Response to reviewer comments on 'Numerical modeling of CO_2 fracturing by the phase field approach'

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We thank the reviewers again for their constructive comments. Below we respond to each of the comments, along with the corresponding changes of the manuscript. All changes to the previous submission are marked in blue in the revised version.

Comments from Reviewer #2

To the best of my knowledge, the proposed method is novel and the paper is well written. All my questions have been answered in this revision. From my opinion, this paper is acceptable for publication without further revision.

Comments from Reviewer #3

The modification does not solve my major concerns.

1. Regarding Question 4: Mandel's problem is a typical poroelastic problem. It is not for incompressible fluid. The principles behind the poroelastic borehole are the same as those in the Mandel's problem. The authors did not demonstrated typical poroelastic response through borehole pressurization.

We did try to solve Mandel's problem with our algorithm, and the results are satisfactory. Nevertheless, we do not plan to include it in the manuscript, because in some sense it repeats the numerical example in Section 4.2, as both couple porous flow and the displacement of the porous medium.

In the sequel, we first describe how we solved Mandel's problem with our algorithm. Consider a 2D rectangular domain of width $2a$ and height $2b$ occupied by a saturated poroelastic material. Constant compression forces are applied on

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rigid impermeable plates $y = \pm b$ with a magnitude of $2F$, see the configuration in Figure 1. The load is applied instantaneously at $t = 0^+$. At the right edge ($x = \pm a$) the sample can be drained while the lateral boundaries are free of stress.

We only model a quarter of the sample due to symmetry. The sample is assumed to be under plane strain conditions. The following boundary conditions are imposed:

$$\begin{aligned} p = 0 &\quad \text{on} \quad x = a, \\ u_2 = U_2(b,t) &\quad \text{on} \quad y = b, \\ u_1 = 0 &\quad \text{on} \quad x = 0, \\ u_2 = 0 &\quad \text{on} \quad y = 0. \end{aligned}$$

where $U_2(b,t))$ is the value of Mandel's closed form solution at $y = b$ [Y. Abousleiman, A.-D. Cheng, L. Cui, E. Detournay, J.-C. Roegiers, Mandel's problem revisited, Geotechnique 46 (2) (1996) 187–195]. The remaining boundary conditions are traction free boundary conditions.

Based on the analytical solution, as the compression load $2F$ is imposed, an instantaneous pressure increase and the following deformation responses are expected:

$$\begin{aligned} p(x,y,0^+) &= \frac{FB(1+\nu_u)}{3a}, \\ u_1(a,y,0^+) &= \frac{F\nu_u}{2G}, \\ u_2(x,b,0^+) &= -\frac{Fb(1-\nu_u)}{2Ga}. \end{aligned}$$

The material parameters for rock and fluid are given in Table 1. Using our proposed algorithm in combination with the fixed-stress split method (see [43][44], which we have now also cited in the manuscript, see Section 3), we solved the numerical problem for $t_f = 6$s in 600 equal time steps. Figure 2 shows our numerical results and the analytical solution for pore pressure. Excellent agreement is observed. Figure 3 shows the pore pressure developed in the sample during consolidation.

A comment on Mandel's problem follows here. It is true that this problem is a typical poroelastic problem. However, it would somehow repeat our second verification example (see Section 4.2) where the porous flow is coupled with the porous medium's displacement. The example in Section 4.2 solved by Detourney and Cheng [56] has been used in a few works, e.g., [11][57], to verify the solution to the poroelastic response of a pressurized borehole. For these reasons, we preferred not to include Mandel's problem in the manuscript.

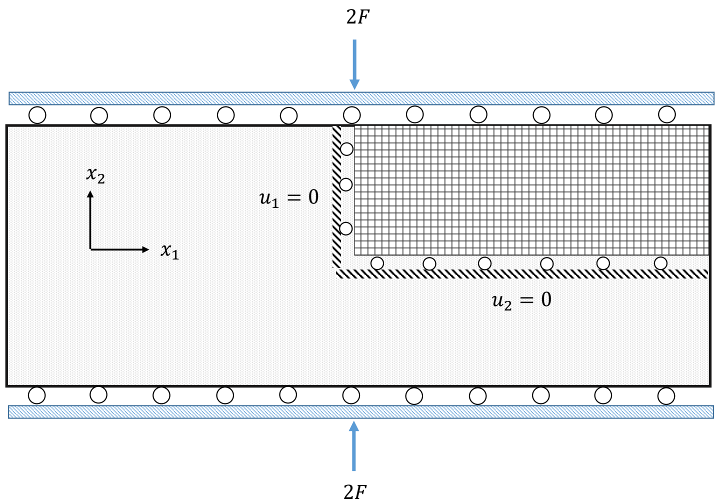


Figure 1: Schematic view of Mandel's problem. An instantaneous compression force $2F$ is applied on the horizontal edges. Due to symmetry, only a quarter of the sample is modeled with proper boundary conditions.

Table 1: Mandel's problem: Input parameters according to [43].

Parameters	symbol	unit	value
Young's modulus	E	MPa	1
Poisson's ratio	ν	–	0.2
Biot coefficient	α	–	1.
Permeability	k_0	m^2	1
Viscosity	μ	MPa·s	1
Length	a	m	2.5
Height	b	m	1
Load	F	MN	2.5
Skempton coefficient	B	–	1
Drained Poisson's ratio	ν_u	–	0.5
Biot's modulus	M	MPa	∞

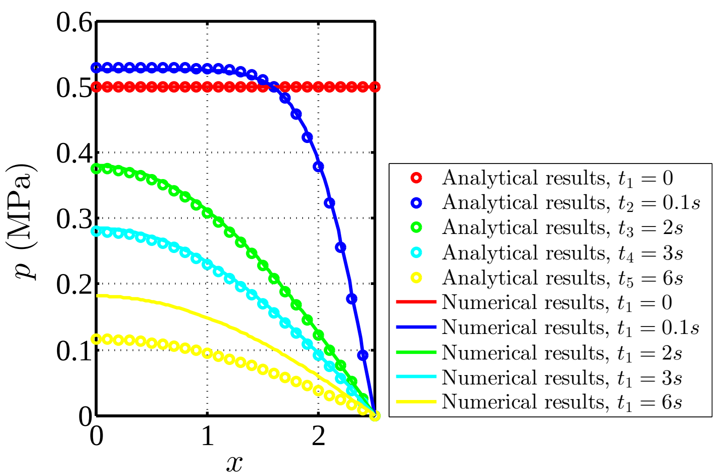
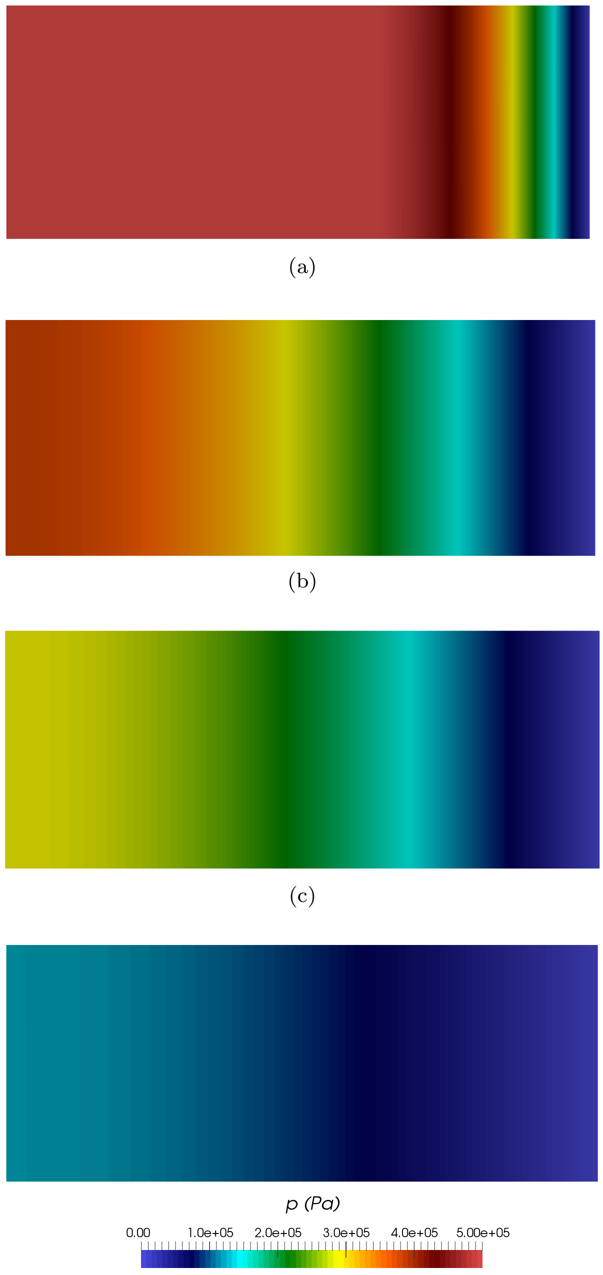


Figure 2: Mandel's problem: change of pore pressure in time. Excellent agreement is observed with the analytical solution.



(a) (b) (c) (d)

Figure 3: Pressure profile at different stages (3a) $t = 0.1s$, (3b) $t = 2s$, (3c) $t = 3s$, and (3d) $t = 6s$). At very early stages, the pressure rises rapidly in response to the instantaneous loading, then gradually decays to zero.

The phase field method for hydraulic fracture modeling is indeed successfully verified through semi-analytical solutions in other researchers' work. However, this does not indicate all the models utilizing the phase field method can be correctly verified through the semi-analytical solutions. The authors have to demonstrate that the model in this paper can match the semi-analytical solutions. I cannot understand why the authors think it is not necessary to verify their model just because the verification of the same method is already performed by other researchers.

Let us reiterate that our model for CO_2 treats it as a *compressible* fluid. None of existing papers that models fracturing with compressible CO_2 verifies with asymptotic semi-analytical solutions with hydraulic fracturing, the latter of which are based on an *incompressible* assumption of the fluid. Compressible fluids behave very differently from incompressible ones.

For verifying our model, we have not only checked the convergence of the discretization parameters for space and time, but also compared our examples in the manuscript against various solutions: the Hubbert-Willis (H-W) solution [60], the Haimson-Fairhurst (H-F) solution [61], the Sneddon and Lowengrub solution [58], and the Detourney and Cheng solution [56].

In response to the reviewer's request, we have reached out by applying our compressible formulation to the equation of state of water [R. T. Fernandez. Natural Convection from Cylinders Buried in Porous Media. PhD thesis, University of California; 1972] for a fracturing problem in the toughness-dominated regime (K-vertex), in order to compare our solution with the KGD model [J. Geertsma and F. De Klerk, A rapid method of predicting width and extent of hydraulically induced fractures, Journal of Petroleum Technology 21 (12) (1969) 1–571]. The parameters are given in Table 2. Figure 4 plots the evolution of the maximum fracture aperture of our numerical results and the KGD analytical solution. A good agreement is observed. Nevertheless, the fracture length evolution and the injection pressure do not agree well (not shown). This might be due to the over-extrapolation of the formulation to a fracturing fluid (water) that the formulation was not intended to model.

2. Regarding Question 2: All the given three references assume that grains are incompressible, which could be used approximately for soil. But this study is not about soil, it is about reservoir rock. There would not be too many people in the petroleum industry using this assumption in simulations.

It is true that including more complex assumptions for grains can lead to a more realistic model. However, we can cite a few works in which the grains for the reservoir are assumed incompressible: [M. Sheng, G. Li, S. N. Shah, X. Jin, Extended finite element modeling of multi-scale flow in fractured shale gas reservoirs, in: SPE Annual Technical Conference and Exhibition, Society of Petroleum Engineers, 2012, pp. SPE-159919-MS] and [A. Verruijt, Theory and Problems of Poroelasticity, 2013]. Thus, we believe the assumption adopted here is not too much out of the norm.

Table 2: Parameters for the comparison of KGD solution using the equation of state of water (a compressible formulation).

Parameters	symbol	unit	value
Young's modulus	E	MPa	6×10^{3}
Poisson's ratio	ν	–	0.34
Critical energy release rate	g_c	MPa·mm	0.306
Biot coefficient	α	–	0.85
Porosity	ϕ	–	0.01
Initial permeability	k_0	mm^2	1×10^{-12}
Dynamic viscosity of water	μ	MPa·s	7.9×10^{-3}
Initial pressure	p_0	MPa	0.1
Final time	t_f	s	500
Flow rate	$\boldsymbol{v}\cdot n$	mm^2/s	5

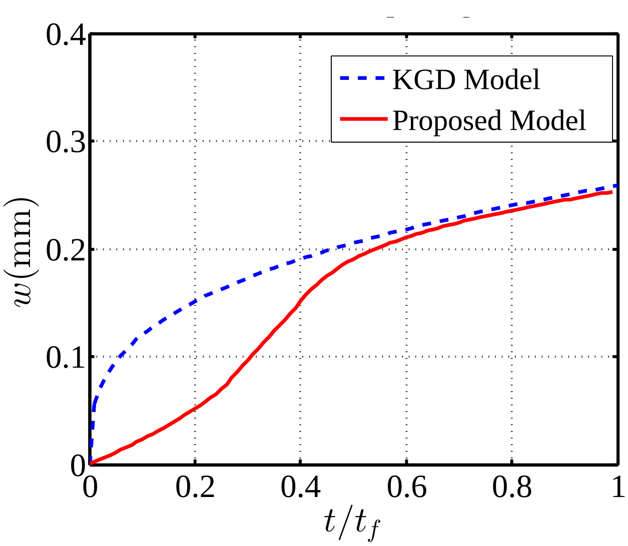


Figure 4: Verification of maximum fracture width for a toughness-dominated regime with KGD analytical solution [Geertsma and de Klerk, 1969].

3. Regarding Question 3: The discretization of the weak form is for solid part of a poroelastic model. The fluid flow part is entirely ignored.

In Appendix A.2 of the first revision, the discretization for fluid part was already provided. Moreover, we added one extra reference at the beginning of Appendix A for the implementation details.

In summary, the sequential coupling through volume strain is not correct; the model is not verified correctly, the correctness of the numerical model presented here is unknown.

We hope our response has clarified most of the issues.