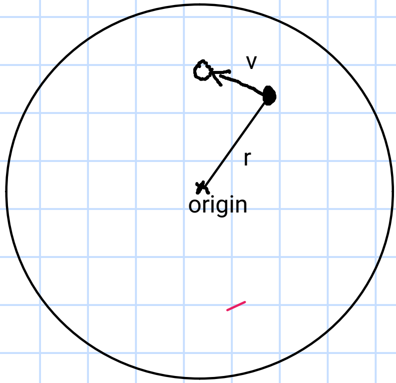
Tracer particle orbiting in spherical gravitational potential:



Initial position at (x1,y1,z1), with initial velocity (vx1,vy1,vz1)

It feels an initial acceleration from the gravitational potential (ax1, ay1, az1)

$$\mathrm{accel}(r) = -\frac{d\phi}{dr}$$

or

$$\mathrm{accel}(r) = -\frac{GM(<r)}{r^2}$$

(for spherical mass distribution)

Time stepping along the orbit in a loop:

In a tiny time step dt, the new velocity due to acceleration will be:

vx2=vx1+ax1*dt
vy2=vy1+ay1*dt
vz2=vz1+az1*dt

and the new position due to the particle velocity will be:

x2=x1+vx1*dt
y2=y1+vy1*dt
z2=z1+vz1*dt

Loop

At the new position, we will have a new acceleration so restart loop

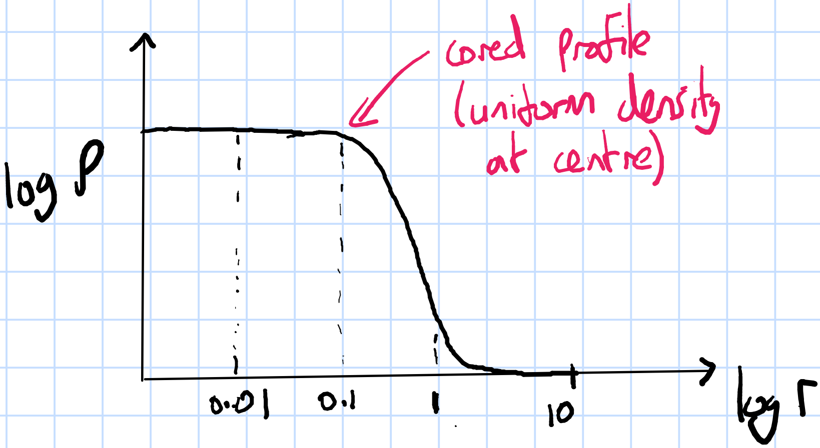
Density Profile: Plummer

paper: Aarseth+1974

$$\rho(r) = \underbrace{\frac{3M_{pl}}{4\pi r_{pl}^3}}_{\rho_0}\left(1+\frac{r^2}{r_{pl}^2}\right)^{-5/2}$$

M_{pl} : Total mass

r_{pl} : scale radius of plummer



Potential:

$$\phi(r) = -\frac{GM_{pl}}{(r^2+r_{pl}^2)^{1/2}}$$

Enclosed mass:

$$M(<r) = \frac{M_{pl}\, r^3}{(r^2+r_{pl}^2)^{1.5}}$$

Density Profile: NFW halo (dark matter)

paper: Lokas & Mamon 2001

$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

r_s is radial scale length

Truncated at R_{200} (200 times critical density of Universe)

$$M_{200} = \frac{4}{3}\pi R_{200}^3 \times 200 \times \underbrace{\rho_{crit}}_{\frac{3H_0^2}{4\pi G}}$$

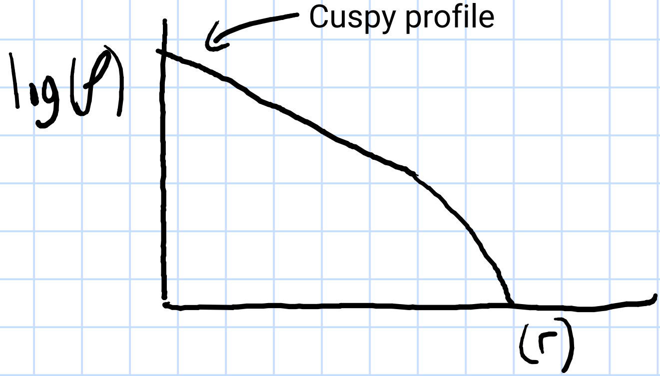
$\sim 9.8 \times 10^{-27}$ kg/m^3 (at $z=0$)

NFW halo concentration:

$$c = \frac{R_{200}}{r_s}$$

$c \sim 2-4$ (clusters)

$c \sim 10-15$ (galaxies)



NFW potential:

$$\phi(r) = -\frac{4\pi G \rho_0 r_s^3}{r} \ln\left(1+\frac{r}{r_s}\right)$$

NFW enclosed mass:

$$M(s) = M_{200}\, g(c) \left[\ln(1+cs) - \frac{cs}{1+cs}\right]$$

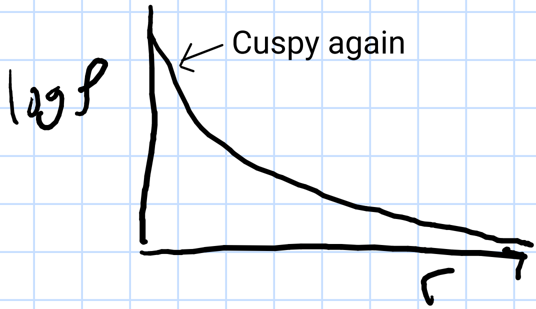
where $s = \frac{r}{R_{200}}$ and $g(c) = \frac{1}{\ln(1+c) - c/(1+c)}$

Paper: Hernquist+1990

Hernquist density profile:
(spherical galaxies, bulges)

$$\rho(r) = \frac{M}{2\pi}\frac{a}{r}\frac{1}{(r+a)^3}$$

a is radial scalelength



Potential:

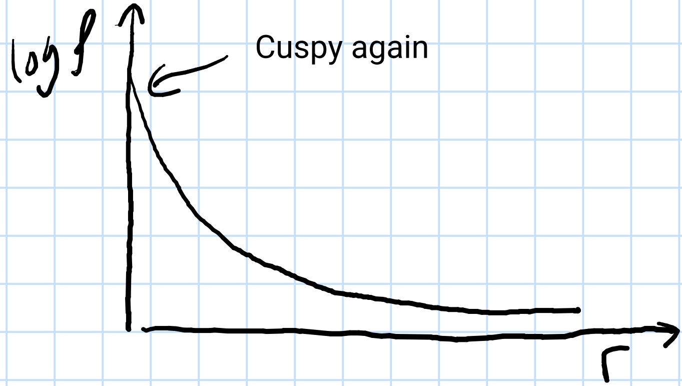
$$\phi = -\frac{GM}{(r+a)}$$

Enclosed mass:

$$M(<r) = M\frac{r^2}{(r+a)^2}$$

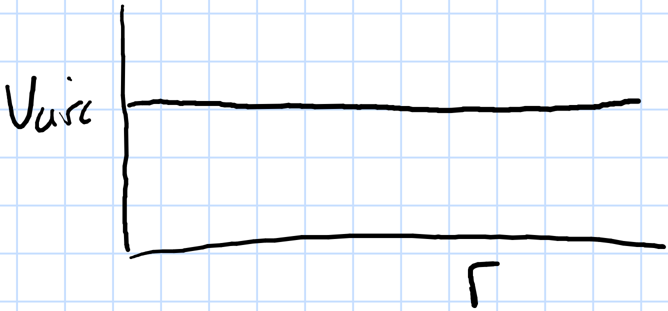
Isothermal sphere (simple DM halo)

$$\rho = \rho_0 \left(\frac{r}{a}\right)^2 \qquad \phi = 4\pi G \rho_0 a^2 \ln\left(\frac{r}{a}\right)$$



$$V_{circ}^2 = 4\pi G \rho_0 a^2$$

Note, V_{circ} is a constant



Easy way to model flat rotation curves of observed galaxies

Gravitational acceleration and potential:

$$\text{accel} = -\frac{\partial \phi}{\partial r} \quad \text{or} \quad \text{accel}(r) = \frac{-GM(<r)}{r^2}$$

(for spherical mass distribution)

Vcirc example:

Isothermal halo:

$$\phi = 4\pi G \rho_0 a^2 \ln\left(\frac{r}{a}\right)$$

$$\frac{\partial \phi}{\partial r} = 4\pi G \rho_0 a^2 \frac{1}{\cancel{r}} = \frac{V_{circ}^2}{\cancel{r}}$$

$$\therefore \quad V_{circ}^2 = 4\pi G \rho_0 a^2$$

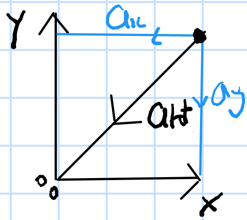
constant (as we saw before)

Acceleration components: (assuming origin at (0,0))

$$a_x = -\frac{\partial \phi(r)}{\partial r}\left(\frac{x}{r}\right), \quad a_y = -\frac{\partial \phi(r)}{\partial r}\left(\frac{y}{r}\right), \quad a_z = -\frac{\partial \phi(r)}{\partial r}\left(\frac{z}{r}\right)$$

Quadrature:

$$a_{tot}^2 = a_x^2 + a_y^2 + a_z^2$$
$$= \left(-\frac{\partial \phi}{\partial r}\right)^2 \underbrace{\frac{(x^2 + y^2 + z^2)}{r^2}}_{1}$$
$$= \left(-\frac{\partial \phi}{\partial r}\right)^2$$



Single Particle Timestepper:

$$x = x_i, \quad y = y_i, \quad z = z_i$$
$$V_x = V_{xi}, \quad V_y = V_{yi}, \quad V_z = V_{zi}$$

Choose initial position & velocity

$$t = 0, \quad dt = t_{total}/N_{step}$$

Initial time & size of time step

$$\text{Do } i = 1, N_{step}$$

$$t = i \times dt$$

Time evolves forward

$$r = (x^2 + y^2 + z^2)^{1/2}$$
$$a_x = -\frac{\partial \phi(r)}{\partial r}\left(\frac{x}{r}\right)$$
$$a_y = -\frac{\partial \phi(r)}{\partial r}\left(\frac{y}{r}\right)$$
$$a_z = -\frac{\partial \phi(r)}{\partial r}\left(\frac{z}{r}\right)$$

Calculate accelerations at current position

$$V_x = V_x + a_x\, dt$$
$$V_y = V_y + a_y\, dt$$
$$V_z = V_z + a_z\, dt$$

Update velocities using accelerations

$$x = x + V_x\, dt$$
$$y = y + V_y\, dt$$
$$z = z + V_z\, dt$$

Update positions using velocities

$$\text{Enddo}$$

G=1 units

Purpose:
To avoid tiny numbers in calculations that result in rounding errors

Approach:
Choose mass unit and length unit.
This decides the velocity unit and time unit

Useful formulae:

$$T_{sim}(Myr) = 14.9 \left[\frac{L_{sim}^3 (pc)}{M_{sim}(M_\odot)} \right]^{1/2}$$

$$V_{sim}(m/s) = 980.4 \left[\frac{L_{sim}(pc)}{T_{sim}(Myr)} \right]$$

Example: $M_{sim} = 1 M_\odot$, $L_{sim} = 1 pc$

$\therefore$ $T_{sim} = 14.9$ Myr,

$V_{sim} = 65.8$ m/s

Application: Use positions in Lsim units, velocities in Vsim units
Acceleration equations in code uses G=1
Simulation runs for time in Tsim units

e.g. $x = 10pc = 10 L_{sim}$

$V_x = 100 m/s = 1.52 V_{sim}$

$t_{end} = 149 Myr = 10 T_{sim}$

What you must do:

Part 1: Code preparation and testing

(i) Demonstrate that these equations result in G=1 units

Hint: $G_{SI} = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$

$= 6.67 \times 10^{-11} [L_{SI}]^3 [M_{SI}]^{-1} [T_{SI}]^{-2} = 1 [L_{sim}]^3 [M_{sim}]^{-1} [T_{sim}]^{-2}$

(ii) Write your own single particle time stepper in G=1 units including the analytical potential of a point mass, hernquist sphere and NFW density distribution

(iii) Test your code using a point mass potential. Set up the particle on a circular orbit and confirm that the velocity is as expected for the chosen radius.

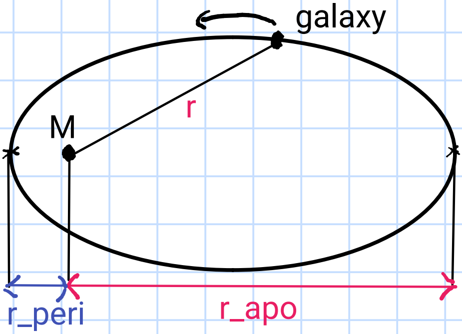
(iv) Again, for a point mass potential, show that an elliptical orbit satisfies Kepler's law:

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

Period → T

semi major axis ← a

where, $a = 0.5(r_{peri} + r_{apo})$

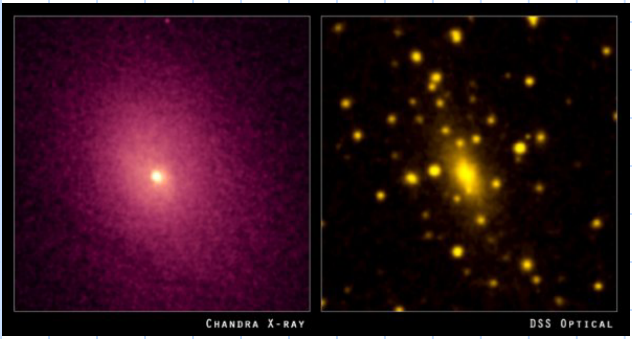


r_peri: closest distance to M along elliptical orbit

r_apo: largest distance to M along elliptical orbit

(v) For a hernquist potential, with M=1e12 Msol and scaleradius a=10 kpc, set up a circular orbit at r=20 kpc and r=100 kpc, and show that the orbital velocity is as expected.

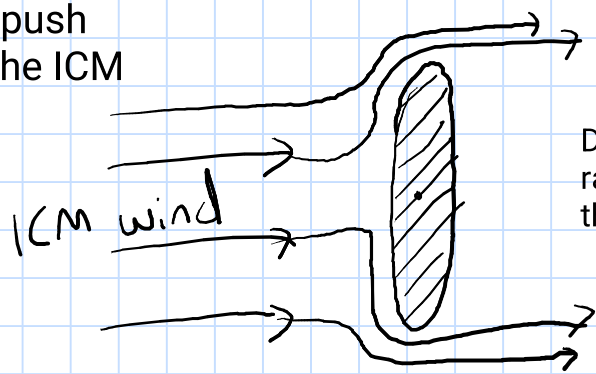
Ram pressure stripping (RPS) in clusters



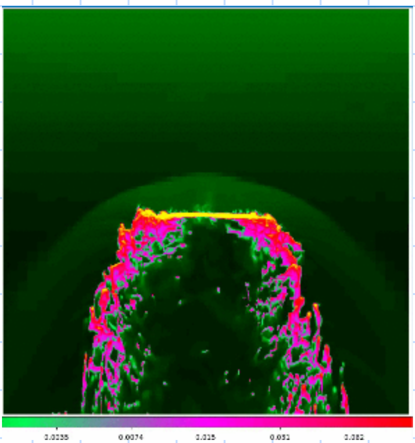
$$\rho_{ICM} = \rho_0 \left[1 + \left(\frac{r}{r_{core}} \right)^2 \right]^{-3\beta/2}$$

Beta profile of ICM in clusters

Galaxies push through the ICM



Disk gas feels a ram pressure from the wind



Where galaxy gravity is too weak, disk gas is stripped (outside inwards)

Galaxies known as "Jellyfish galaxies" for their shape

If ram pressure gets stronger, disk is truncated more until all gas is stripped

What you must do:

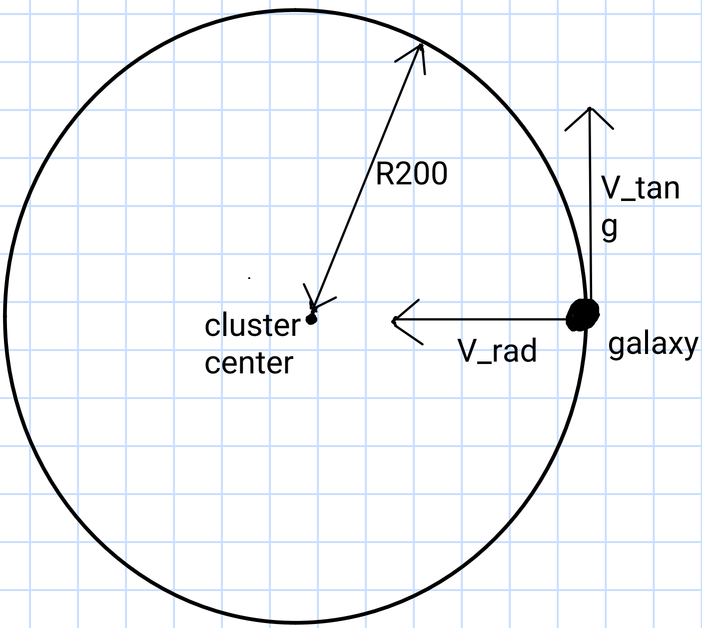
Part 2: Application to Ram pressure stripping in clusters

(i) For a rough model of the Virgo cluster, use an NFW halo analytical potential with M200= 5e14 Msol, and c=4. What is the value of R200 and the scaleradius rs?

(ii) The particle is tracing the orbit of an infalling galaxy. Place it at R200 initially. Measure the orbit if the galaxy initially has no radial motion (Vrad=0), and initially a tangential motion Vtang=100 km/s.

Repeat for Vrad=0 and Vtang=300 km/s. Measure the time to reach first pericentre (tperi), and the radius and orbital velocity at first pericentre (Rperi & Vperi).

How does tperi, Rperi and Vperi change if you use initial velocities of Vrad=500 km/s and Vtang=300 km/s?



(iii) Assume a beta profile for the hot gas content of the cluster that is similar to the Virgo cluster:

For Virgo: $\beta \approx 0.5$

$r_{core} \approx 50$ kpc

$\rho_0 \approx 2 \times 10^{-26}$ g/cc

For the Vrad=0 and Vtang=300 km/s orbit, calculate the density of the ICM which the galaxy passes through along its orbit. Also measure its orbital velocity along the orbit. Use both to calculate the ram pressure the galaxy is subjected to along its orbit and make plots of their time evolution:

Ram pressure, $P_{RPS} = \rho_{ICM} V_{orb}^2$

(iv) Ram pressure will strip the gas from the galaxy centre when the following condition occurs:

$$\overbrace{P_{RPS}}^{\text{Ram pressure}} > \overbrace{2\pi G \Sigma_{0,star} \Sigma_{0,gas}}^{\text{Restoring force from disk}}$$

Central surface density of stars & gas

where, $\Sigma_{0,star} = \dfrac{M_{star}}{2\pi r_d^2}$, $\Sigma_{0,gas} = \dfrac{M_{gas}}{2\pi r_d^2}$

Scalelength of galaxy disk

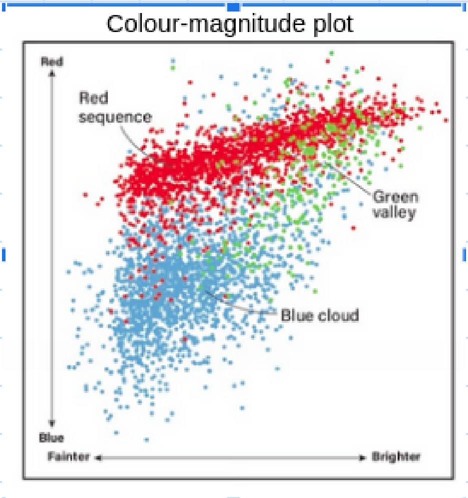
For a massive galaxy, and low mass galaxy, calculate at which radius and time the central disk gas is stripped during first infall

	M_{star} ($M_\odot$)	M_{gas} ($M_\odot$)	r_d (kpc)
Massive:	1×10^{10}	1×10^{9}	4.0
Low mass:	1×10^{9}	1×10^{9}	2.5

Galaxies on the colour-magnitude diagram:

Plot of colour on y axis versus luminosity on x-axis

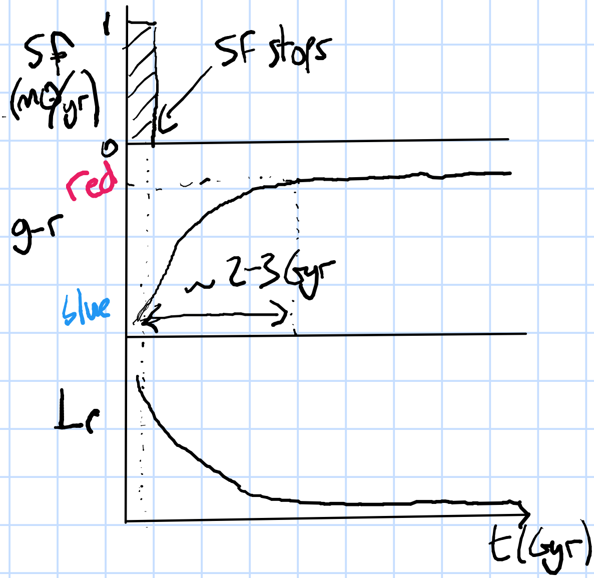
E.g. colour (g-r) versus r (apparent magnitude in r-band)



Galaxies that stop forming stars eventually finish on red sequence

Galaxies still forming stars are typically found on blue cloud

When galaxies stop forming stars, they transition from blue cloud to red sequence (across the green valley)

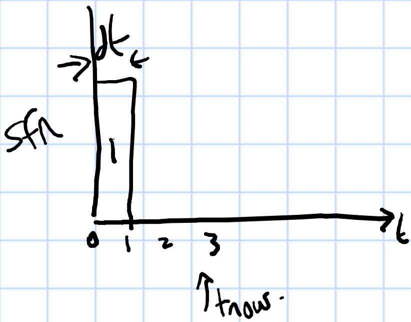


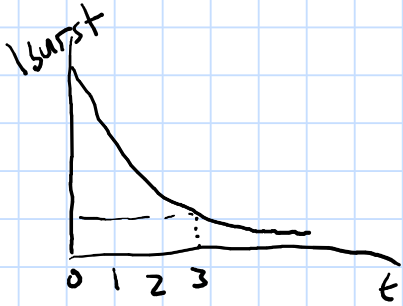
After SF ends, galaxies reddens and fades

Initial change is rapid but then slows down

Takes ~several Gyr to fully transition

Single Burst

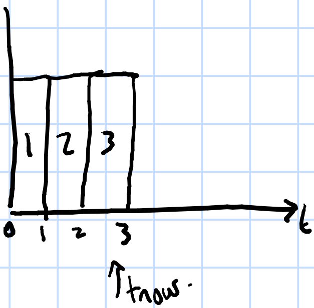




At tnow, need luminosity from burst 1

$$L_1 = \underbrace{SFR(t=1) \times dt}_{\text{Stellar mass produced (Msol)}} \times \underbrace{L_{burst}(t=3)}_{\text{Luminosity from 1 Msol burst}}$$

Multiple Bursts



At tnow need luminosity from all bursts combined

$$L_{now} = L_1 + L_2 + L_3$$

$$L_1 = SFR(\underline{t=1})\, dt \times L_{burst}(\underline{t=3})$$
$$L_2 = SFR(\underline{t=2})\, dt \times L_{burst}(\underline{t=2})$$
$$L_3 = SFR(\underline{t=3})\, dt \times L_{burst}(\underline{t=1})$$

decreasing

Implementation

```
Do i=1,3
 | Do j=i,1        <- backwards
 |  | SFR(i) dt × Lburst(j)
 | Enddo
Enddo
```

2. SFH code: assume matching dt (0.1 Myr)

Read in burst file

```
Open singleburst.dat
Do i=1,ntbin
   Read t, Mgb, Mrb
   lgb(i) = 10^(-Mgb/2.5)
   lrb(i) = 10^(-Mrb/2.5)
   t(i) = t
Enddo
```

Convert Mabs to Luminosity

t in years

Read SFH file

```
Open SFH.dat
Do i=1,ntbin
   Read SFR(i)
Enddo
```

Read backwards from tnow for luminosity from previous bursts

```
Do i=1,ntbin
   tnow = i x tstep
   K = i+1
   lgtot = lrtot = 0
   Do j=1,i
      K = K-1
      lgtot = lgtot + SFR(j) x tstep x lgb(K)
      lrtot = lrtot + SFR(j) x tstep x lrb(K)
   Enddo
   Mgtot = -2.5 lg(lgtot)
   Mrtot = -2.5 log(lrtot)
   Print, Mrtot, Mgtot-Mrtot, tnow
Enddo
```

Step size in SFR and burst file

Counter moves backwards

Sums luminosity from bursts

Convert back to absolute mags

What you must do:

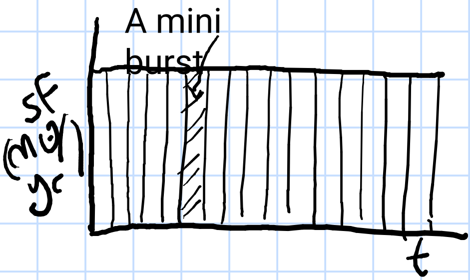
Part 3: Colour evolution of stripped galaxies

After the disk gas is lost, galaxies will stop forming stars and change colour from blue to red colours. We will use simple stellar population models to predict the colour evolution:

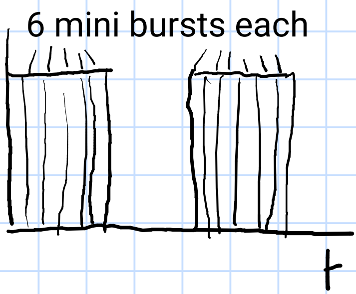
(i) Plot the g-r colour evolution and luminosity of the single burst of stars (Mstr=1 Msol) from Bruzual & Charlot+2016 (singleburst.dat)

(ii) A star formation history (SFH) of a galaxy can be considered a series of individual mini star bursts combined:

E.g. continuous star formation:



Or two bursts:



By summing the light from each burst, you can calculate the colour and luminosity evolution of any SFH. Write a code to give the g-r color evolution and r band luminosity evolution for a given SFH.

(iii) Test out your code for a continuous SFH of 3.0 Msol/yr for 14 Gyr. The answer provided by Bruzual & Charlot is given for comparison (constSF_colevol.dat).

(iv) Plot the evolution of the low mass galaxy from part 2 on a colour-magnitude diagram and compare its track to the position of observed galaxies (colmagdata_obsgals.dat) on this plot. Assume it fell into the cluster 5 Gyr ago.