MODELING HEAT EXTRACTION FROM A FRACTURE IN HOT DRY ROCK USING AN INTEGRAL EQUATION METHOD

A. H.-D. Cheng[†], A. Ghassemi[‡], E. Detournay[††]

[†] Department of Civil & Environmental Engineering
University of Delaware
Newark, DE 19716

[‡]Department of Geology & Geological Engineering
University of North Dakota
Grand Forks, ND 58202

[††]Department of Civil Engineering
University of Minnesota
Minneapolis, MN 55455

ABSTRACT

In the study of heat extraction by circulating water in a fracture embedded in geothermal reservoir, the heat conduction in the reservoir is typically assumed to be one-dimensional and perpendicular to the fracture. In this paper we demonstrate that by an integral equation formulation utilizing Green's function, the two-dimensional heat flow in the reservoir can be modeled. In the resulting numerical solution system, the discretization of reservoir geometry is entirely eliminated, leading to a much more efficient scheme. The two-dimensional heat conduction effect as compared to its one-dimensional simplification is studied.

INTRODUCTION

The Hot Dry Rock (HDR) concept of geothermal exploitation involves drilling two or more wells to suitable depths to connect permeable fractures of natural or man-made origin, injecting cold water into one well, and recovering hot water from the other. Comprehensive review of the HDR concept can be found in Hayashi, et al., 1999. Declining fluid and energy output at a given site will be unavoidable. Therefore, effective physical/mathematical modeling is necessary in order to forecast and control the production of energy.

A number of analytical and numerical solutions exist for modeling heat extraction from a fracture. See, for example, Willis-Richards and Wallroth (1995) for a comprehensive review. Depending on the type of fracture system, reservoir, and operation conditions, some physical models can be quite complicated. Mechanisms such as coupling of fracture pressure with reservoir elasticity, reservoir thermoelastic and poroelastic effects, nonlinear and temperature dependent fluid flow, etc., have been modeled. However, with few exceptions, such as the finite element solution by Kolditz (1995), the heat conduction in the reservoir is typically modeled as one-dimensional and perpendicular to the fracture surface. The primary reason for such simplification is due to the difficulty in modeling an unbounded domain by numerical discretization.

In this paper, we demonstrate that the heat conduction in the unbounded domain can be handled using the Green's function approach. Particularly, by transforming the governing equations of heat extraction from a single fracture embedded in an infinite geothermal reservoir into an integral equation, the need for discretizing the reservoir is entirely eliminated. We hence find a much more efficient numerical solution.

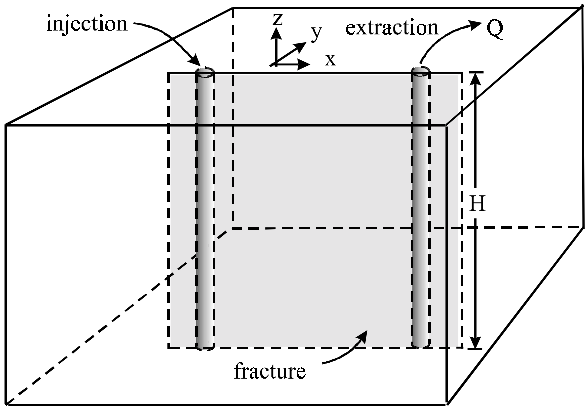

Figure 1: Idealized view of heat extraction from a hot dry rock reservoir.

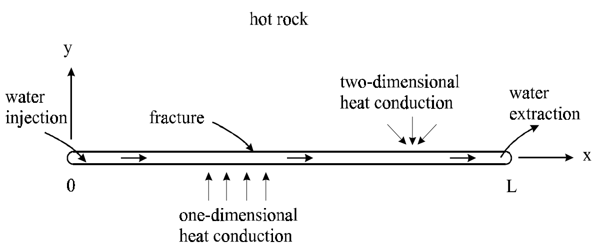

Figure 2: Solution domain of the mathematical problem.

MATHEMATICAL MODEL

The present solution is based on an idealized geometry as depicted in Figure 1. The reservoir is assumed to be of constant height and infinite horizontal extent. It is insulated at the top and the bottom. The fracture is a vertical plane penetrating the entire height of the reservoir. Hence, the solution geometry is two-dimensional, as shown in Figure 2. It is further assumed that the fracture walls are impermeable and the fracture aperture is constant. All thermal properties, such as heat capacity and conductivity, are considered constant and water is assumed to be incompressible.

The heat transport in the fracture includes heat storage, advection, longitudinal dispersion, and conduction from the fracture walls, can be expressed as:

$$\frac{\partial T(x,0,t)}{\partial t} + v\frac{\partial T(x,0,t)}{\partial x} - D_L\frac{\partial^2 T(x,0,t)}{\partial x^2} = \frac{2K_r}{\rho_w c_w b}\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=0} \quad (1)$$

where ρ_w is the water density, c_w is the specific heat of water, D_L is the longitudinal dispersion coefficient, K_r is the rock thermal conductivity, v is the average flow velocity, b is the fracture aperture width, and $T(x,y,t)$ is the temperature. We note that T is used to denote temperature in both rock and the fracture fluid, because the temperature of both media must be continuous.

The heat conduction in the rock is governed by

$$\frac{\partial^2 T(x,y,t)}{\partial x^2} + \frac{\partial^2 T(x,y,t)}{\partial y^2} = \frac{\rho_r c_r}{K_r}\frac{\partial T(x,y,t)}{\partial t} \quad (2)$$

where ρ_r is the rock density, and c_r is the rock specific heat. However, as commented earlier, the heat conduction in the rock is typically simplified to be one-dimensional (Figure 2):

$$\frac{\partial^2 T(x,y,t)}{\partial y^2} = \frac{\rho_r c_r}{K_r}\frac{\partial T(x,y,t)}{\partial t} \quad (3)$$

The initial temperature is assumed to be a constant, $T(x,y,0) = T_{ro}$ everywhere. At $t = 0^+$, water is injected at $x = 0$ at a flow rate Q and temperature $T(0,0,t) = T_{wo}$. From continuity, the average velocity is $v = Q/bH$. As the water travels through the fracture it is heated up in the process. The extraction temperature $T(L,0,t)$ is a part of the solution.

INTEGRAL EQUATION FORMULATION

In this section we shall deal with the case of two-dimensional heat conduction in the reservoir given in (2). It can be shown that under the typical laminar flow condition, the heat transport is dominated by advection in the fracture and diffusion in the reservoir. Therefore, both the dispersion and the heat storage effects can be neglected without causing significant error. Hence, the fracture heat transport equation (1) can be simplified to:

$$\frac{dT(x,0,t)}{dx} = \alpha \left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=0} \tag{4}$$

where $\alpha = \frac{2K_r}{vb\rho_w c_w}$. For the purpose of applying Laplace transform, it is more desirable to solve for temperature deficit, which has a null initial condition, than the temperature itself. Based on the definition $T_D(x,y,t) = \frac{T_{ro}-T(x,y,t)}{T_{ro}-T_{wo}}$, where T_{ro} is the initial rock temperature, (4) and (2) are expressed as:

$$\frac{\partial \widetilde{T}_D(x,0,s)}{\partial x} = \alpha \left.\frac{\partial \widetilde{T}_D(x,y,s)}{\partial y}\right|_{y=0} \tag{5}$$

$$\nabla^2 \widetilde{T}_D(x,y,s) = \frac{\rho_r c_r s}{K_r}\widetilde{T}_D(x,y,s) \tag{6}$$

In the above, the Laplace transform has been applied. Equations (5) and (6) are subject to the following boundary condition:

$$\widetilde{T}_D(0,0,s) = \frac{1}{s} \tag{7}$$

The solution system, (5) to (7), is defined in two spatial dimensions. It shall now be demonstrated that by utilizing Green's function, the system can be converted into a one-dimensional integral equation.

The temperature (deficit) in the reservoir due to a continuous point heat source of unit magnitude located at (x', y') is given by the Green's function in the Laplace transform domain:

$$G(x,y,x',y',s) = \frac{1}{2\pi K_r s} K_0(\beta r) \tag{8}$$

where K_0 is the modified Bessel function of the second kind of order zero, $\beta = \sqrt{\frac{\rho_r c_r s}{K_r}}\, r$, and $r = \|\mathbf{x} - \mathbf{x}'\|$ is the Euclidean distance between the field point $\mathbf{x}$ and the source point $\mathbf{x}'$. The temperature in the reservoir due to the distribution of sources along the fracture trajectory is given by:

$$\widetilde{T}_D(x,y,s) = \frac{1}{2\pi K_r}\int_0^L \widetilde{q}(x',s)\, K_0(\beta r)\, dx' \tag{9}$$

where $\widetilde{q}$ is the Laplace transform of source intensity. The source intensity is equal to the heat flux into the fracture:

$$\widetilde{q}(x,s) = -2K_r \left.\frac{\partial \widetilde{T}_D(x,y,s)}{\partial y}\right|_{y=0^+} \tag{10}$$

Using (5), the source intensity can be expressed as:

$$\widetilde{q}(x,s) = -vb\rho_w c_w \frac{d\widetilde{T}_D(x,0,s)}{dx} \tag{11}$$

Substituting (11) into (9), yields:

$$\widetilde{T}_D(x,y,s) = -\frac{1}{\pi\alpha}\int_0^L \frac{d\widetilde{T}_D(x',0,s)}{dx'} K_0(\beta r)\, dx' \tag{12}$$

Performing integration by parts, the above equation is transformed into:

$$\begin{aligned}\pi\alpha\widetilde{T}_D(x,y,s) &= \frac{1}{s}K_0(\beta r) \\ &-\widetilde{T}_D(L,0,s)\,K_0(\beta r') \\ &+\beta\int_0^L \widetilde{T}_D(x',0,s)\,\frac{x-x'}{r'}K_1(\beta r)\, dx'\end{aligned} \tag{13}$$

where $r\prime = \sqrt{(x-x')^2 + y^2}$. In the above the boundary condition (7) were utilized. Applying the above equation along the fracture trajectory, $y = 0$ and $0 \le x \le L$, results in:

$$\begin{aligned}&\pi\alpha\widetilde{T}_D(x,0,s) = \\ &\frac{1}{s}K_0(\beta x) - \widetilde{T}_D(L,0,s)\,K_0(\beta\cdot(L-x)) + \\ &\beta\int_0^L \widetilde{T}_D(x',0,s)\,\frac{x-x'}{|x-x'|}K_1(\beta\cdot|x-x'|)\, dx'\end{aligned} \tag{14}$$

Equation (14) is defined in one-dimension in the interval $0 \le x \le L$. It incorporates all the initial and boundary conditions and without including the reservoir temperature as a variable. It can be solved for the water temperature in the fracture $\widetilde{T}_D(x,0,s)$. Once the fracture temperature is known, it can be used in (13) to evaluate the temperature $\widetilde{T}_D(x,y,s)$ everywhere.

NUMERICAL IMPLEMENTATION

Equation (14) is identified as a Fredholm integral equation of the second kind (Polyanin and Manzhirov, 1998). It is Cauchy singular due to the $1/r$ singularity contained in the Bessel function K_1, which needs to be regularized before a numerical integration can be performed. For the purpose of regularization, we first replace $\widetilde{T}_D(x',0,s)$ in the singular integral in (14) by $\widetilde{T}_D(x,0,s)$ and perform the integration analytically:

$$\beta \int_0^L \widetilde{T}_D(x,0,s) \frac{x-x'}{|x-x'|} \cdot K_1\left(\beta\,|x-x'|\right)\,dx' = \widetilde{T}_D(x,0,s)\left[K_0\left(\beta\cdot(L-x)\right) - K_0\left(\beta\,x\right)\right] \quad (15)$$

Then (15) can be subtracted from (14) to give:

$$\left[\pi\alpha - K_0\left(\beta\cdot(L-x)\right) + K_0\left(\beta x\right)\right]\widetilde{T}_D(x,0,s) = \frac{1}{s}K_0\left(\beta x\right) - \widetilde{T}_D(L,0,s)\,K_0\left((\beta\cdot(L-x)\right) + \beta\int_0^L\left[\widetilde{T}_D(x',0,s) - \widetilde{T}_D(x,0,s)\right] \frac{x-x'}{|x-x'|}K_1\left(\beta\cdot|x-x'|\right)\,dx' \quad (16)$$

We note that the integral is no longer singular; hence the equation is regularized.

To approximately solve (16), we shall apply a quadrature rule. Normally, the Gaussian quadrature is the most efficient. However, in order to capture the solution accurately at the end points $x=0$, L without extrapolation, we shall use Simpson's rule (Press et al.,1998). Hence (16) becomes:

$$\left[\pi\alpha - K_0(\beta(L-x_i)) + K_0(\beta x_i)\right] T_i = T_0\,K_0\left(\beta x_i\right) - T_n\,K_0\left(\beta(L-x_i)\right) + \beta\,\Delta x \sum_{j=0} w_j\left(T_j - T_i\right)\cdot \frac{x_i-x_j}{|x_i-x_j|}K_1\left(\beta\,|x_i-x_j|\right);\ \text{for } i=1,\ldots,n \quad (17)$$

where n is the number of equal intervals, $\Delta x = L/n$ is the size of the intervals, $x_i = i\,\Delta x$ for $i=0,\ldots,n$ are the abscissas, w_i are weights, and T_i are used to denote the discrete values $\widetilde{T}_D(x_i,0,s)$. The first value T_0 is given by the boundary condition $T_0 = 1/s$. Using Simpson's $\frac{3}{8}$-rule, the weights are:

$$w_j = \frac{3}{8},\frac{7}{6},\frac{23}{24},1,1,\ldots,1,1,\frac{23}{24},\frac{7}{6},\frac{3}{8};\ \text{for } j=0,\ldots,n \quad (18)$$

Note that (17) contains $n+1$ discrete values of T_i. Only n of them are unknown because T_0 is given by the boundary condition. Hence there exist n unknowns and n equations for $i=1,\ldots,n$. This forms a linear system:

$$[A]\{T\} = \{b\} \quad (19)$$

which can be solved for T_i. Upon the application of approximate Laplace inversion, the temperature distribution in the fracture is known in space and time.

If the temperature in the geothermal reservoir is needed, the quadrature rule can be applied to (13) such that the temperature at any location (x,y) can be found:

$$\widetilde{T}_D(x,y,s) = \frac{1}{\pi\alpha}\left[T_0\,K_0\left(\beta r\right) - T_n\,K_0\left(\beta r_l\right) + \beta\,\Delta x \sum_{j=0} w_j T_j\,\frac{x-x_j}{r_j}K_1\left(\beta r_j\right)\right] \quad (20)$$

where:

$$r_j = \left[(x-x_j)^2 + y^2\right]^{\frac{1}{2}} \text{ and } r_l = \left[(L-x)^2 + y^2\right]^{\frac{1}{2}}$$

The above formula does not involve any unknown quantity; hence can be straightforwardly evaluated. The solution needs to be inverted to the time domain. This can be performed using an approximate Laplace inversion algorithm known as the Stehfet method (Stehfest, 1970; Cheng et al., 1994).

EXAMPLE

The effect of two-dimensional versus one-dimensional heat conduction is examined using a numerical example below. For this case, we choose the following data set:

$$
\begin{aligned}
L &= 300 \text{ m}; \quad b = 0.3 \text{ cm}; \quad v = 0.5 \text{ cm/s} \\
\rho_w &= 1.0 \text{ g/cm}^3; \quad \rho_r = 2.65 \text{ g/cm}^3; \\
K_r &= 6.2 \times 10^{-3} \text{ cal/cm} \cdot \text{s} \cdot {}^\circ\text{C}; \\
c_w &= 1.0 \text{ cal/g} \cdot {}^\circ\text{C}; \quad c_r = 0.25 \text{ cal/g} \cdot {}^\circ\text{C}
\end{aligned}
$$

Figure 3 presents the temperature profile along the fracture length at three different times, 1, 5, and 15 years after production. The temperature shown is the normalized temperature $T_w = (T - T_{wo})/(T_{ro} - T_{wo}) = 1 - T_D$. The two-dimensional heat conduction solution (solid lines) is computed based on (17), while the one-dimensional solution (dash line) is given by (Lowell, 1976). It can be observed that the 2-D case always predicts a higher temperature for the obvious reason that more heat is supplied from the additional hot rock region. The increase in temperature is more significant near the inlet than outlet, and at large than small time.

The most useful information for the application is the extraction temperature. Figure 4 shows a plot of the normalized extraction temperature $T_w(L, 0, t)$ for 30 years of production time. It can be observed that the 2-D heat conduction case always predicts a higher extraction temperature. The difference is more pronounced at large production time. At the end of 30 years, the predicted temperature T_w of the 2-D case is about 10% higher than the 1-D case. Another way to envision the difference is that the 1-D case predicts that the heat extraction efficiency drops to below 50% after 23 years of production, while the 2-D case predicts 29 years.

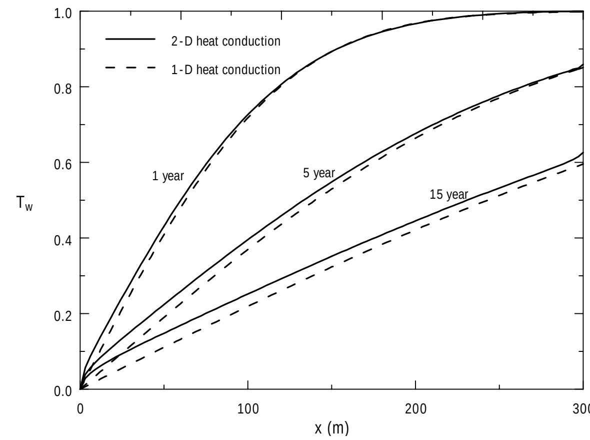

Figure 3: Normalized temperature distribution in the fracture based on 1-D and 2-D heat conduction model.

In reporting the above result we are aware that in the FEM solution of Kolditz the multidimensional heat conduction effect is even more pronounced when the 1-D is compared with the 3-D model. Hence we conclude that in the modeling of heat conduction in geothermal reservoir, the use of correct reservoir geometry could be important in predicting the life of HDR reservoir.

SUMMARY AND CONCLUSION

In this paper we have examined the effect of two dimensional heat conduction in a geothermal reservoir. An integral equation formulation has been derived such that the need for modeling the reservoir geometry is eliminated in the discrete solution system. Numerical investigations have shown that the two-dimensional heat conduction can significantly

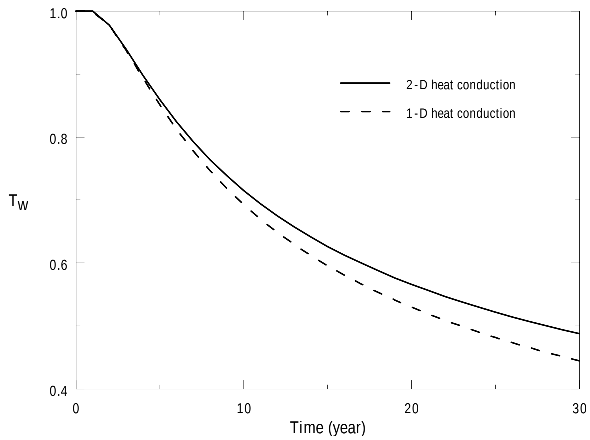

Figure 4: Normalized extraction temperature based on 1-D and 2-D heat conduction model..

alter the prediction of the extracted water temperature and the reservoir life.

Although the present integral equation solution is two-dimensional, the same concept can be applied in a three-dimensional geometry to eliminate the discretization of geothermal reservoir. Other improvements include modeling of reservoir elasticity, thermoelasticity, and poroelasticity, which can cause the fracture width to become dependent on the fracture fluid pressure, temperature, and reservoir compliance. Efforts in these directions are underway.

ACKNOWLEDGMENT

The financial support of the US Department of Energy (DE-FG07-99ID13855) is gratefully acknowledged.

REFERENCES

Abe, H., Mura, T. and Keer, L.M., 1976, "Heat extraction from a hydraulically fractured penny-shaped crack in hot dry rock," Proc. 2nd Workshop Geothermal Reservoir Engineering, Stanford University, CA, 200-212.

Cheng, A.H.-D., Sidauruk, P. and Abousleiman, Y., 1994, "Approximate inversion of the Laplace transform," Mathematica J., 4, 76-82.

Hayashi, K., Willis-Richards, J., Hopkirk, R.J. and Niibori, Y., 1999, "Numerical models of HDR geothermal reservoirs—a review of current thinking and progress," Geothermics, 28, 507-518.

Kolditz, O., 1995, "Modelling flow and heat transfer in fractured rocks: Dimensional effect of matrix heat diffusion," Geothermics, 24, 421-437.

Lowell, R.P., 1976, "Comments on 'Theory of heat extraction from fracture hot dry rock' by A.C. Gringarten, P.A. Witherspoon, and Y. Ohnishi," J. Geophys. Res., 81, 359-360.

Polyanin, A.D. and Manzhirov, A.V., 1998, Handbook of Integral Equations, CRC Press.

Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P., 1992, Numerical Recipe in Fortran, the Art of Scientific Computing, 2nd ed., Cambridge Univ. Press.

Stehfest, H., 1970, "Numerical inversion of Laplace transforms," Comm. ACM, 13, 47-49 and 624.

Willis-Richards, J. and Wallroth, T., 1995. "Approaches to the modeling of HDR reservoirs: a review," Geothermics, 24, 307-332.