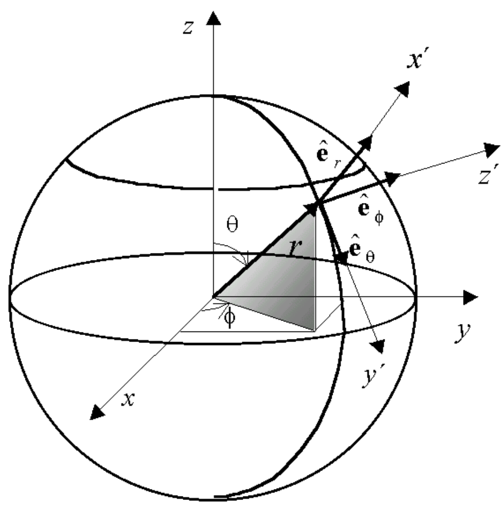
$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial \mathrm{u}_r}{\partial \theta} + \frac{\partial \mathrm{u}_\theta}{\partial r} - \frac{\mathrm{u}_\theta}{r}\right)$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial \mathrm{u}_r}{\partial z} + \frac{\partial \mathrm{u}_z}{\partial r}\right)$$

$$\varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial \mathrm{u}_\theta}{\partial z} + \frac{1}{r}\frac{\partial \mathrm{u}_z}{\partial \theta}\right)$$

$$\mathbf{x}(r,\theta,z) \equiv \begin{cases} x = r\cos\theta \\ y = r\,sin\theta \\ z = z \end{cases}$$

Spherical Coordinates



$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{rr} & \varepsilon_{r\theta} & \varepsilon_{r\phi} \\ \varepsilon_{r\theta} & \varepsilon_{\theta\theta} & \varepsilon_{\theta\phi} \\ \varepsilon_{r\phi} & \varepsilon_{\theta\phi} & \varepsilon_{\phi\phi} \end{bmatrix}$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \qquad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial \mathrm{u}_\theta}{\partial \theta} + \frac{\mathrm{u}_r}{r}$$

$$\varepsilon_{\phi\phi} = \frac{1}{r\cdot\sin\theta}\frac{\partial \mathrm{u}_\phi}{\partial \phi} + \frac{\mathrm{u}_\theta}{r}\cot\theta + \frac{\mathrm{u}_r}{r}$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial \mathrm{u}_r}{\partial \theta} + \frac{\partial \mathrm{u}_\theta}{\partial r} - \frac{\mathrm{u}_\theta}{r}\right)$$

$$\varepsilon_{r\phi} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial \mathrm{u}_r}{\partial \phi} + \frac{\partial \mathrm{u}_\phi}{\partial r} - \frac{\mathrm{u}_\phi}{r}\right)$$

$$\varepsilon_{\theta\phi} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial \mathrm{u}_\theta}{\partial \phi} + \frac{1}{r}\frac{\partial \mathrm{u}_\phi}{\partial \theta} - \frac{\mathrm{u}_\phi}{r}\cot\phi\right)$$

$$\mathbf{x}(r,\theta,\phi) \equiv \begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

Deformation Rate Tensor

Cylindrical Coordinates

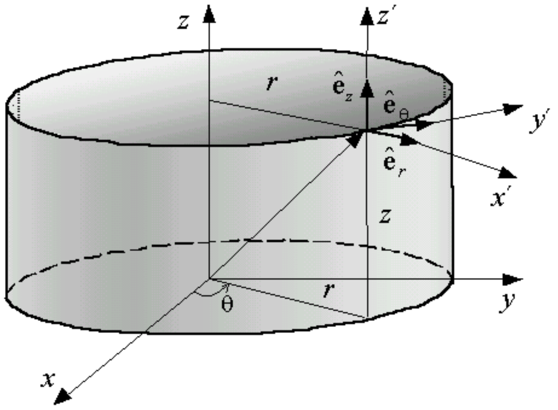
$$\mathbf{d} = \begin{bmatrix} d_{rr} & d_{r\theta} & d_{rz} \\ d_{r\theta} & d_{\theta\theta} & d_{\theta z} \\ d_{rz} & d_{\theta z} & d_{zz} \end{bmatrix}$$

$$d_{rr} = \frac{\partial \mathrm{v}_r}{\partial r} \qquad d_{\theta\theta} = \frac{1}{r}\frac{\partial \mathrm{v}_\theta}{\partial \theta} + \frac{\mathrm{u}_r}{r} \qquad d_{zz} = \frac{\partial \mathrm{v}_z}{\partial z}$$

$$d_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial \mathrm{v}_r}{\partial \theta} + \frac{\partial \mathrm{v}_\theta}{\partial r} - \frac{\mathrm{v}_\theta}{r}\right)$$

$$d_{rz} = \frac{1}{2}\left(\frac{\partial \mathrm{v}_r}{\partial z} + \frac{\partial \mathrm{v}_z}{\partial r}\right)$$

$$d_{\theta z} = \frac{1}{2}\left(\frac{\partial \mathrm{v}_\theta}{\partial z} + \frac{1}{r}\frac{\partial \mathrm{v}_z}{\partial \theta}\right)$$



$$\mathbf{x}(r,\theta,z) \equiv \begin{cases} x = r\cos\theta \\ y = r\,sin\theta \\ z = z \end{cases}$$

Spherical Coordinates

$$\mathbf{d} = \begin{bmatrix} d_{rr} & d_{r\theta} & d_{r\phi} \\ d_{r\theta} & d_{\theta\theta} & d_{\theta\phi} \\ d_{r\phi} & d_{\theta\phi} & d_{\phi\phi} \end{bmatrix}$$

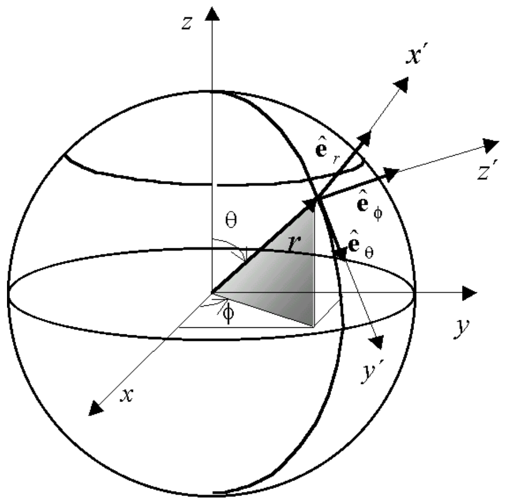
$$d_{rr} = \frac{\partial \mathrm{v}_r}{\partial r} \qquad d_{\theta\theta} = \frac{1}{r}\frac{\partial \mathrm{v}_\theta}{\partial \theta} + \frac{\mathrm{v}_r}{r}$$

$$d_{\phi\phi} = \frac{1}{r\cdot\sin\theta}\frac{\partial \mathrm{v}_\phi}{\partial \phi} + \frac{\mathrm{v}_\theta}{r}\cot\theta + \frac{\mathrm{v}_r}{r}$$

$$d_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial \mathrm{v}_r}{\partial \theta} + \frac{\partial \mathrm{v}_\theta}{\partial r} - \frac{\mathrm{v}_\theta}{r}\right)$$

$$d_{r\phi} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial \mathrm{v}_r}{\partial \phi} + \frac{\partial \mathrm{v}_\phi}{\partial r} - \frac{\mathrm{v}_\phi}{r}\right)$$

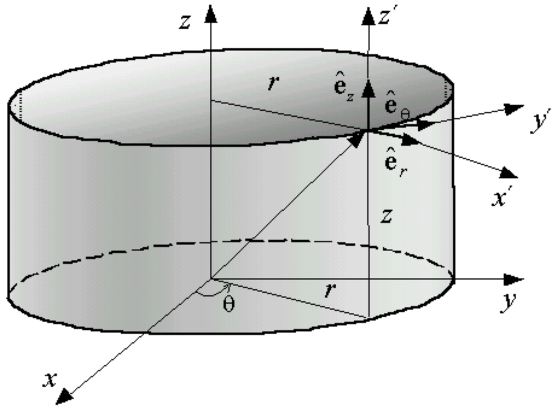
$$d_{\theta\phi} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial \mathrm{v}_\theta}{\partial \phi} + \frac{1}{r}\frac{\partial \mathrm{v}_\phi}{\partial \theta} - \frac{\mathrm{v}_\phi}{r}\cot\phi\right)$$



$$\mathbf{x}(r,\theta,\phi) \equiv \begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

Navier Equations

Cylindrical Coordinates



$$(\lambda + 2G)\frac{\partial e}{\partial r} - \frac{2G}{r}\frac{\partial \omega_z}{\partial \theta} + 2G\frac{\partial \omega_\theta}{\partial z} + \rho\, b_r = \rho\frac{\partial^2 u_r}{\partial t^2}$$

$$(\lambda + 2G)\frac{1}{r}\frac{\partial e}{\partial \theta} - 2G\frac{\partial \omega_r}{\partial z} + 2G\frac{\partial \omega_z}{\partial r} + \rho\, b_\theta = \rho\frac{\partial^2 u_\theta}{\partial t^2}$$

$$(\lambda + 2G)\frac{\partial e}{\partial z} - \frac{2G}{r}\frac{\partial}{\partial r}\left(r\omega_\theta\right) + \frac{2G}{r}\frac{\partial \omega_r}{\partial \theta} + \rho\, b_z = \rho\frac{\partial^2 u_z}{\partial t^2}$$

with

$$\omega_r = -\Omega_{\theta z} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)$$

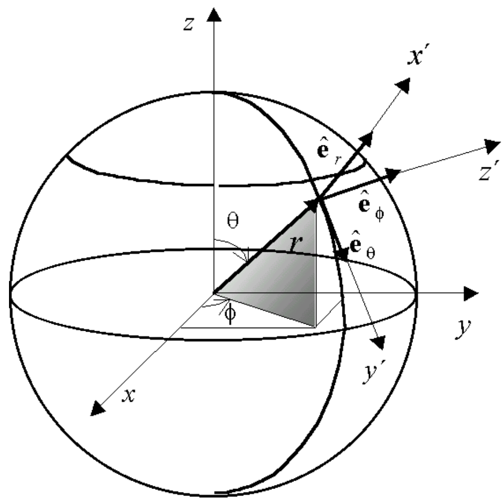
$$\omega_\theta = -\Omega_{zr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)$$

$$\omega_z = -\Omega_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial (r u_\theta)}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$

$$e = \frac{1}{r}\frac{\partial}{\partial r}\left(r u_r\right) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}$$

$$\mathbf{x}(r,\theta,z) \equiv \begin{cases} x = r\cos\theta \\ y = r\, sin\theta \\ z = z \end{cases}$$

Spherical Coordinates



$$(\lambda + 2G)\frac{\partial e}{\partial r} - \frac{2G}{r\, sin\theta}\frac{\partial}{\partial \theta}\left(\omega_\phi sin\theta\right) + \frac{2G}{r\, sin\theta}\frac{\partial \omega_\theta}{\partial \phi} + \rho\, b_r = \rho\frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{(\lambda + 2G)}{r}\frac{\partial e}{\partial \theta} - \frac{2G}{r\, sin\theta}\frac{\partial \omega_r}{\partial \phi} + \frac{2G}{r\, sin\theta}\frac{\partial}{\partial r}\left(r\omega_\phi sin\theta\right) + \rho\, b_\theta = \rho\frac{\partial^2 u_\theta}{\partial t^2}$$

$$\frac{(\lambda + 2G)}{r\, sin\theta}\frac{\partial e}{\partial \phi} - \frac{2G}{r}\frac{\partial}{\partial r}\left(r\omega_\theta\right) + \frac{2G}{r}\frac{\partial \omega_r}{\partial \theta} + \rho\, b_\phi = \rho\frac{\partial^2 u_\phi}{\partial t^2}$$

with

$$\omega_r = -\Omega_{\theta\phi} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi \sin\theta\right) - \frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi}\right)$$

$$\omega_\theta = -\Omega_{\phi r} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \phi} - \frac{1}{r}\frac{\partial (r u_\phi)}{\partial r}\right)$$

$$\omega_\phi = -\Omega_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r u_\theta\right) - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$

$$\mathbf{x}(r,\theta,\phi) \equiv \begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

$$e = \frac{1}{r^2 sin\theta}\left[\frac{\partial}{\partial r}\left(r^2 u_r sin\theta\right) + \frac{\partial}{\partial \theta}\left(r u_\theta sin\theta\right) + \frac{\partial}{\partial \phi}\left(r u_\phi\right)\right]$$

Cauchy Stress Tensor for a Newtonian Fluid

Cartesian Coordinates

$$\sigma_x = \mu\left[2\frac{\partial \mathrm{v}_x}{\partial x} - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + \mathrm{K}\ \nabla\cdot\mathbf{v}$$

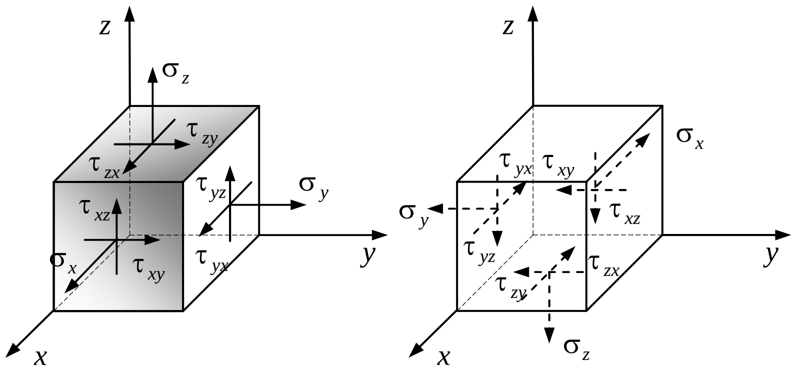
$$\sigma_y = \mu\left[2\frac{\partial \mathrm{v}_y}{\partial y} - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + \mathrm{K}\ \nabla\cdot\mathbf{v}$$

$$\sigma_z = \mu\left[2\frac{\partial \mathrm{v}_z}{\partial z} - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + \mathrm{K}\ \nabla\cdot\mathbf{v}$$

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial \mathrm{v}_x}{\partial y} + \frac{\partial \mathrm{v}_y}{\partial x}\right]$$

$$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial \mathrm{v}_y}{\partial z} + \frac{\partial \mathrm{v}_z}{\partial y}\right]$$

$$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial \mathrm{v}_z}{\partial x} + \frac{\partial \mathrm{v}_x}{\partial z}\right]$$



Cylindrical Coordinates

$$\sigma_r = \mu\left[2\frac{\partial \mathrm{v}_r}{\partial r} - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + \mathrm{K}\ \nabla\cdot\mathbf{v}$$

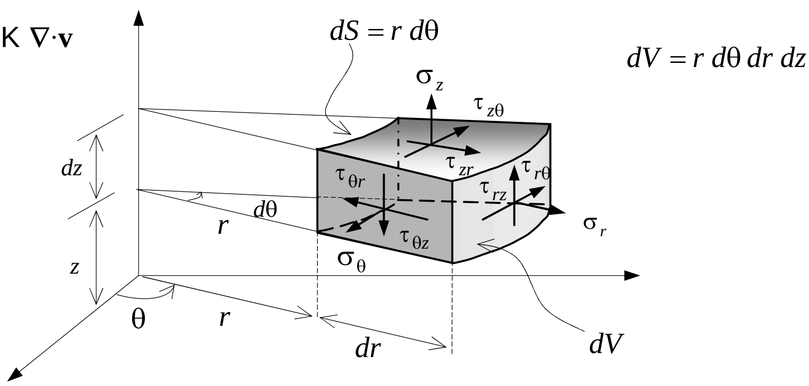
$$\sigma_\theta = \mu\left[2\left(\frac{1}{r}\frac{\partial \mathrm{v}_\theta}{\partial \theta} + \frac{\mathrm{v}_r}{r}\right) - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + \mathrm{K}\ \nabla\cdot\mathbf{v}$$

$$\sigma_z = \mu\left[2\frac{\partial \mathrm{v}_z}{\partial z} - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + \mathrm{K}\ \nabla\cdot\mathbf{v}$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{\mathrm{v}_\theta}{r}\right) + \frac{1}{r}\frac{\partial \mathrm{v}_r}{\partial \theta}\right]$$

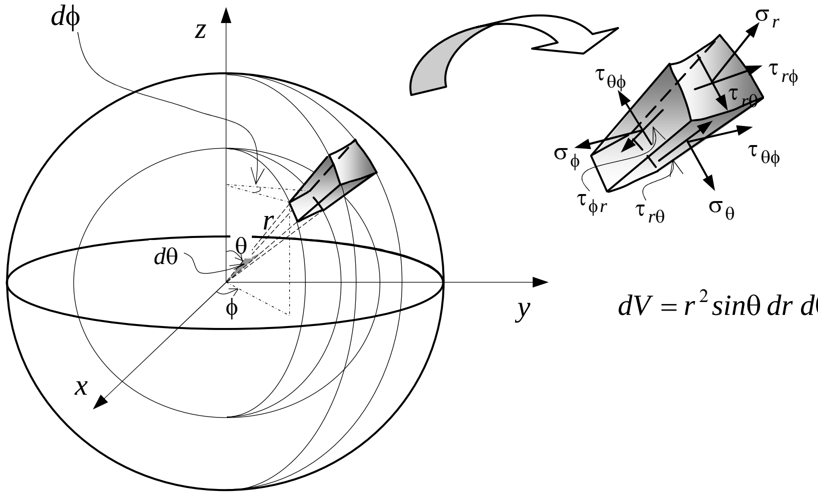
$$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial \mathrm{v}_\theta}{\partial z} + \frac{1}{r}\frac{\partial \mathrm{v}_z}{\partial \theta}\right]$$

$$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial \mathrm{v}_z}{\partial r} + \frac{\partial \mathrm{v}_r}{\partial z}\right]$$



with $\nabla\cdot\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\mathrm{v}_r) + \frac{1}{r}\frac{\partial \mathrm{v}_\theta}{\partial \theta} + \frac{\partial \mathrm{v}_z}{\partial z}$

Spherical Coordinates



$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\sigma_r = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + K\,\nabla\cdot\mathbf{v}$$

$$\sigma_\theta = \mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + K\,\nabla\cdot\mathbf{v}$$

$$\sigma_\phi = \mu\left[2\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta \,cotg\,\theta}{r}\right) - \frac{2}{3}(\nabla\cdot\mathbf{v})\right] - p + K\,\nabla\cdot\mathbf{v}$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial\theta}\right]$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = \mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$\tau_{\phi r} = \tau_{r\phi} = \mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

with $\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$

Mass Continuity Equation

Cartesian Coordinates

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho\, v_x\right)+\frac{\partial}{\partial y}\left(\rho v_y\right)+\frac{\partial}{\partial z}\left(\rho v_z\right)=0$$

Cylindrical Coordinates

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(\rho\, r v_r\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\rho\, v_\theta\right)+\frac{\partial}{\partial z}\left(\rho\, v_z\right)=0$$

Spherical Coordinates

$$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho\, r^2 v_r\right)+\frac{1}{r\, sin\theta}\frac{\partial}{\partial \theta}\left(\rho\, v_\theta sin\theta\right)+\frac{1}{r\, sin\theta}\frac{\partial}{\partial \phi}\left(\rho v_\phi\right)=0$$

Navier-Stokes Equations for a Newtonian Incompressible Fluid

Cartesian Coordinates

$$\rho\left(\frac{\partial \mathrm{v}_x}{\partial t}+\mathrm{v}_x\frac{\partial \mathrm{v}_x}{\partial x}+\mathrm{v}_y\frac{\partial \mathrm{v}_x}{\partial y}+v_z\frac{\partial \mathrm{v}_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 \mathrm{v}_x}{\partial x^2}+\frac{\partial^2 \mathrm{v}_x}{\partial y^2}+\frac{\partial^2 \mathrm{v}_x}{\partial z^2}\right)+\rho b_x$$

$$\rho\left(\frac{\partial \mathrm{v}_y}{\partial t}+\mathrm{v}_x\frac{\partial \mathrm{v}_y}{\partial x}+\mathrm{v}_y\frac{\partial \mathrm{v}_y}{\partial y}+\mathrm{v}_z\frac{\partial \mathrm{v}_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 \mathrm{v}_y}{\partial x^2}+\frac{\partial^2 \mathrm{v}_y}{\partial y^2}+\frac{\partial^2 \mathrm{v}_y}{\partial z^2}\right)+\rho b_y$$

$$\rho\left(\frac{\partial \mathrm{v}_z}{\partial t}+\mathrm{v}_x\frac{\partial \mathrm{v}_z}{\partial x}+\mathrm{v}_y\frac{\partial \mathrm{v}_z}{\partial y}+\mathrm{v}_z\frac{\partial \mathrm{v}_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 \mathrm{v}_z}{\partial x^2}+\frac{\partial^2 \mathrm{v}_z}{\partial y^2}+\frac{\partial^2 \mathrm{v}_z}{\partial z^2}\right)+\rho b_z$$

Cylindrical Coordinates

$$\rho\left(\frac{\partial \mathrm{v}_r}{\partial t}+\mathrm{v}_r\frac{\partial \mathrm{v}_r}{\partial r}+\frac{\mathrm{v}_\theta}{r}\frac{\partial \mathrm{v}_r}{\partial \theta}-\frac{\mathrm{v}_\theta^2}{r}+\mathrm{v}_z\frac{\partial \mathrm{v}_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\mathrm{v}_r)\right)+\frac{1}{r^2}\frac{\partial^2 \mathrm{v}_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial \mathrm{v}_\theta}{\partial \theta}+\frac{\partial^2 \mathrm{v}_r}{\partial z^2}\right)+\rho b_r$$

$$\rho\left(\frac{\partial \mathrm{v}_\theta}{\partial t}+\mathrm{v}_r\frac{\partial \mathrm{v}_\theta}{\partial r}+\frac{\mathrm{v}_\theta}{r}\frac{\partial \mathrm{v}_\theta}{\partial \theta}+\frac{\mathrm{v}_r\mathrm{v}_\theta}{r}+\mathrm{v}_z\frac{\partial \mathrm{v}_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\mathrm{v}_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 \mathrm{v}_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial \mathrm{v}_r}{\partial \theta}+\frac{\partial^2 \mathrm{v}_\theta}{\partial z^2}\right)+\rho b_\theta$$

$$\rho\left(\frac{\partial \mathrm{v}_z}{\partial t}+\mathrm{v}_r\frac{\partial \mathrm{v}_z}{\partial r}+\frac{\mathrm{v}_\theta}{r}\frac{\partial \mathrm{v}_z}{\partial \theta}+\mathrm{v}_z\frac{\partial \mathrm{v}_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \mathrm{v}_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \mathrm{v}_z}{\partial \theta^2}+\frac{\partial^2 \mathrm{v}_z}{\partial z^2}\right)+\rho b_z$$

Spherical Coordinates

$$\rho\left(\frac{\partial \mathrm{v}_r}{\partial t}+\mathrm{v}_r\frac{\partial \mathrm{v}_r}{\partial r}+\frac{\mathrm{v}_\theta}{r}\frac{\partial \mathrm{v}_r}{\partial \theta}+\frac{\mathrm{v}_\phi}{r\sin\theta}\frac{\partial \mathrm{v}_r}{\partial \phi}-\frac{\mathrm{v}_\theta^2+\mathrm{v}_\phi^2}{r}\right)=-\frac{\partial p}{\partial r}+$$
$$+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\mathrm{v}_r\right)\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial \mathrm{v}_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\mathrm{v}_r}{\partial\phi^2}-\frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\mathrm{v}_\theta\sin\theta)-\frac{2}{r^2\sin\theta}\frac{\partial \mathrm{v}_\phi}{\partial\phi}\right)+\rho b_r$$

$$\rho\left(\frac{\partial \mathrm{v}_\theta}{\partial t}+\mathrm{v}_r\frac{\partial \mathrm{v}_\theta}{\partial r}+\frac{\mathrm{v}_\theta}{r}\frac{\partial \mathrm{v}_\theta}{\partial \theta}+\frac{\mathrm{v}_\phi}{r\sin\theta}\frac{\partial \mathrm{v}_\theta}{\partial \phi}+\frac{\mathrm{v}_r\mathrm{v}_\theta}{r}-\frac{\mathrm{v}_\phi^2 cotg\ \theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+$$
$$+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \mathrm{v}_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\mathrm{v}_\theta\sin\theta)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\mathrm{v}_\theta}{\partial\phi^2}+\frac{2}{r^2}\frac{\partial \mathrm{v}_r}{\partial\theta}-\frac{2\ cotg\ \theta}{r^2\sin\theta}\frac{\partial \mathrm{v}_\phi}{\partial\phi}\right)+\rho b_\theta$$

$$\rho\left(\frac{\partial \mathrm{v}_\phi}{\partial t}+\mathrm{v}_r\frac{\partial \mathrm{v}_\phi}{\partial r}+\frac{\mathrm{v}_\theta}{r}\frac{\partial \mathrm{v}_\phi}{\partial \theta}+\frac{\mathrm{v}_\phi}{r\sin\theta}\frac{\partial \mathrm{v}_\phi}{\partial \phi}+\frac{\mathrm{v}_\phi\mathrm{v}_r}{r}+\frac{\mathrm{v}_\theta\mathrm{v}_\phi}{r}cotg\ \theta\right)=-\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}+$$
$$+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \mathrm{v}_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\mathrm{v}_\phi\sin\theta)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\mathrm{v}_\phi}{\partial\phi^2}+\frac{2}{r^2\sin\theta}\frac{\partial \mathrm{v}_r}{\partial\phi}+\frac{2\ cotg\ \theta}{r^2\sin\theta}\frac{\partial \mathrm{v}_\theta}{\partial\phi}\right)+\rho b_\phi$$