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Propagation of a system of cracks under thermal stress

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ABSTRACT: The propagation of thermally driven cracks as a means of enhancing reservoir permeability is of interest in geothermal reservoir stimulation. A numerical model based on the complex hypersingular integral equation is developed and used to examine the plausibility and the conditions necessary for the success of thermal stimulation. Two example problems, namely the sudden cooling of the surface of infinite half-space and injection/extraction process in two-dimensional fractures are considered to illustrate the impact of cooling on fractures and their propagation. Interaction of multiple secondary thermal fractures and their trajectory is also considered. Results indicate that length of the secondary cracks is mainly determined by the temperature distribution in the geothermal reservoir, and under suitable conditions of in-situ stress and cooling thermal stimulation can lead to significant fracture propagation. However, a timely increase in reservoir permeability may require pressurization of the thermal cracks.

1. INTRODUCTION

When rocks are heated/cooled, the bulk solid and pore fluid tend to undergo expansion/contraction. A volumetric expansion can result in significant pressurization of the pore fluid depending on the degree of containment and the thermal and hydraulic properties of the fluid as well as the solid. The net effect is a coupling of thermal and poromechanical processes when developing a geothermal reservoir. These processes occur on various time scales and the significance of their interaction and coupling is dependent upon the problem of interest. In hydraulic fracturing, the fluid-rock mechanics coupling evolves rapidly (on the scale of minutes, hours to possibly a few days) compared to the induced thermal processes, thus the thermal effects may not have a large influence on the fluid-mechanical processes in fracture propagation. However, during a long term injection phase (time scale of months to years), the thermo-mechanical coupling can no longer be neglected.

The influences of thermal processes on fracture opening have been previously addressed in [1, 2]. These studies and a poro-thermoelastic stress analysis [3] around a uniformly cooled crack surfaces indicates that large thermally-induced thermal stresses occur around a cooled geothermal well giving rise to tensile cracking. Also, experimental works [4] and results of 3D simulations of cold water injection into hot fractures [5] predict developing of high tensile stresses in the vicinity of cooled surface, indicating a potential for development of secondary thermal fractures. Cooling induced stresses may exceed the in-situ stresses of the geothermal reservoir [2], resulting in formation of system of secondary cracks perpendicular to main fracture. These cracks propagate into the rock matrix, increasing the permeability of the reservoir. To increase the effectiveness of the heat extraction, the secondary thermal fractures should be sufficiently long and open, to allow the fluid flow deep inside the reservoir matrix where the heat is stored (higher temperatures).

The formation and propagation of thermal fractures in response to cooling of their surfaces has been treated theoretically [6, 7]. The stability analysis in [6, 7] predicts that many roughly equal small cracks appear shortly after surface cooling, however, some of them will stop upon further cooling. The bifurcation analysis predicts that at some point the growth of one crack will suppress the propagation of its nearest neighbors and only every second crack will grow further until next bifurcation point is reached (Fig. 1).

Murphy [8] used the simple analytical model of a single crack to estimate the extent of penetration and spacing of thermal cracks. Barr [9] numerically studied the effect of thermal crack penetration into geothermal reservoir. Finnie et al. [4] investigated, both theoretically and experimentally, fracture propagation in rocks due to thermal shock. In this paper, the development and propagation of system of secondary thermal cracks in geothermal reservoir due to cold water injection is studied. The extent of the thermal fractures and the average spacing between them will be estimated using a combination of the real boundary integral equation method for the temperature solution, and the complex variable boundary integral equation for fracture propagation solution.

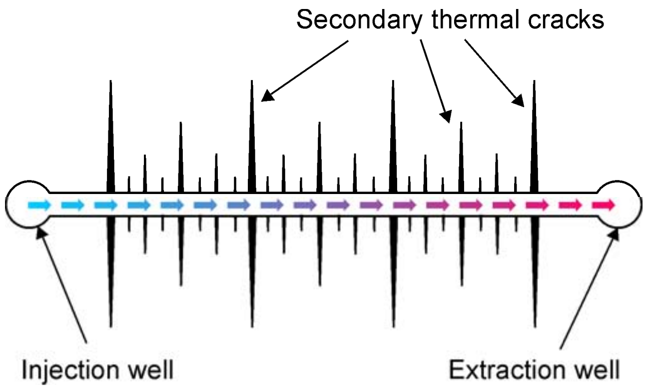

Fig 1. Formation of secondary thermal cracks perpendicular to the main fracture cooled by injected water.

2. TEMPERATURE AND THERMOELASTIC STRESS

The problem of cold-water injection into one or several two-dimensional cracks can be solved using the boundary integral method. It is assumed that rock matrix permeability is low and fluid leak-off is negligible. The newly formed secondary thermal cracks are small and are filled with fluid of the same temperature as the rock matrix at that location along the main crack, and do not influence the heat conduction. As long as the secondary cracks are not connected to other cracks, the fluid flow inside the secondary cracks is low and can be neglected. Therefore, the rock is cooled by the fluid in the main fracture. If an interconnected fracture network forms, then flow in the secondary fractures should be taken into account.

Using the above assumptions, the problem of heat extraction from geothermal reservoir by cold water injection can be solved with integral equation in the Laplace domain as described in Cheng et al. [10]. The temperature in the reservoir is found by solving the integral equation on the fracture plane:

$$\tilde{T}(x,y,s) = -\frac{Q\rho_w c_w}{2\pi K_r}\int_0^L \frac{\partial \tilde{T}(x',0,s)}{\partial x'}\mathrm{K}_0(\xi)\,\mathrm{d}x' \quad (1)$$

where $\tilde{T}$ is the normalized temperature in Laplace domain, s is Laplace transform parameter, $\xi = \sqrt{\rho_r c_r s / k_r}R$ with R as the distance between the source point and the field point, Q is injection rate, k_r is rock thermal conductivity and ρ_w, c_w, ρ_r and c_r are density and heat capacity of water and rock, respectively. The integration is performed along fracture of length L, the integral equation (1) is solved using a collocation technique.

Once the temperature inside the fracture is known, the thermally induced stresses are found using the Green's function for the thermal stresses due to a continuous heat source [11]:

$$\tilde{\sigma}_{ij}(x,y,s) = -Q\rho_w c_w \int_0^L s\tilde{\sigma}_{ij}^{cs}(R,s)\frac{\partial \tilde{T}(x',0,s)}{\partial x'}dx' \quad (2)$$

where the fundamental solution $\tilde{\sigma}^{cs}$ is given as [11]:

$$\begin{aligned}\tilde{\sigma}_{rr}^{cs} &= \frac{-E\alpha_T}{4\pi(1-\nu)k_r}\frac{1}{s}\left(\frac{2}{\xi^2} - \mathrm{K}_2(\xi) + \mathrm{K}_0(\xi)\right)\\ \tilde{\sigma}_{\theta\theta}^{cs} &= \frac{E\alpha_T}{4\pi(1-\nu)k_r}\frac{1}{s}\left(\frac{2}{\xi^2} - \mathrm{K}_2(\xi) - \mathrm{K}_0(\xi)\right)\end{aligned} \quad (3)$$

where E is Young's modulus, ν is Poisson's ratio, α_T is the rock linear thermal expansion coefficient.

The problem of fracture propagation caused by a non-stationary thermal field was solved using the complex variable boundary integral equation of elasticity theory [12] with thermally induced stresses used as the boundary conditions on the crack faces. For closed cracks, the penalty method is employed to maintain proper contact of crack faces without penetration.

3. INITIATION OF THERMAL CRACKING

Experimental observations and theoretical analysis predict that in the absence of large defects in the cooled body, a system of thermal cracks appears some time after cooling initiates. The formation of such array of thermal cracks may be explained by the energy balance between the energy required to generate new surface and energy released by the crack propagation. Thermal cracks appear when the strain energy stored in the body due to thermal stress is sufficient for the formation of a number of cracks of a certain length. A simple theoretical model was proposed in [7, 13] for estimating the minimal size and spacing between thermal cracks formed by the instantaneous uniform cooling of an infinite surface. However, their analysis did not include the high compressive in-situ stresses which are typical for geothermal applications. To account for in-situ stresses, analysis similar to that of Nemat-Nasser et al. [7] can be used. Let us assume that system of vertical parallel cracks is formed due to cooling of the main fracture surface. To simplify calculations, the temperature profile can be replaced by an equivalent parabolic profile:

$$\Delta T(y) = \left(1 - \frac{y}{\sqrt{3}L}\right)^2 (T_r - T_w) \quad (4)$$

where L is cooling front penetration and equal $\sqrt{4tk_r / c_r \rho_r}$ (see Bažant et al. [13]), T_r and T_w are temperature of rock and water respectively and t is time. The thermal cracks cannot penetrate deep into compressive zone, therefore it can be assumed that length of the cracks roughly equal the depth of the layer with tensile stresses and can be found from condition $\sigma^{th} + \sigma^{\infty} = 0$, where σ^{th} and σ^{∞} are the thermally induced stress and compressive in-situ stress acting in the direction normal to the secondary thermal cracks, respectively. Thermally induced stress for infinite half-plane in plane strain conditions is determined as:

$$\sigma^{th} = \alpha_T \Delta T(y) \frac{E}{1-\nu} \quad (5)$$

This gives the approximate length of thermal cracks, a:

$$a = \sqrt{3}L\left(1 - \sqrt{-\frac{\sigma^{\infty}(1-\nu)}{E\alpha_T (T_r - T_w)}}\right). \quad (6)$$

The strain energy per unit length stored in a layer of thickness a, due to the stress acting in the direction normal to the potential crack faces is:

$$U_0(a) = \int_0^a \frac{1}{2E}\left(\alpha_T \Delta T(y) \frac{E}{1-\nu} + \sigma^{\infty}\right)^2 dy \quad (7)$$

If all stored energy is relieved by cracking, the crack spacing can be estimated from the condition $G_c a = dU_0(a)$, where G_c is critical energy release rate of rock and d is crack spacing. Assuming the crack spacing approximately equals the crack length (as dictated by mechanical crack interference) allows us to estimate the minimal crack length and spacing. Then from Eq. (6) the time at which the cracks will appear can be estimated.

4. CRACK PROPAGATION

Several crack propagation direction criteria have been developed over the years, the most popular are the maximum circumferential stress criterion and the maximum energy release rate criterion. Cotterell and Rice [14] have shown that these criteria have the common feature that K_{II}=0 in the direction of crack growth and, therefore, are just another forms of so-called “principle of local symmetry”, i.e. the crack grows along a smooth path where K_{II}=0. In many numerical simulations, the maximum circumferential stress criterion is used since it requires the knowledge of only the stress intensity factor in the initial configuration, and the accuracy of the crack path prediction increases by reduction of the size of the crack increment. However, for some problems, especially those involving crack growth in the direction of compressive stresses, the application of the maximum circumferential stress criterion leads to an oscillating crack path. To overcome this difficulty, several iterative algorithms have been suggested for simulation of smooth crack path. For example, Mogilevskaya [15] and Dobroskok [16] used an iterative scheme with circular arc crack increments that leads to a smooth curve. Another method has been suggested by Stone and Babuška [17], where the Newton iterative scheme is used to solve equation $K_{II}(\beta)$=0, where β is the kink angle, and the crack increment could be straight or parabolic. We use the algorithm of Stone and Babuška in our work because of its better convergence, and for simplicity use straight crack increments. In the numerical model, we solve the equation $K_{II}(\beta)$=0 simultaneously for all crack tips where the crack propagation criterion is satisfied. This results in a highly effective algorithm and normally takes about 2-3 iterations to obtain a stable crack path.

5. NUMERICAL EXAMPLES

Let us consider two examples patterns of secondary thermal fractures: that of instant cooling of the surface of half-plane, and the one corresponding to injection/extraction process in a line major fracture. The injected water temperature is 20 °C and the initial temperature of the rock is 220 °C. Other material constants are presented in Table 1.

The problem of half-space subjected to instant cooling at the surface (Fig. 2) has an analytic solution for the temperature profile as a function of time [7]. Thermally induced stresses are then calculated using Eq. (5), and are applied as surface tractions on the crack faces to simulate crack propagation in the non-stationary thermal field using the stress superposition principle. To simulate the growth and interaction of thermally driven cracks, a number of small initial cracks at random locations were generated before the simulation. Usually, an array of about 100-200 cracks is used with periodic boundary conditions to simulate infinite array of cracks. The initial cracks are located at random positions, keeping the average crack spacing, d, as constant. Results of preliminary studies indicate that initial cracks locations practically do not influence the final crack pattern in average sense [18].

Table 1. Material data used in simulation.

Young's modulus, E	37.5 GPa
Poisson's ratio, ν	0.25
Fracture toughness, K_{Ic}	10 MPa m$^{1/2}$
Rock density, ρ_r	2650 kg / m^3
Water density, ρ_w	1000 kg / m^3
Rock heat capacity, c_r	790 J / kg K
Water heat capacity, c_w	4200 J / kg K
Rock thermal conductivity, k_r	10.7 W / m K
Rock linear thermal expansion, α_T	8e-6 1 / K
Flow rate, Q	5e-5 m^2 / s
In-situ stress, σ_{xx}	35 Mpa
In-situ stress, σ_{yy}	50 Mpa
Rock temperature, T_r	220 °C
Water temperature, T_w	20 °C

Since fracture toughness of the rock is quite small, the cracks may grow into the compressive zone and so sufficiently small time and crack increments are required to avoid crack closure. Time steps were chosen such that only few cracks would grow at any iteration. Then iterative process was used to find the crack propagation direction, as was described in Section 4. After several time steps, some of the cracks stop to grow, while the remaining fractures continue to grow, resulting in a characteristic crack pattern presented in Fig. 3.

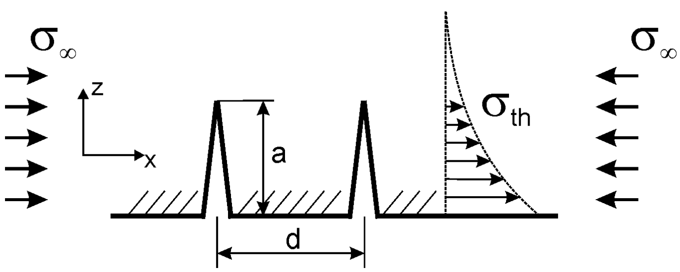

Fig. 2. An array of edge cracks loaded by thermally induced stress σ_{th} and a far-field compressive stress σ_{∞}.

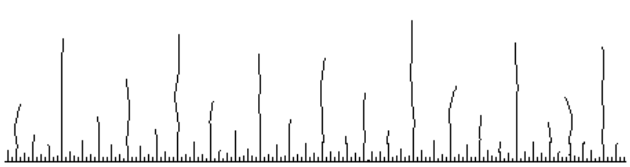

Fig. 3. Results of simulation: typical crack pattern.

The numerically obtained crack patterns were used to estimate the crack length and crack spacing. Several simulations were performed for each combination of parameters and the results were averaged for higher accuracy. Fig. 4 presents the numerical results for crack length for three different compressive in-situ stresses: 20, 35 and 50 MPa. The results show that in the case of instant cooling of the half-space surface, the cracks length is proportional to the cooling depth L, or proportional to the square root of time. The numerical values of the crack length are quite close to the approximation given by Eq. (6). The average crack spacing at different depths is presented in Fig. 5 along with a power law fit.

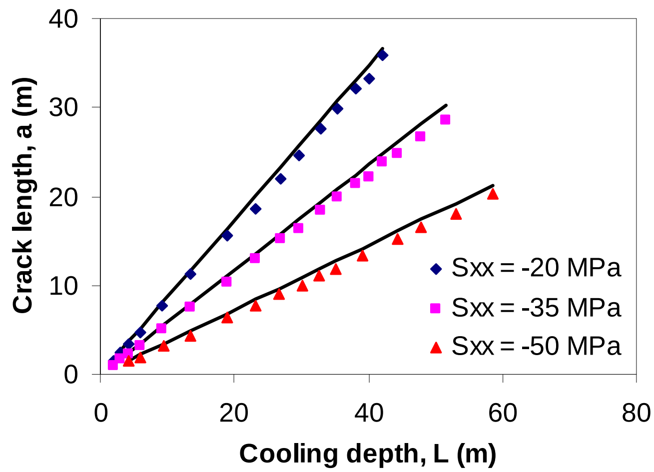

Fig. 4. Crack length as function of cooling depth L. Dots represent results of simulation, lines – approximation by Eq. (6).

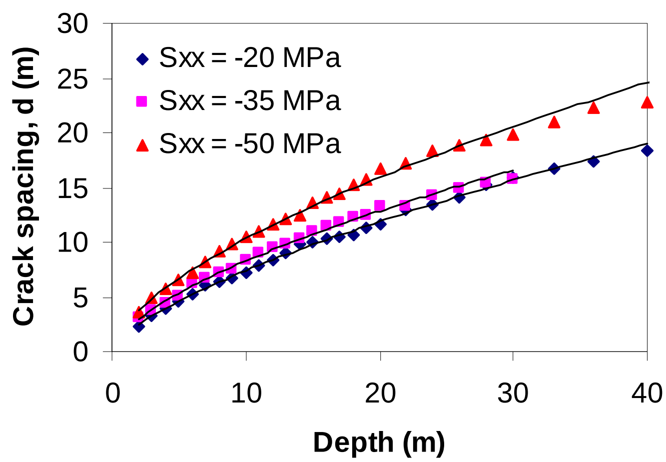

Fig. 5. Crack spacing as function of depth. Dots represent results of simulation, lines – power law fit.

The presented theory of thermally driven crack growth due to instant cooling of half-space predicts that for physical parameters listed in Table 1, the cracks will grow up to 47 m after 10 years. During fluid injection into geothermal reservoir, the cooling rate is limited by the amount of injected fluid and more complex analysis is required, as was described in Section 2. Let us consider a line fracture with a length of 200 m that is cooled by a 20 °C fluid injected at a rate of 5e-5 m^2 / s.

Before the start of simulation, 50 initial small cracks with half-lengths of 2 m were generated along the main fracture. Each crack was subdivided into 10 segments. For each time step the induced thermal stresses were calculated for each segment's collocation points and then used as boundary conditions in the elastic analysis. Then, the crack tips for which the propagation criterion is met were moved by some distance (0.4 m in this simulation) and the iterative algorithm of Stone and Babuška [17] was employed to find the propagation direction for which the K_{II} is zero (thermal stresses were updated for each iteration).

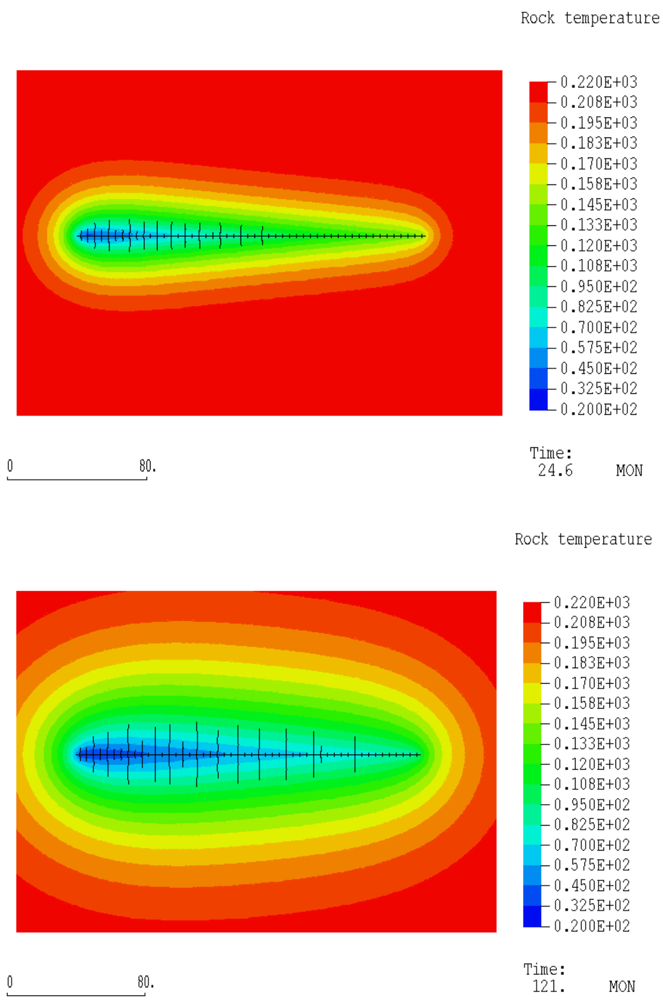

Fig. 6. The temperature field in the rock matrix after two (top) and ten years (bottom) of injection/extraction. Black vertical lines are thermally induced cracks.

Due to the high thermal tensile stresses and low tensile strength of rocks, it can be expected that thermal fractures will appear relatively soon after total stresses (in-situ plus thermally induced stresses) in the direction parallel to main fracture become tensile and exceed tensile strength of material. The theoretical analysis of Bažant et al. [13] predicts that typically, initial cracks as small as several centimeters could be expected. Our analysis which takes into account the compressive in-situ stresses, predicts that the initial crack length and spacing increases with increasing confining stress. However, this effect becomes significant only for sub-critical confining stress, i.e., when the maximum thermal stresses are only slightly larger then in-situ stress. For the parameters used in this work both analyses predict an initial thermal crack length equal to 0.2-0.3 m. In the current study, we are mostly interested in the long-term behavior of the secondary thermal cracks and the initial depth of fractures was chosen as 2 m, therefore, all smaller fractures that may have grown at earlier times are not considered in this simulation.

The initial cracks located near the injection well start to propagate a few days after injection. As the thermal front moves along the main fracture, other fractures start to propagate. The thermal front and the length of the secondary fractures after 2 and 10 years of fluid injection are presented in Fig. 6. As expected, for injection problems the cracks are significantly shorter and have lengths of only about 20 m after 10 years of injection. Fig. 7 shows the results of a similar simulation for two parallel fractures spaced at 50 m. Much longer secondary cracks can be expected in the inner cooler zone, however, cracks that tend to grow outward have lengths similar to the single fracture case (Fig. 6b).

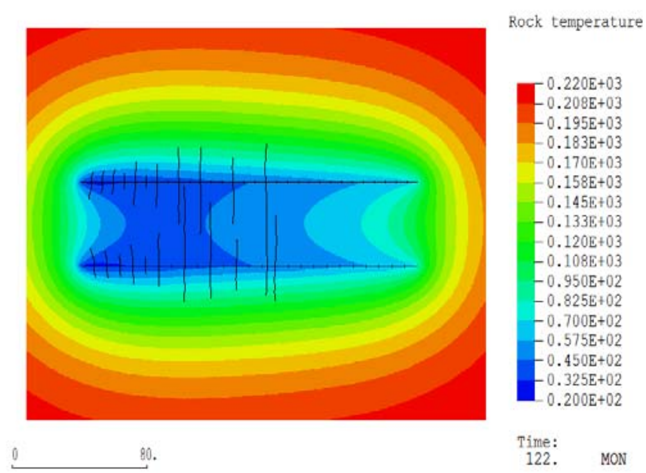

Fig. 7. The temperature field in the rock matrix after ten years of injection/extraction for two fractures system. Black vertical lines are thermally induced cracks.

Fig. 8 shows the total σ_{xx} stress component (in-situ plus thermally induced) and the secondary thermal cracks geometry after 30 years of injection/extraction. All thermal fractures are located inside the zone of tensile total stresses with their tips grown to edge of the compressive stress zone. The length of the longest secondary fractures is about 25 m. The opening of the longest cracks after 30 years is about 8 mm whereas the shorter cracks have an opening of nearly 1.5 mm.

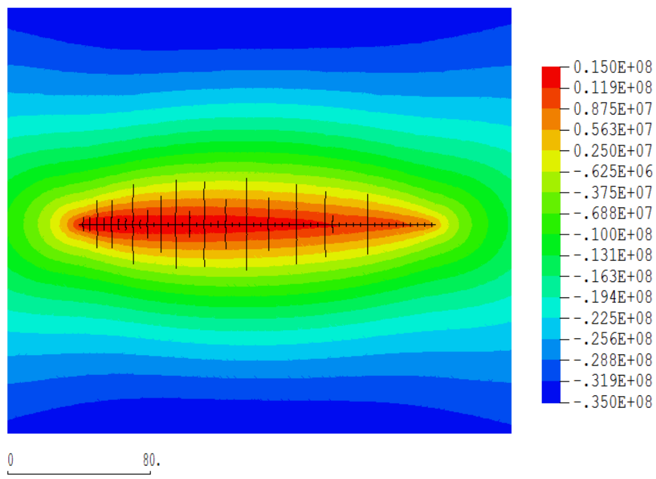

Fig. 8. Distribution of the total stress, σ_{xx} (Pa), and secondary fractures in the reservoir rock matrix after 30 years of operation.

CONCLUSIONS

A boundary element numerical model has been developed for simulating the growth behavior of many cracks and their interaction in a non-stationary thermal field due to cold water injection into a geothermal reservoir. The model has been used to investigate the influence of the main physical parameters of the system on the length and spacing of thermally driven fractures. The results show that in the case of instant cooling of the half-space surface, the cracks length is proportional to the cooling depth *L*. Numerical examples of injection/extraction show that a fracture network can be generated by thermal shock of the reservoir rock. However, results indicate that in the presence of high compressive in-situ stresses, the growth rate of the length of the secondary fractures is rather slow when driven by external stress solely caused by cold water circulation. Additional propagation force such as pressurization by a fluid would be needed to form the fracture network for effective reservoir development.

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