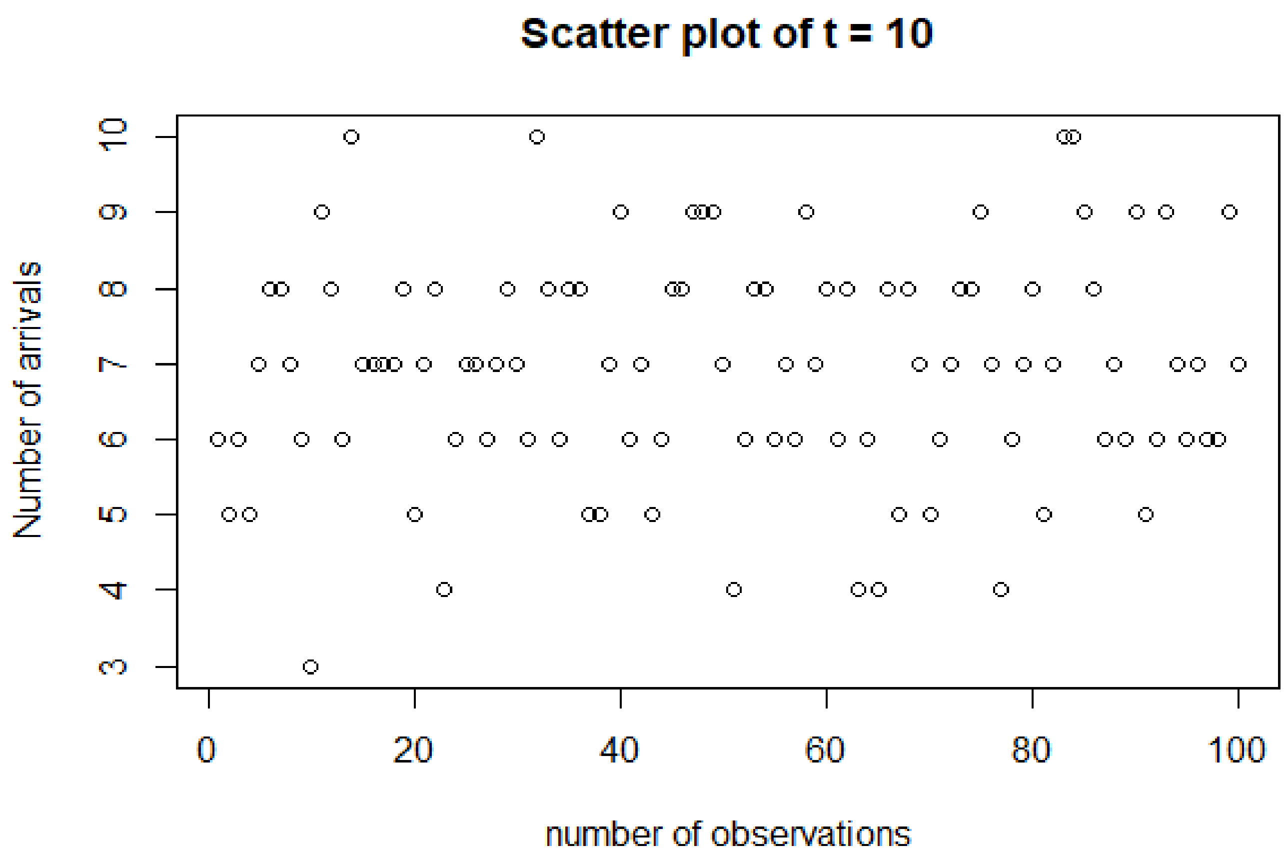
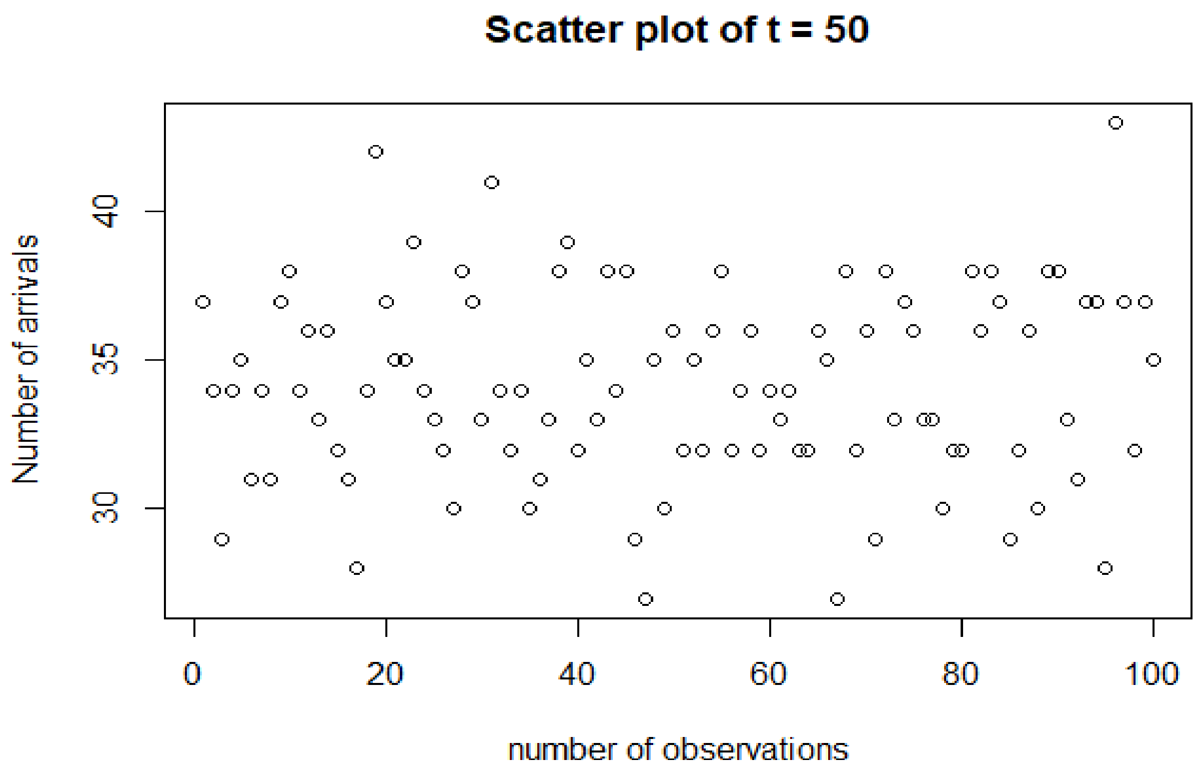
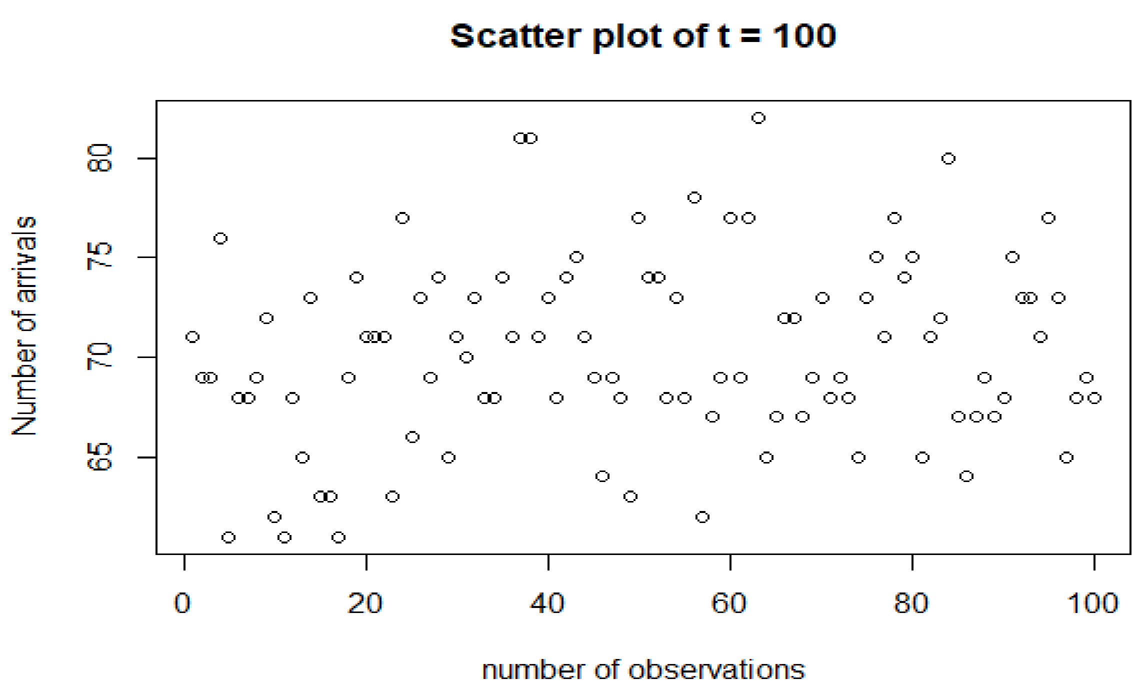
ASSIGNMENT-1

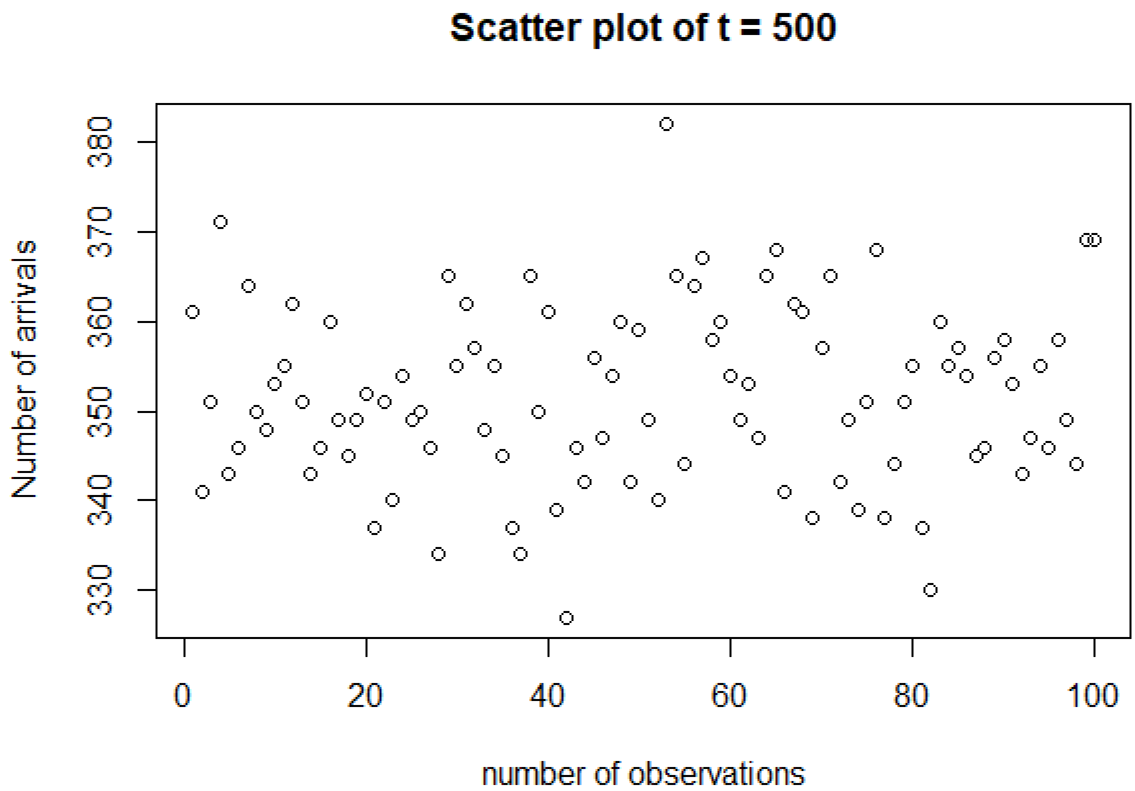
Answer-1
In this we need to study the Bernoulli process and plot the Density and CDF for various distributions.

Generating a bernoulli process for p = 0.7 and t = 10,50,100 and 500. The scatter plots are shown below:




Scatter plot of t = 50
Number of arrivals
30
35
40
0
20
40
60
80
100
number of observations

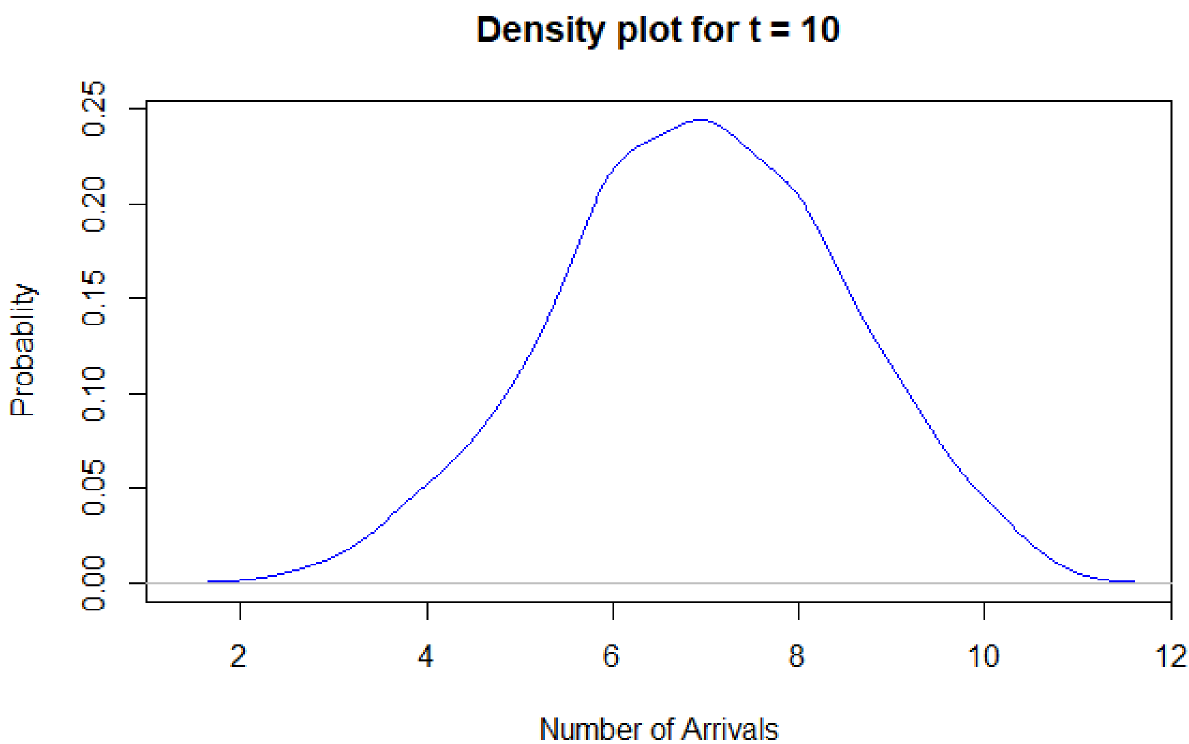

Scatter plot of t = 100
Number of arrivals
65
70
75
80
0
20
40
60
80
100
number of observations

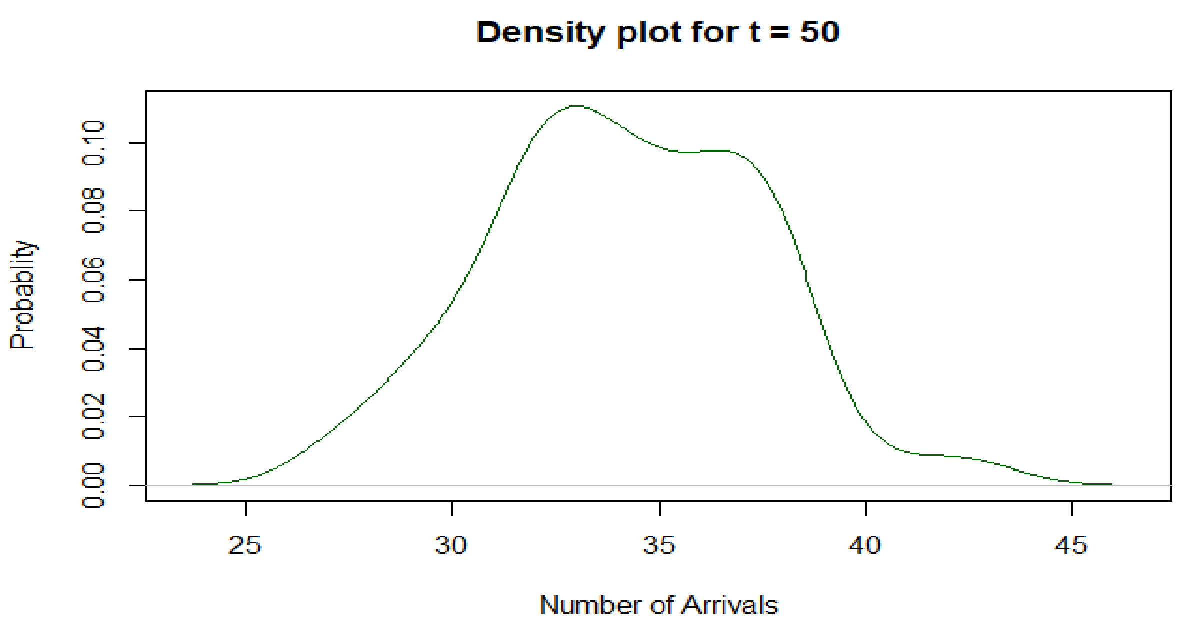


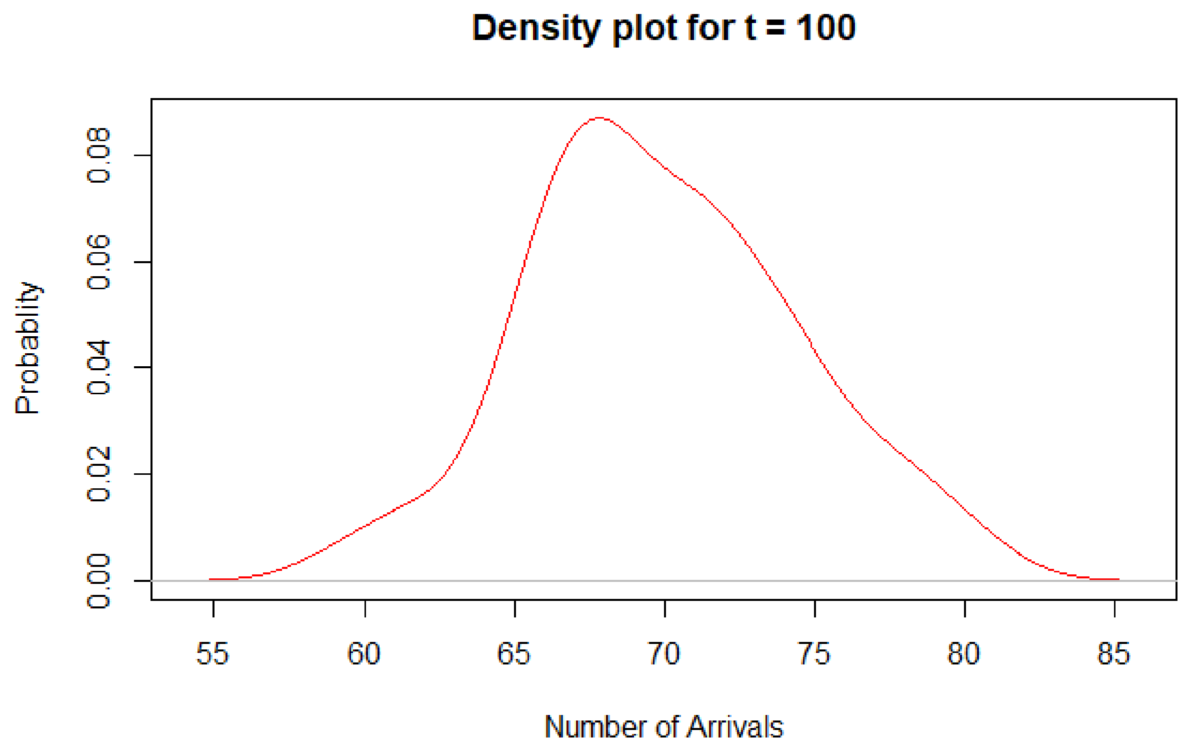
The scatter plots are constructed using the rbinom function present is R. The x axis denotes the number of observations(or we can say the number of trials) and the y axis represents the number of arrival times. The number of observations obtained is equal to the number of Bernoulli processes performed.

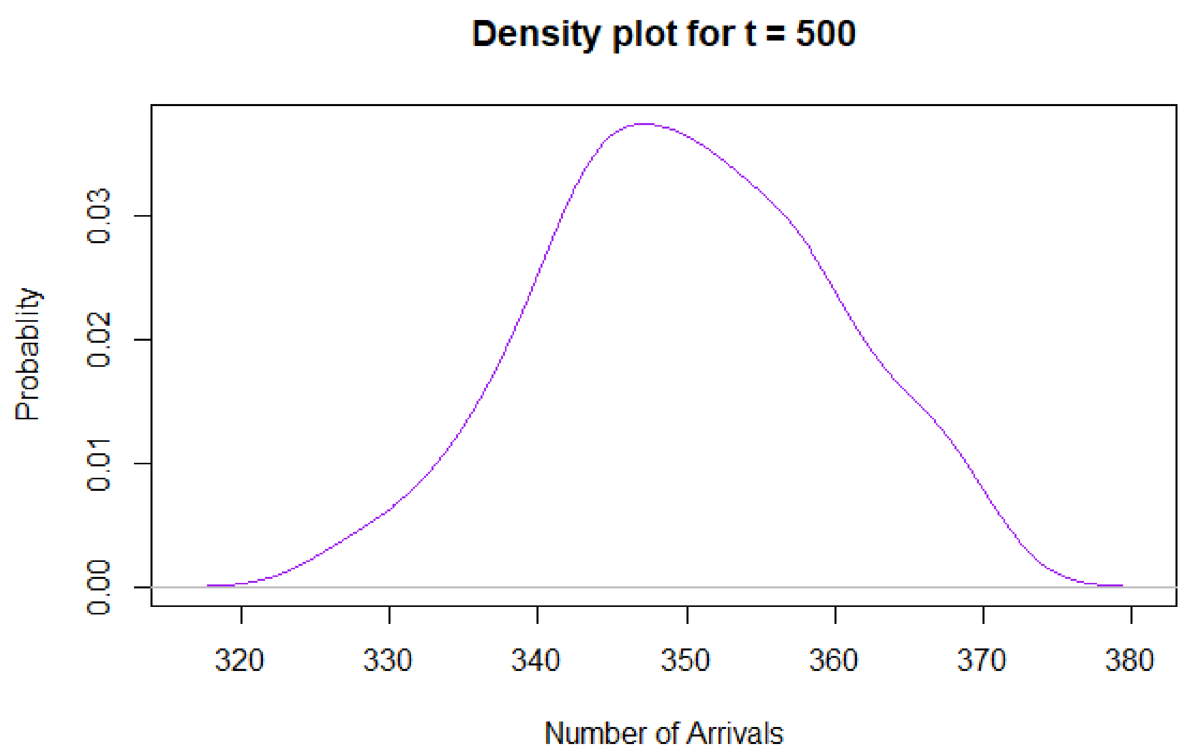
1. For scatter plot with t = 10 the points will be concentrated around 7
2. For scatter plots with t = 50 the points will be concentrated around 35
3. For scatter plot with t = 100 the points will be concentrated around 70
4. For scatter plot with t = 500 the points will be concentrated around 350

The Density function of the given simulation will be as follows:





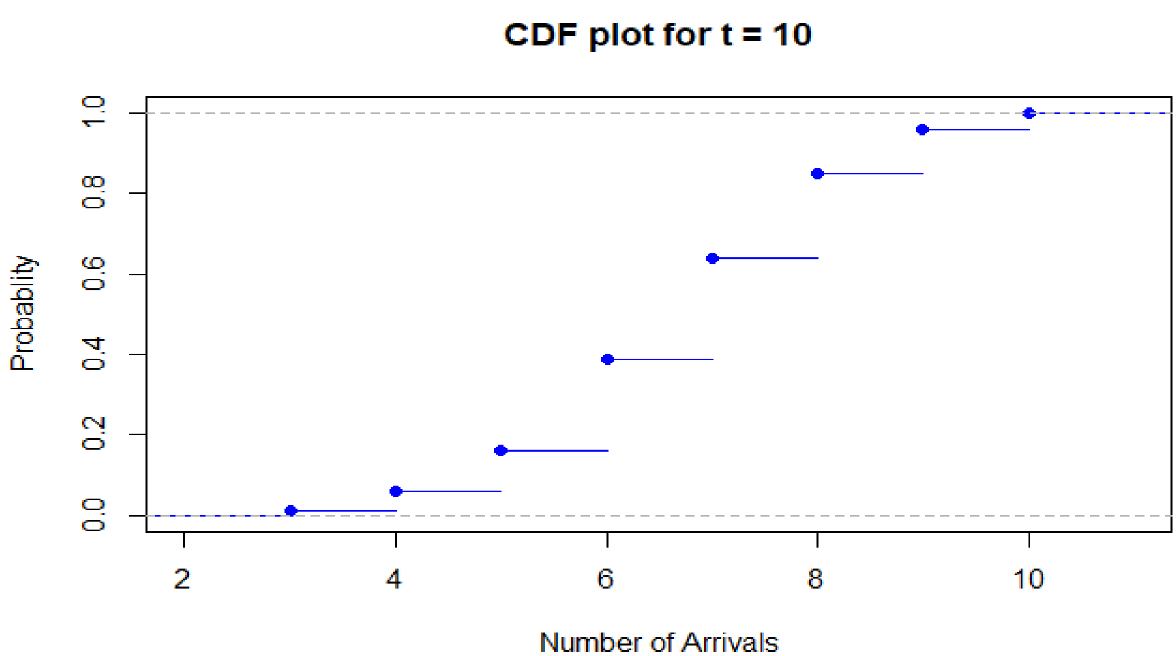

Density plot for t = 100
Probablity
0.00
0.02
0.04
0.06
0.08
55
60
65
70
75
80
85
Number of Arrivals

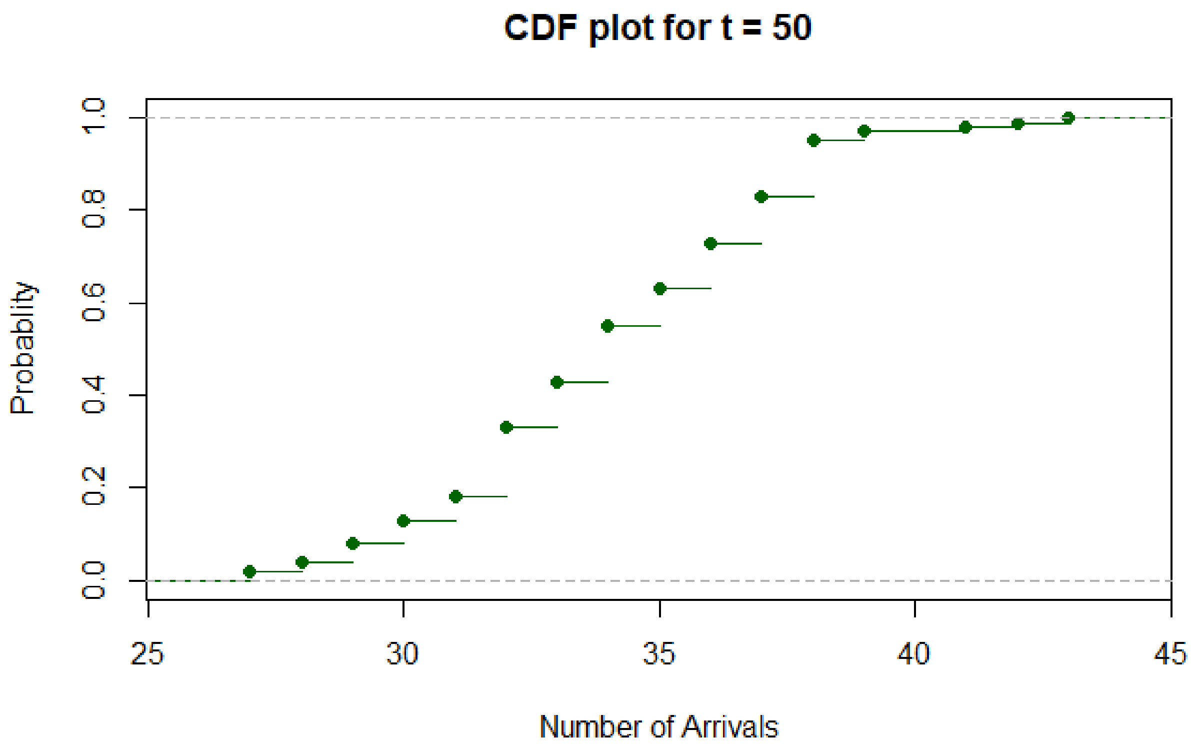

Density plot for t = 500
Probablity
0.00
0.01
0.02
0.03
320
330
340
350
360
370
380
Number of Arrivals

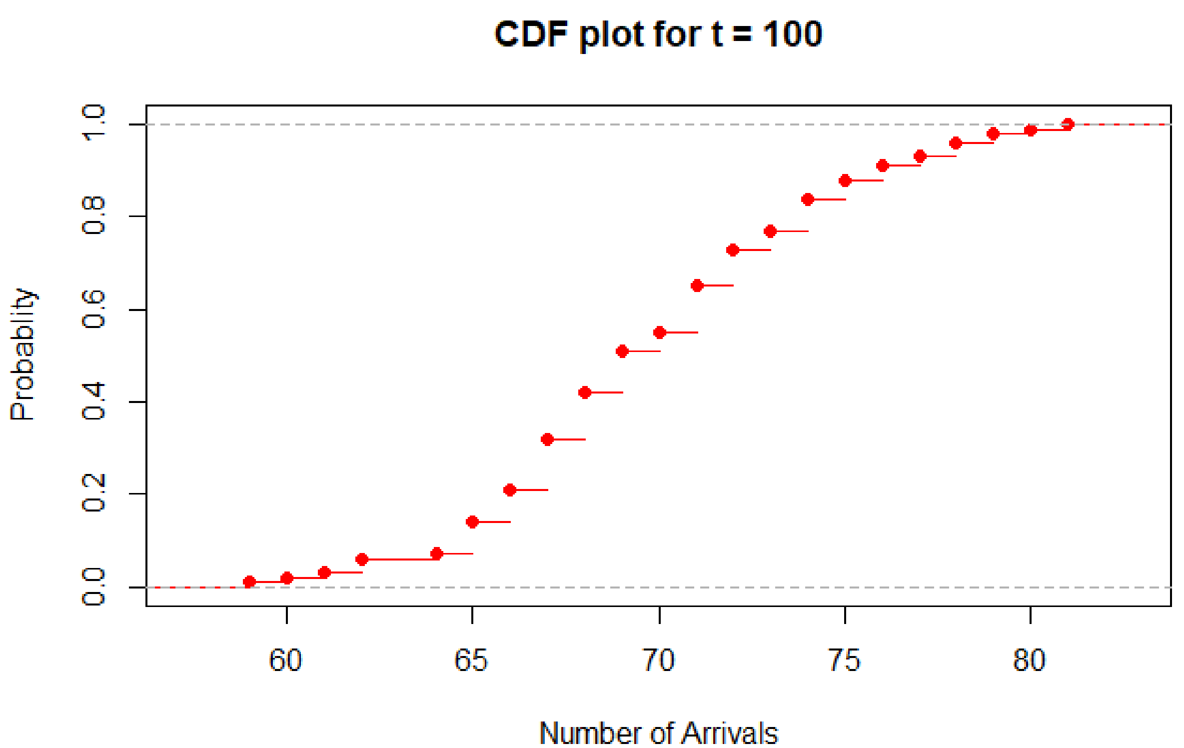
In the density function shown above the graph is between probability and number of arrivals. It can be clearly seen that the maximum probability is obtained for those arrivals that are pXt. Since the values of the probabilities are discrete then the sum of probabilities has to be 1. And hence the value of p decreases on increasing the arrival time. The following table shows the approximate value of arrival time for which the probability peaked for different values of t:

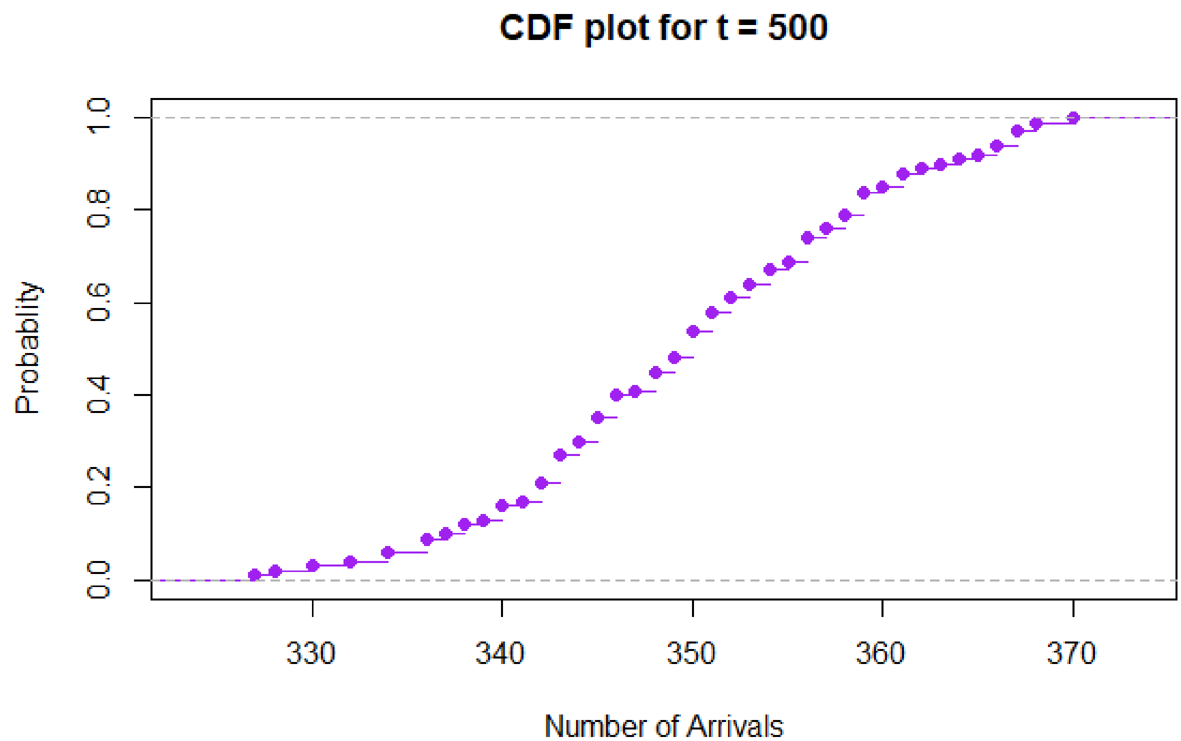
Values of t	**Arrival Time**
10	**10 * 0.7 = 7**
50	**50 * 0.7 = 35**
100	**100 * 0.7 = 70**
500	**500 * 0.7 = 350**

The plots for CDFs are as follows:




CDF plot for t = 50
Probablity
0.0
0.2
0.4
0.6
0.8
1.0
25
30
35
40
45
Number of Arrivals

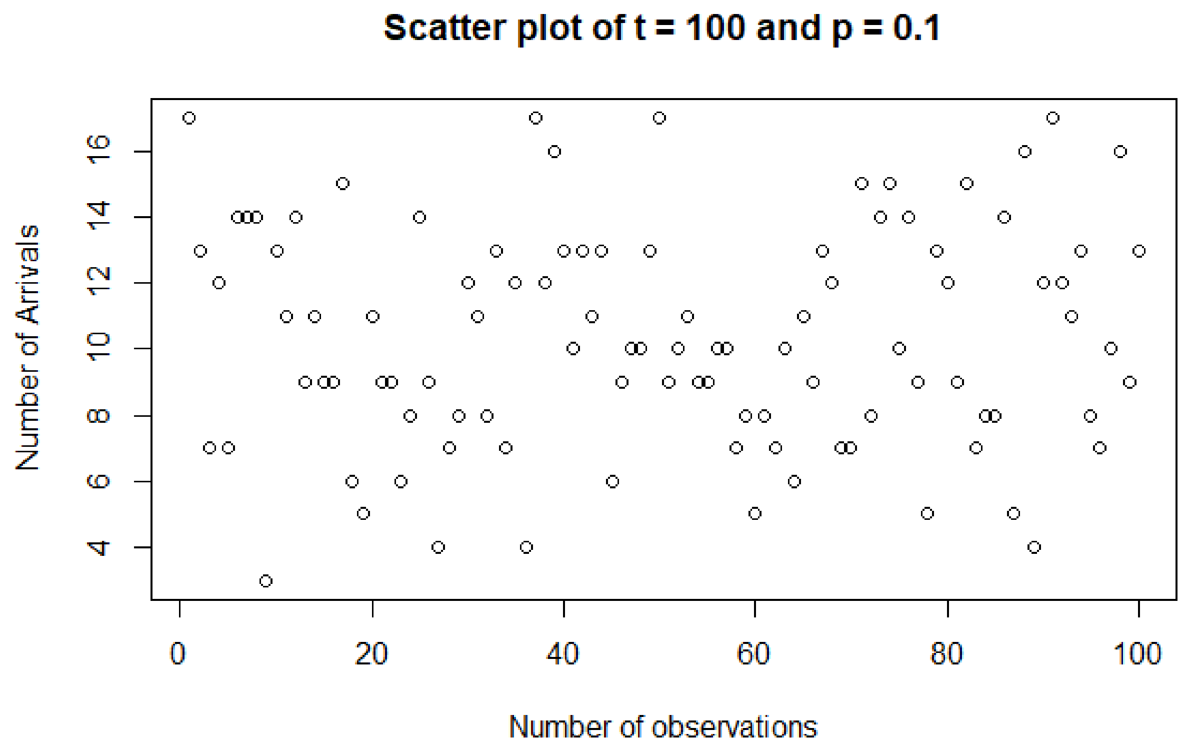

CDF plot for t = 100
Probablity
0.0
0.2
0.4
0.6
0.8
1.0
60
65
70
75
80
Number of Arrivals

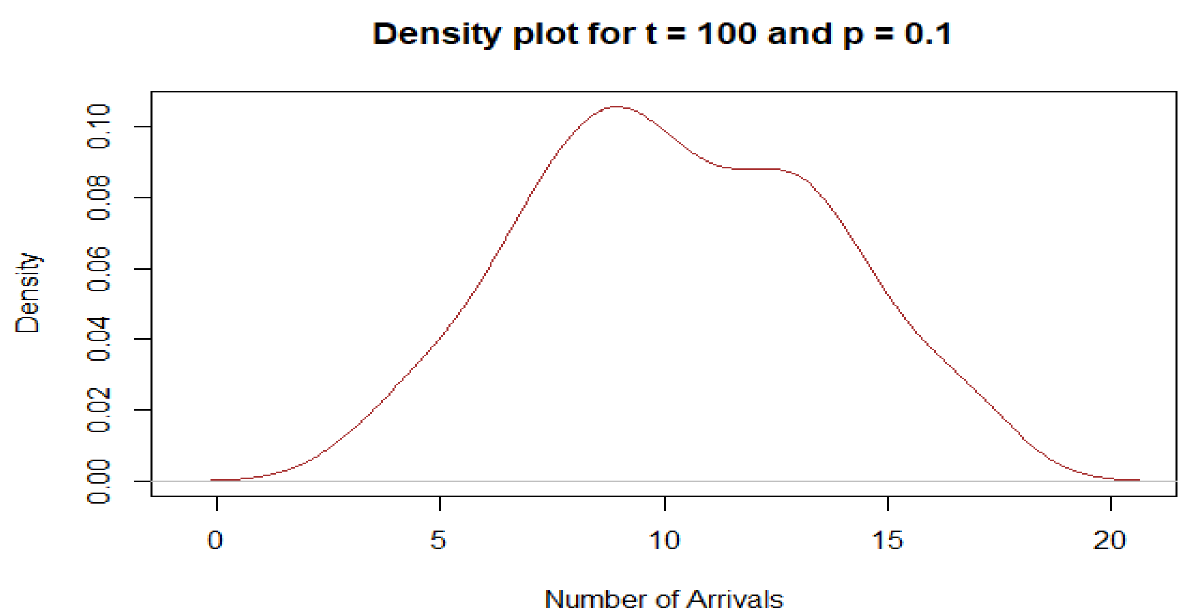


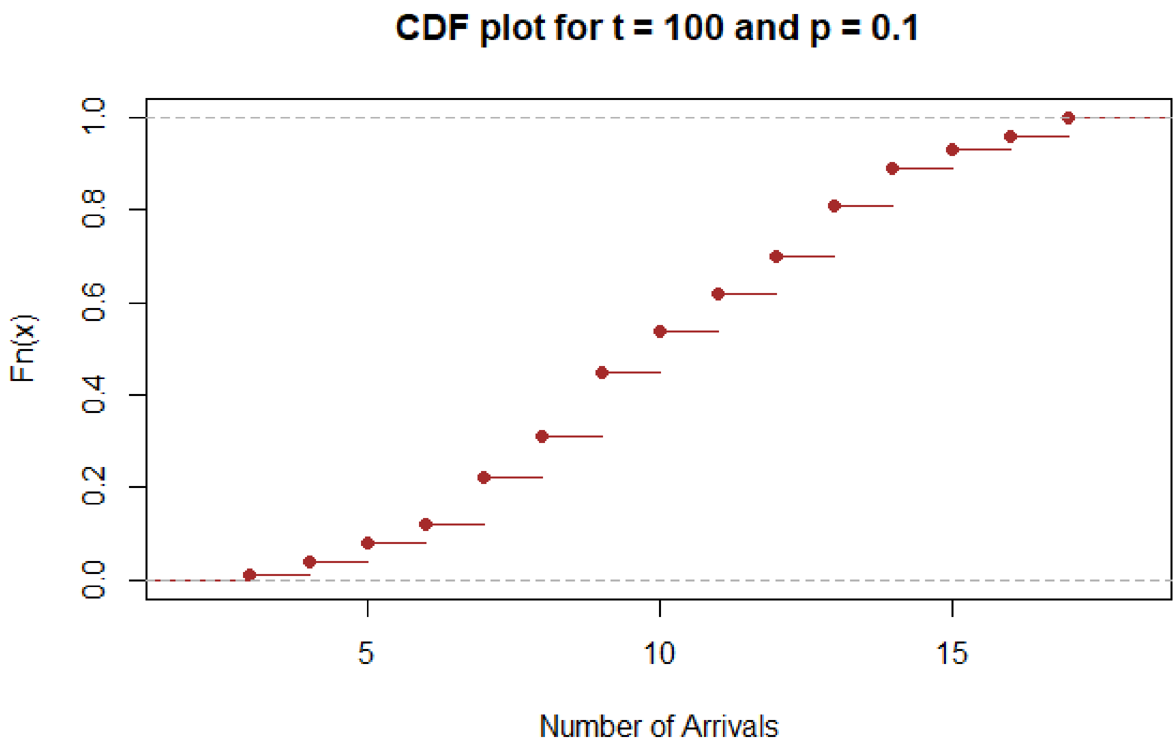
The CDF is defined as follows:

$$F_X(x) = P(X \leq x)$$

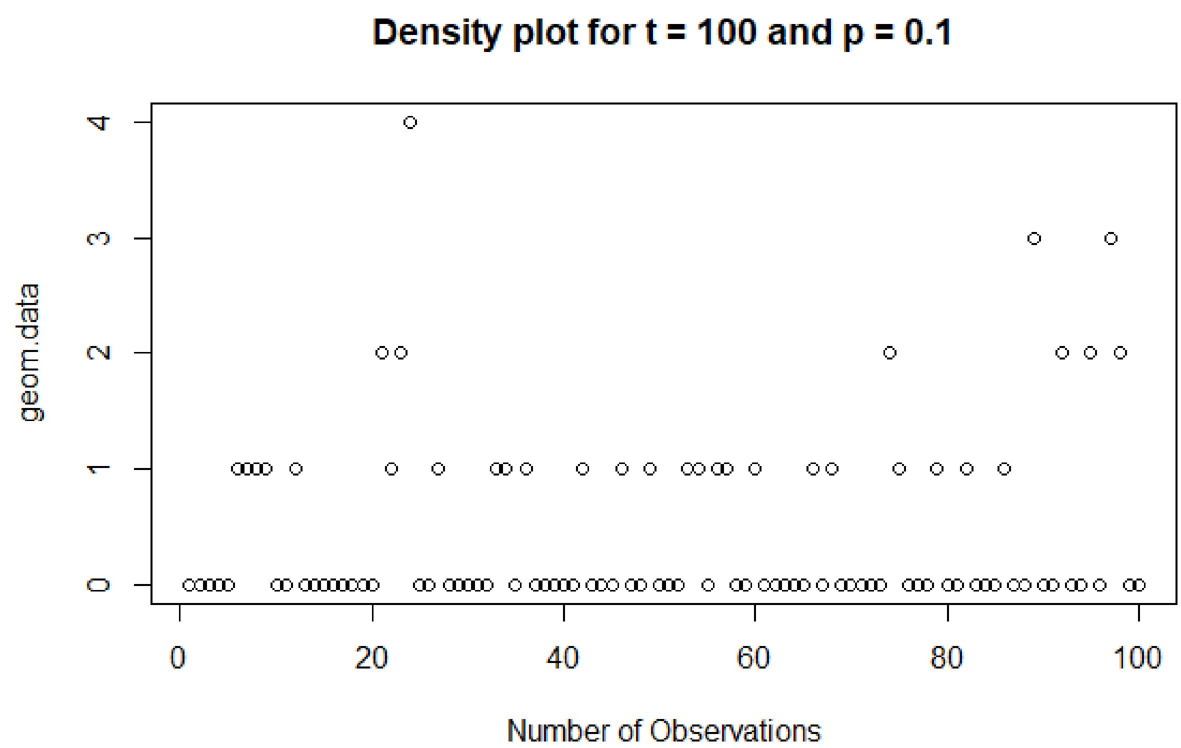
For the CDF values the graph is monotonically increasing. As seen in the density function graph the max probability is obtained around 0.7*t. Hence the rate at which the CDF increases, increases around the value of arrival times at which maximum probability is observed.


Scatter plot of t = 100 and p = 0.1
Number of Arrivals
Number of observations

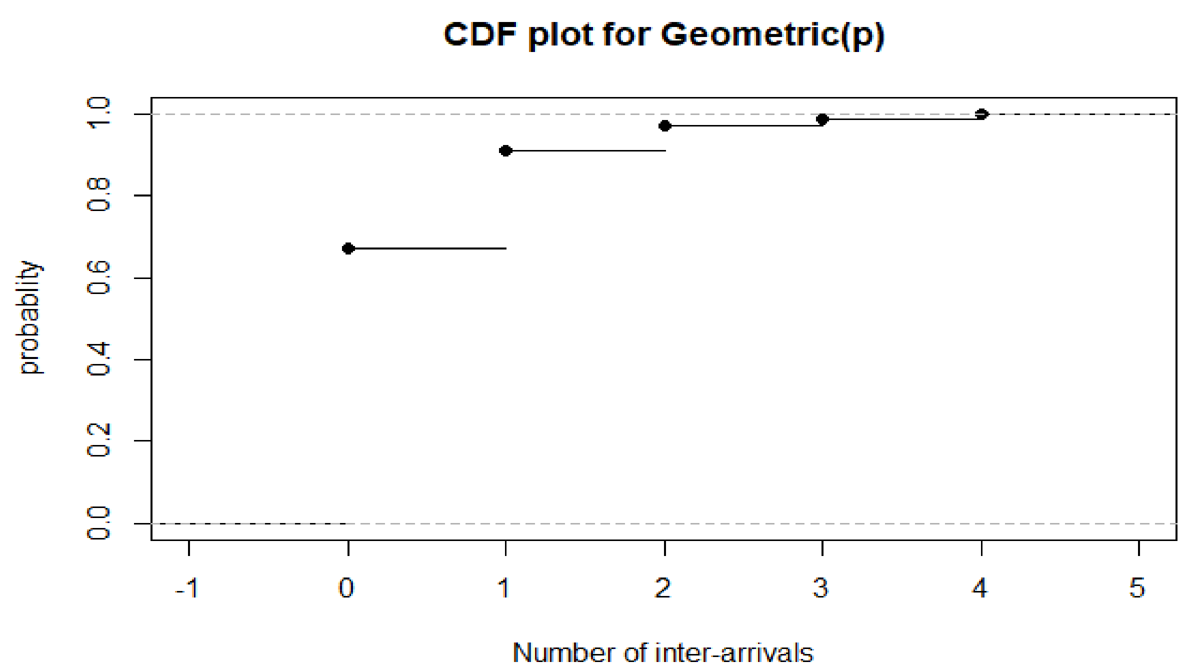

Density plot for t = 100 and p = 0.1
Density
Number of Arrivals



Again in this plot the value of t = 100 and the value of p = 0.1. Hence the maximum value of p is observed for arrival times near 10 and the rate of the graph of the CDF also increases for values around 10

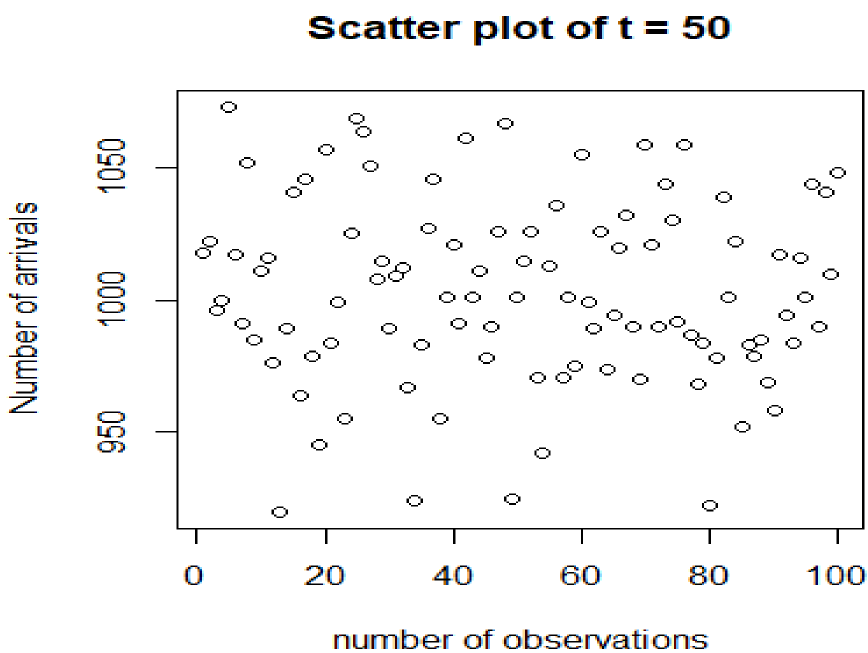


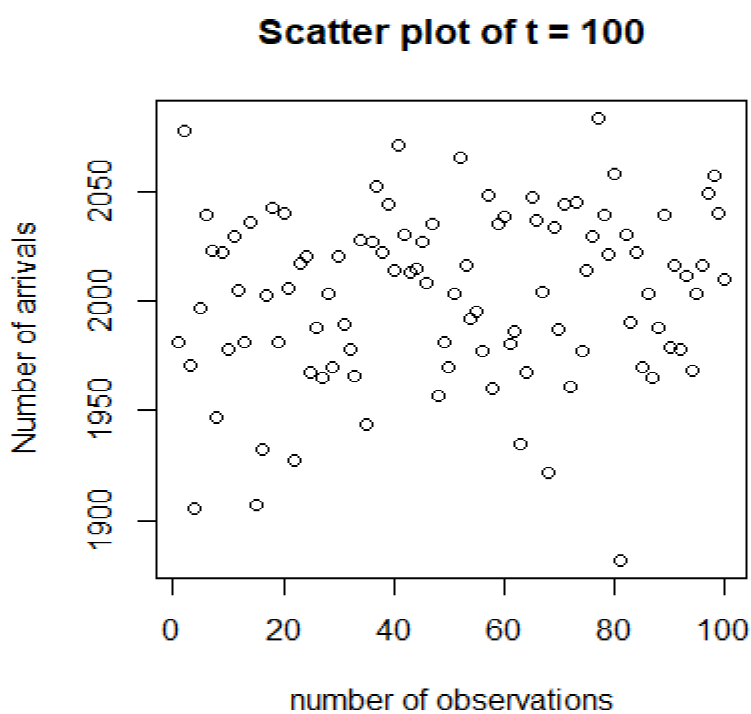
Density plot for t = 100 and p = 0.1
geom.data
0
1
2
3
4
0
20
40
60
80
100
Number of Observations



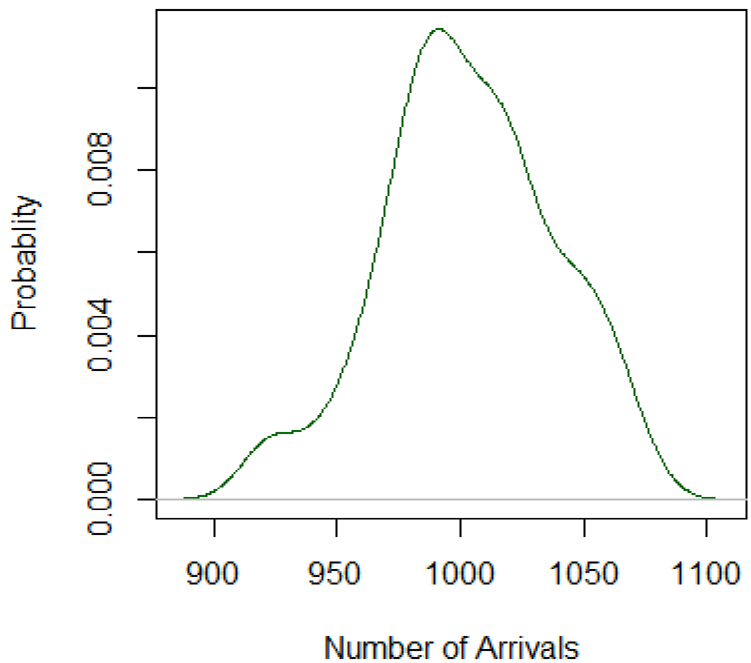
CDF plot for Geometric(p)
probablity
0.0
0.2
0.4
0.6
0.8
1.0
-1
0
1
2
3
4
5
Number of inter-arrivals

ANSWER-2

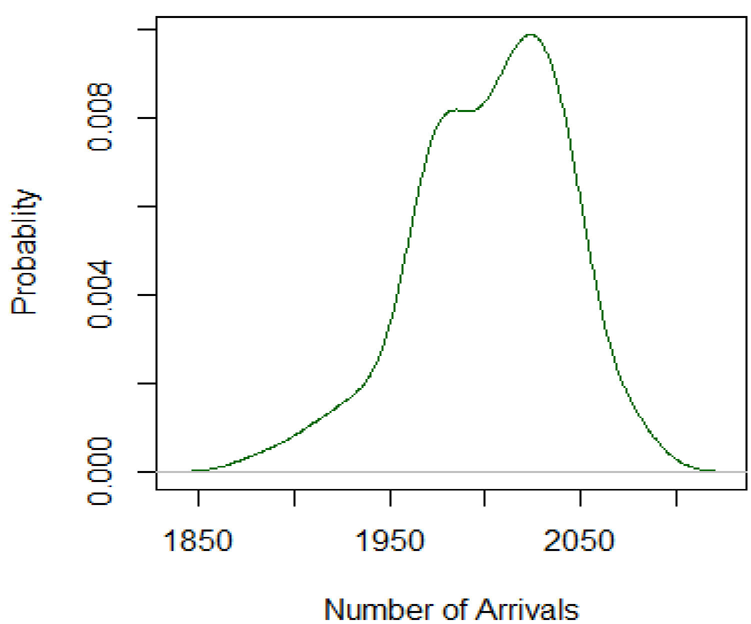

Scatter plot of t = 50
Number of arrivals
950
1000
1050
0
20
40
60
80
100
number of observations


Scatter plot of t = 100
Number of arrivals
1900
1950
2000
2050
0
20
40
60
80
100
number of observations

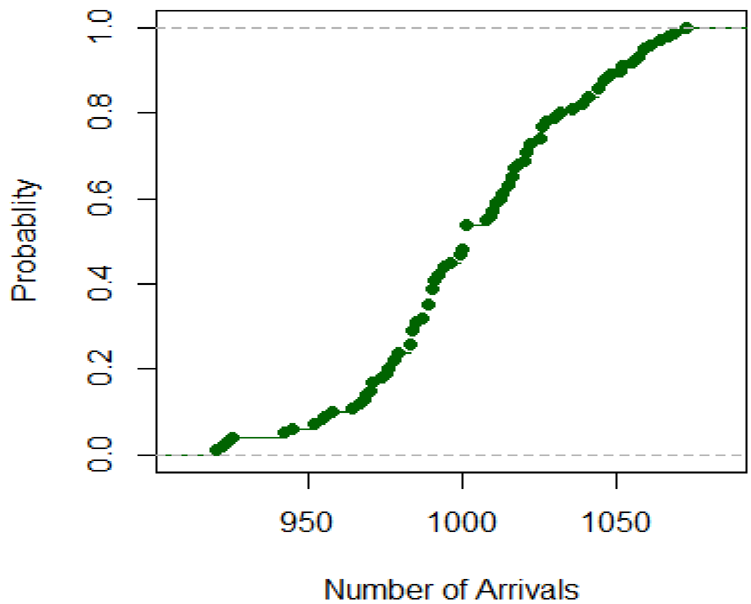
Density plot for t = 50



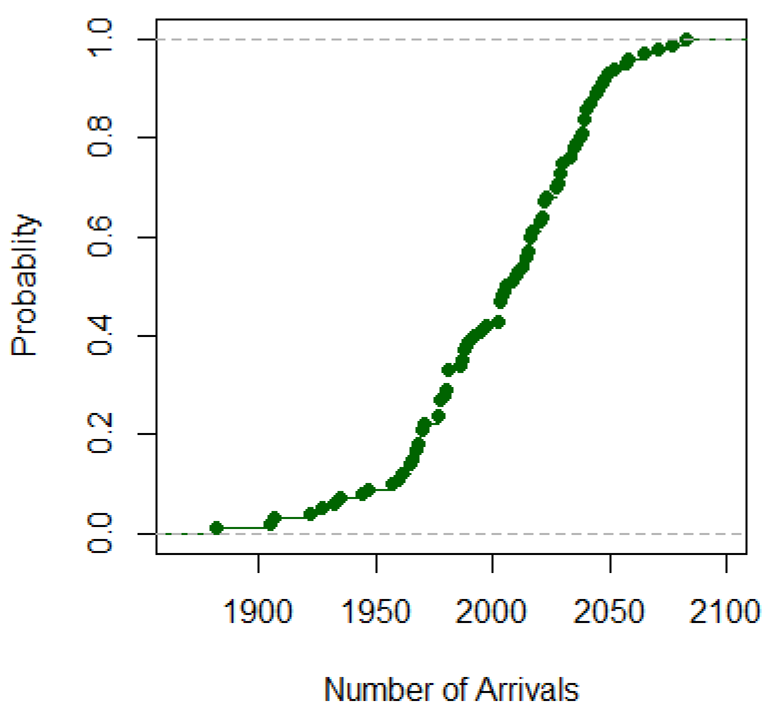
Density plot for t = 100



CDF plot for t = 50



CDF plot for t = 100



The plot for CDF is almost the similar for both t = 50 and 100. The formula for CDF and density in poisson is as follows:

$$f(x, \lambda) = \frac{e^{-\lambda} \lambda^x}{x!} \quad \text{CDF}$$

$$F(x, \lambda) = \sum_{k=0}^{x} \frac{e^{-\lambda} \lambda^x}{k!} \quad \text{PDF}$$

The value of x is continuously increasing and so will be the value of λ^x. But at the same time the value of x! is also increasing. So the effect will be nullified and the graph will be similar, increasing sharply for the middle values of x.So for the higher and lower values of x the graph wouldn't be steep but for the middle values a significant change would be observed