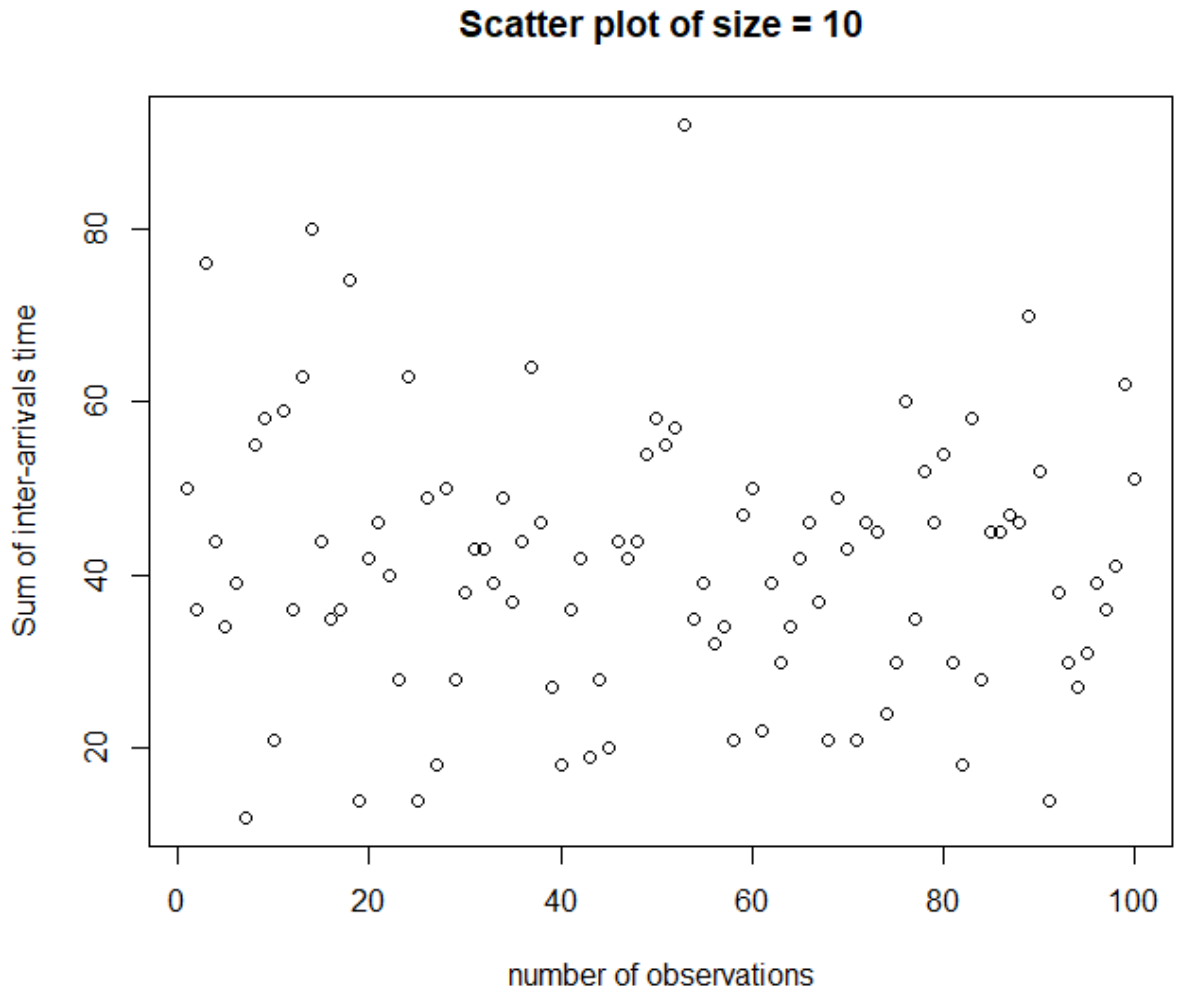
ASSIGNMENT-2

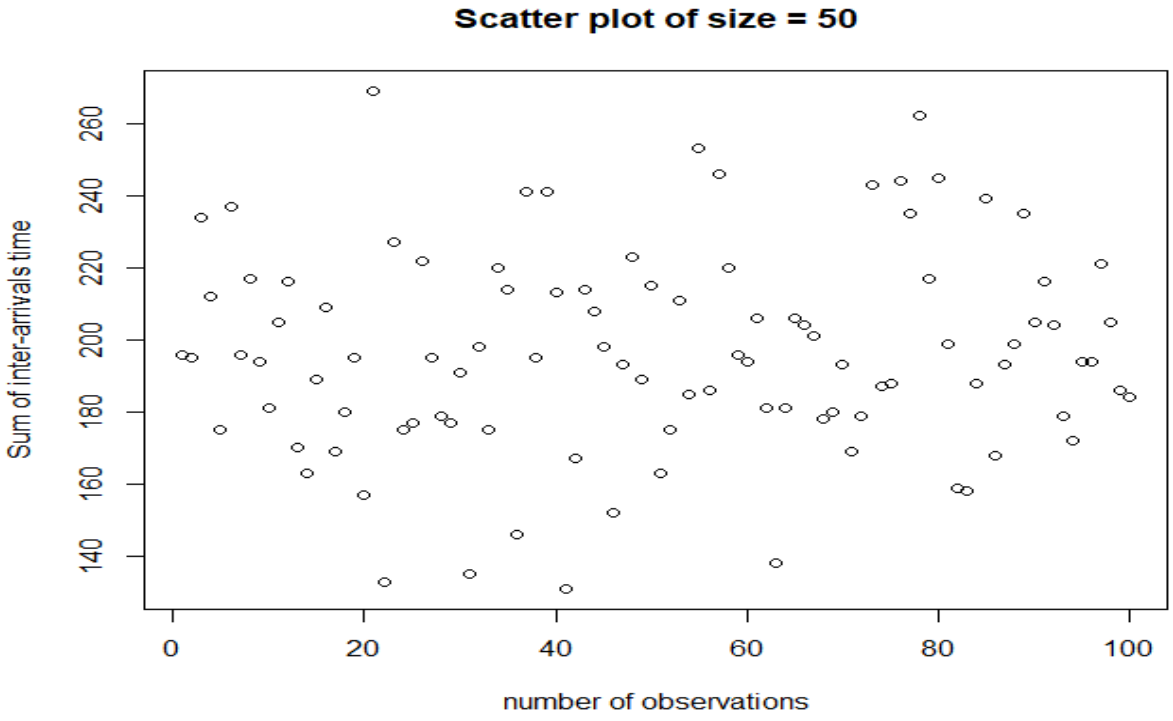
Answer-1

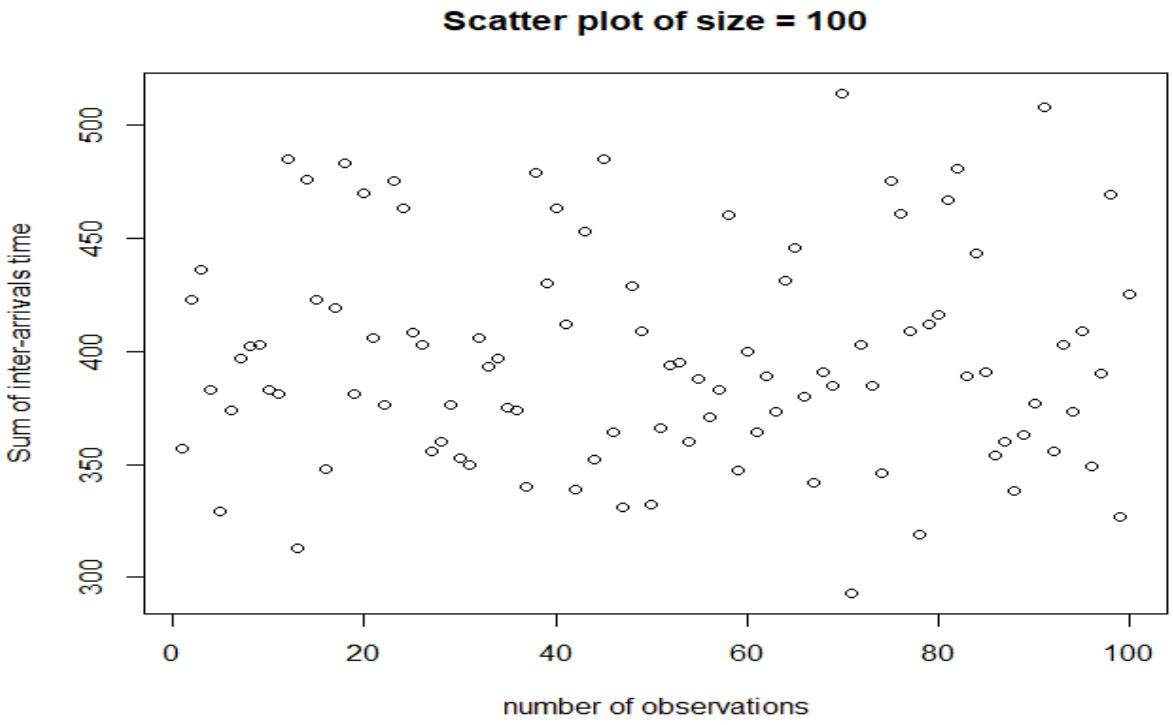
The interarrival time follows a geometric distribution with success probability p. The process is the Bernoulli process. The sum of interarrival time follows negative binomial distribution or pascal distribution.

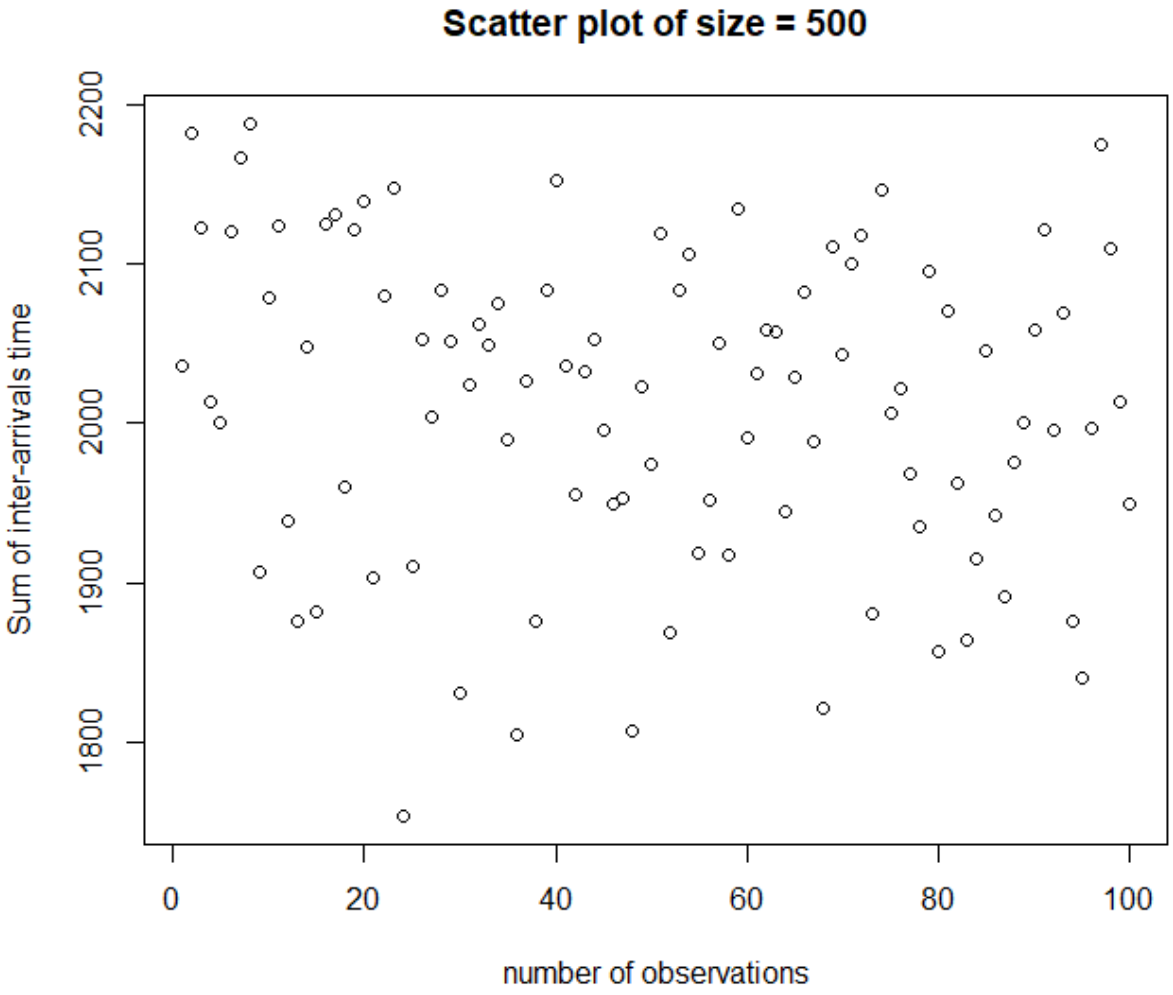
Hence we are required to find the value of n that is the number of time experiments need to be done to obtain k success. (value of k varies in different cases).

For various values of size parameters the scatter plots are shown below



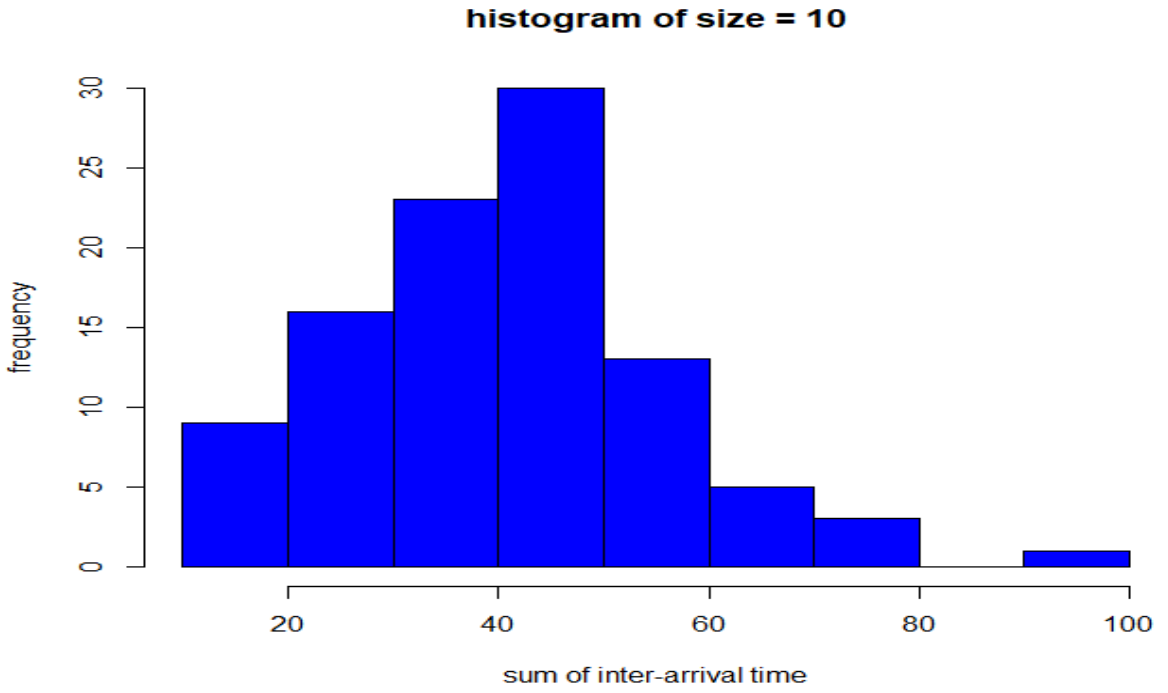

Scatter plot of size = 50
Sum of inter-arrivals time
140
160
180
200
220
240
260
0
20
40
60
80
100
number of observations

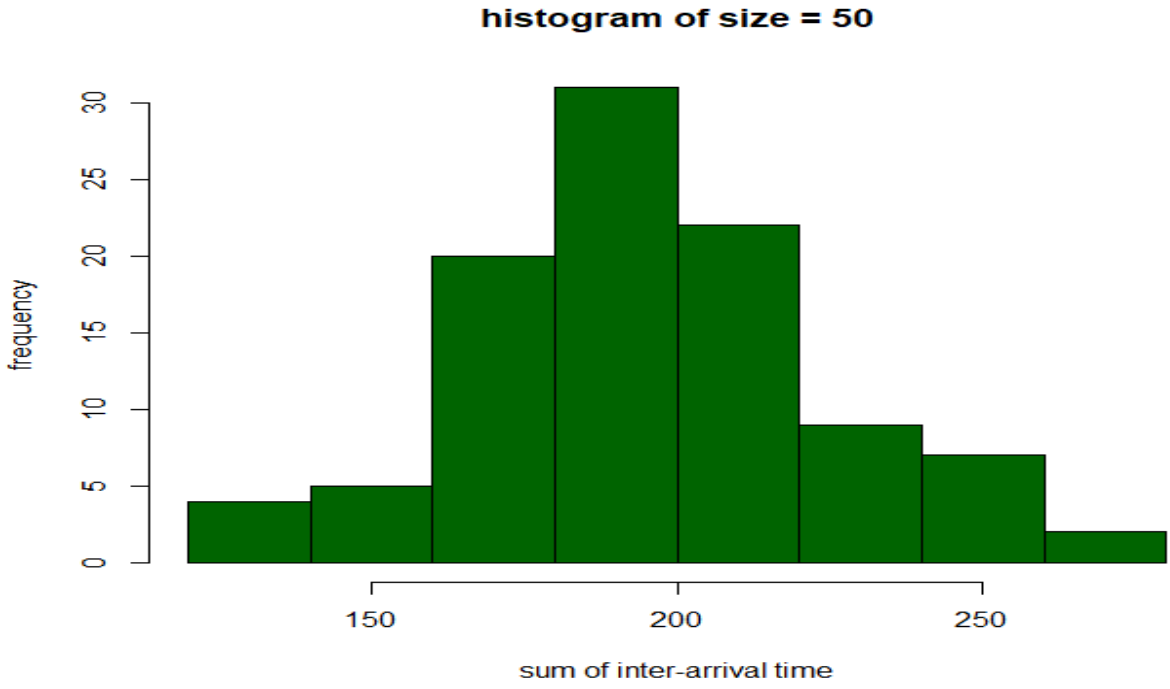

Scatter plot of size = 100
Sum of inter-arrivals time
300
350
400
450
500
0
20
40
60
80
100
number of observations

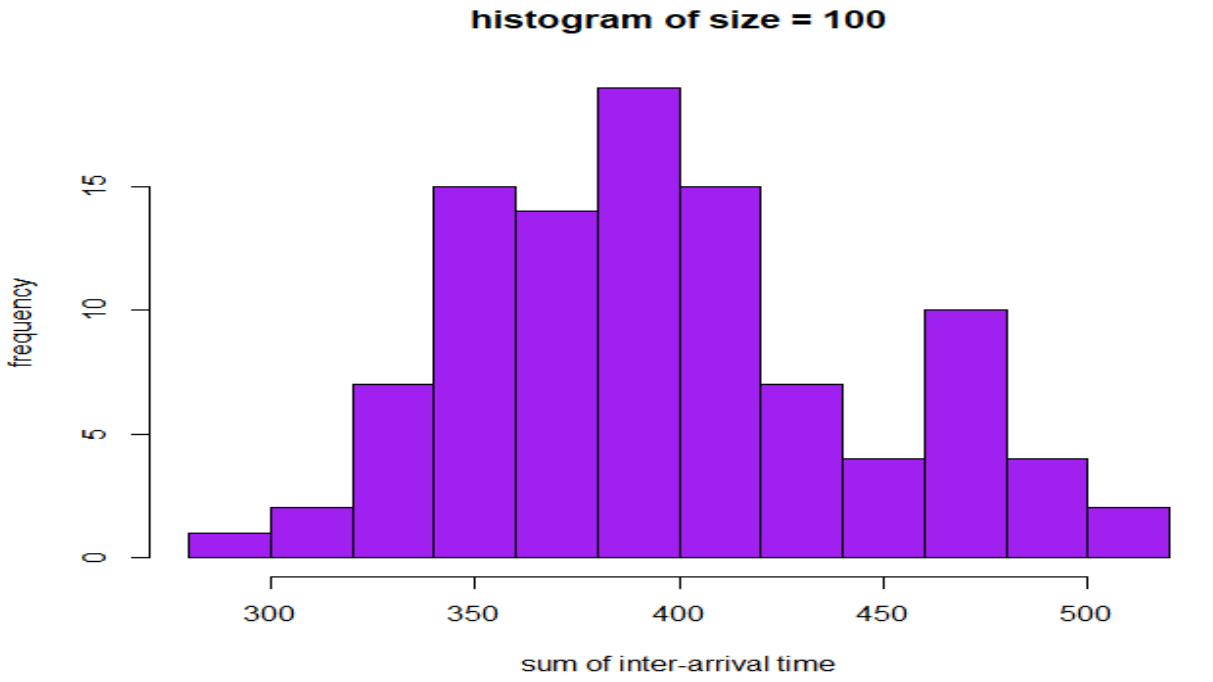


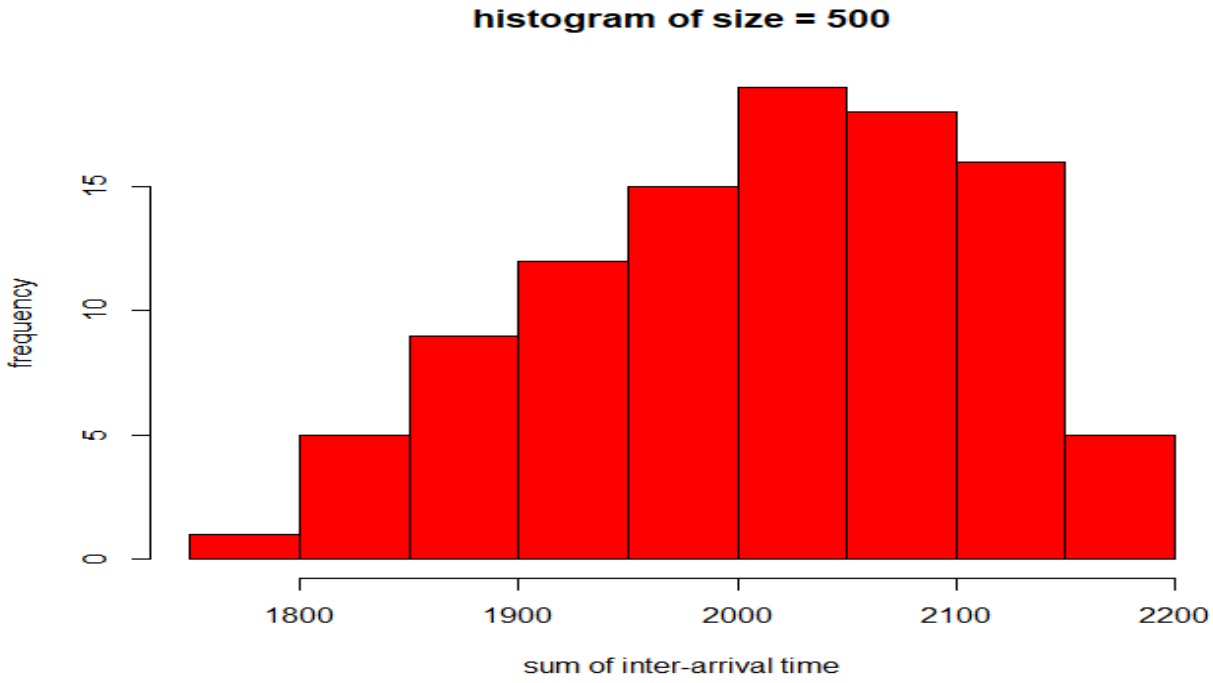
The Scatter plots are constructed using the rnbinom function present in R. The x axis denotes the number of observations(or we can say the number of trials) and the y axis denotes the sum of interarrival times. For each point in the scatter plot denotes the number of values of n that is the number of times the experiment needs to be repeated so as to achieve k number of success.

The histogram of the plots are as shown below


histogram of size = 10
frequency
0
5
10
15
20
25
30
20
40
60
80
100
sum of inter-arrival time


histogram of size = 50
frequency
0
5
10
15
20
25
30
150
200
250
sum of inter-arrival time

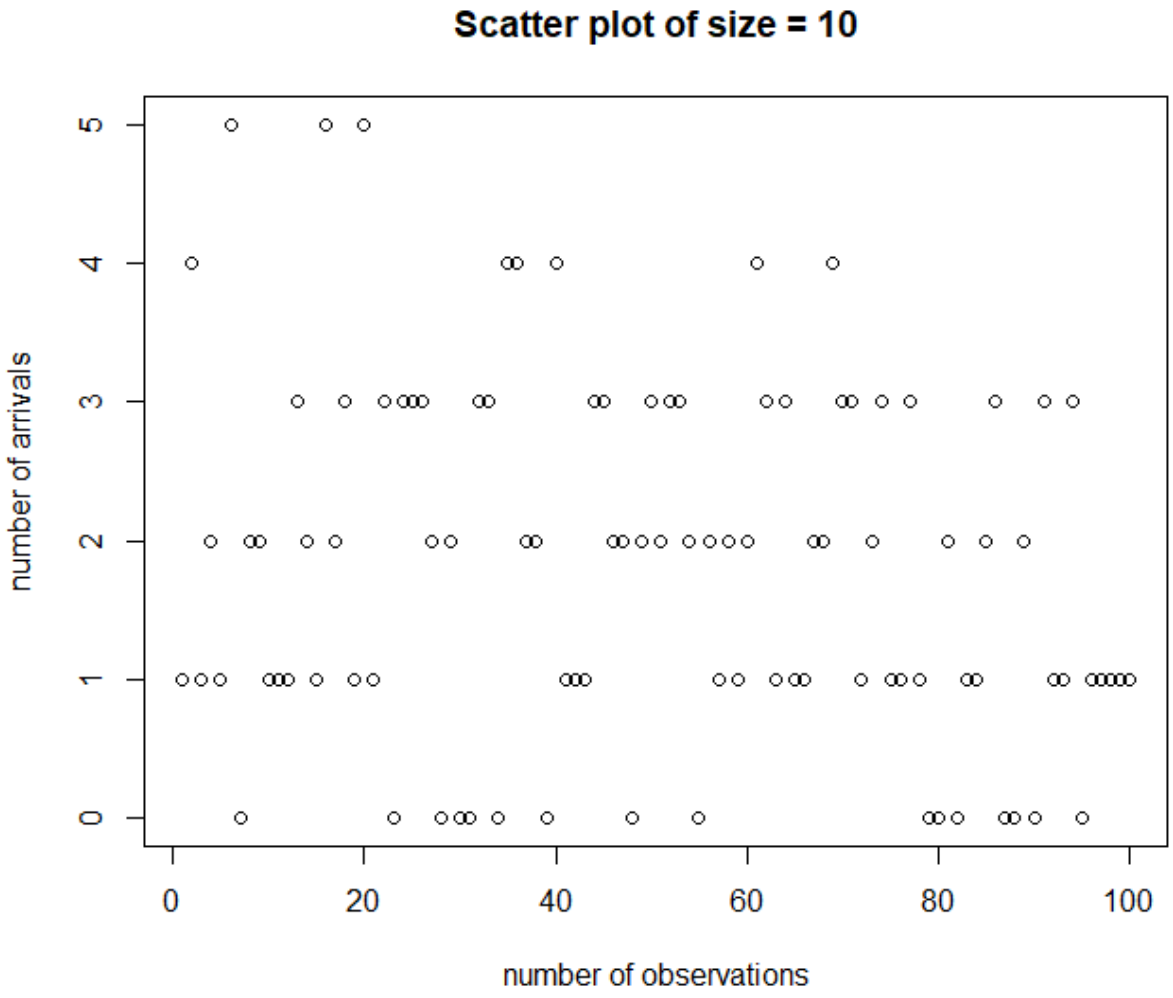

histogram of size = 100
frequency
0
5
10
15
300
350
400
450
500
sum of inter-arrival time

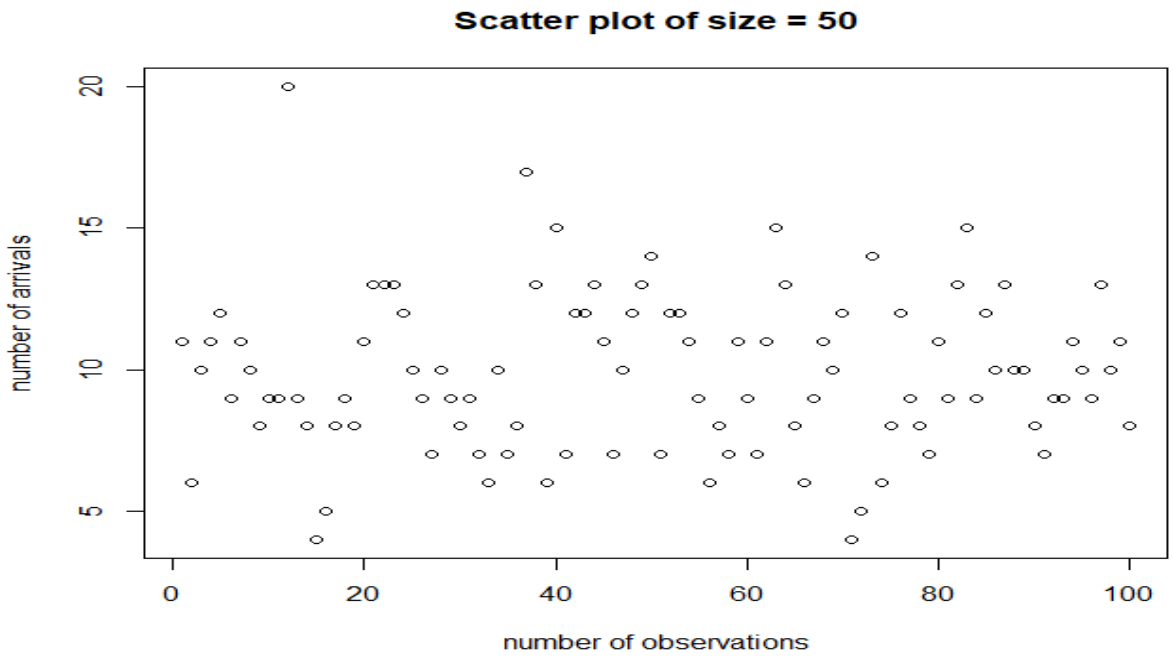

histogram of size = 500
frequency
0
5
10
15
1800
1900
2000
2100
2200
sum of inter-arrival time

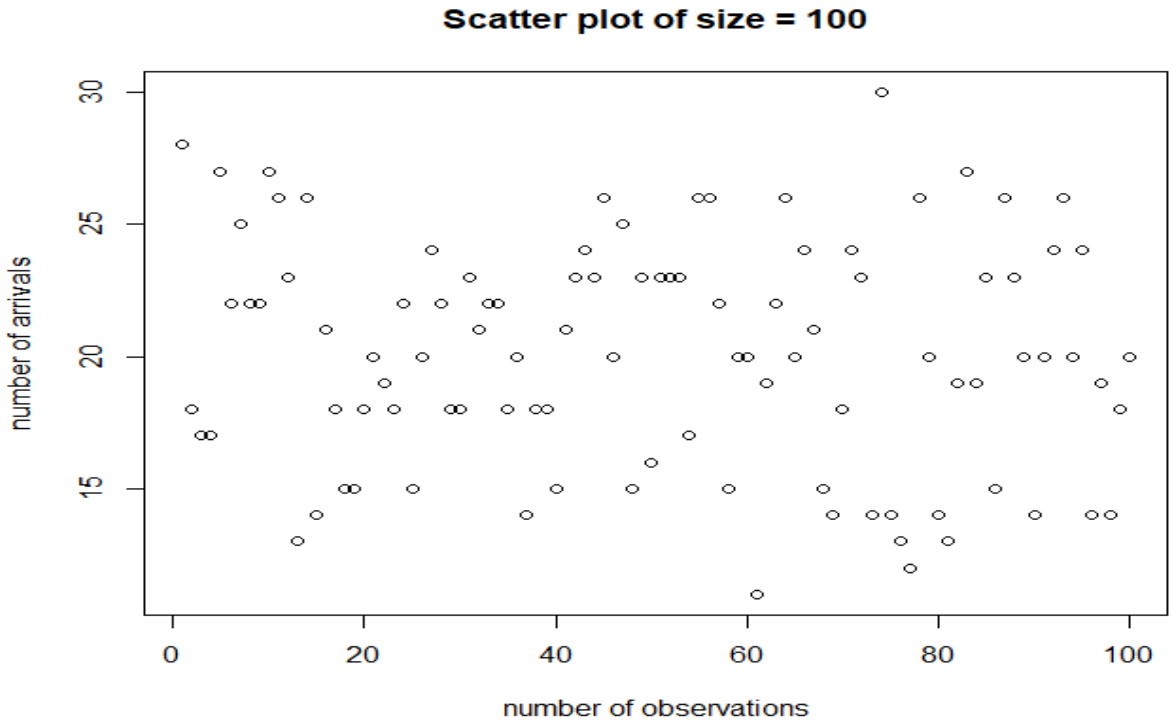
In the histograms shown above the x axis denotes sum of inter-arrival time and the y axis shows the frequency of those sum of inter-arrival times.

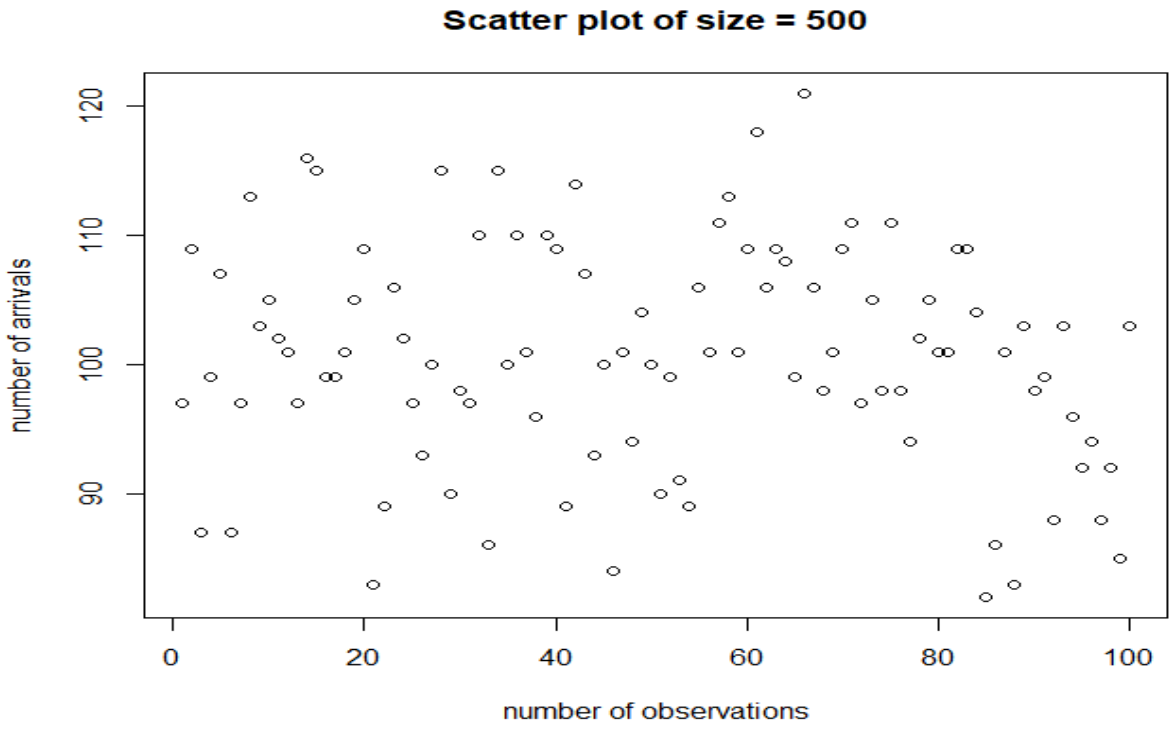
Part-B

For simulating the value of Nt we use the function rbinom as the number of arrivals with the probability value as 0.2. The scatter plots shown below represent the value of the number of arrivals for t = 10, 50, 100, 500 for 100 observations. Since the interarrival times follow geometric distribution hence the process is Bernoulli process and the number of arrivals follow binomial distribution.

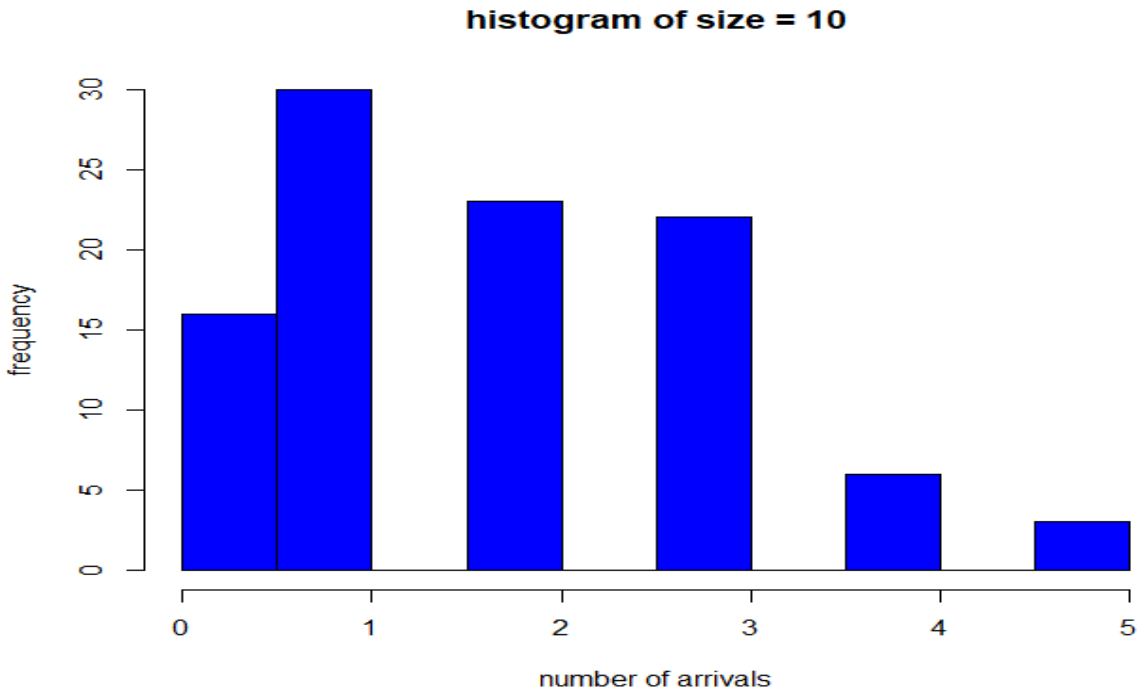


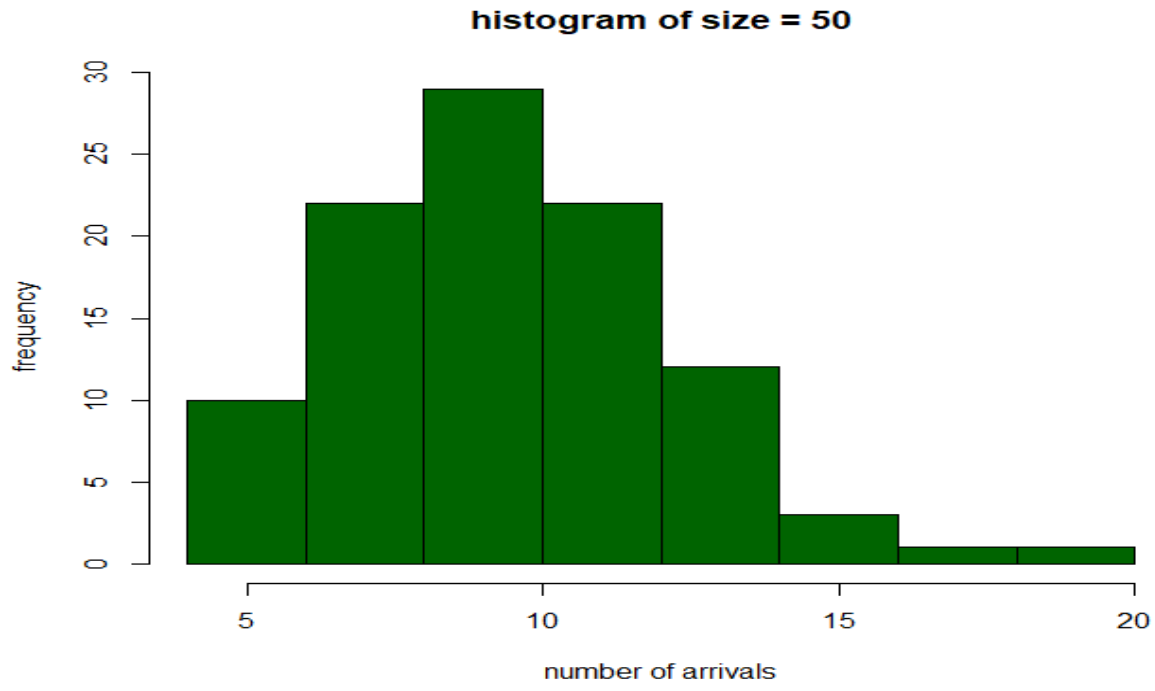

Scatter plot of size = 50
number of arrivals
number of observations
0
20
40
60
80
100
5
10
15
20

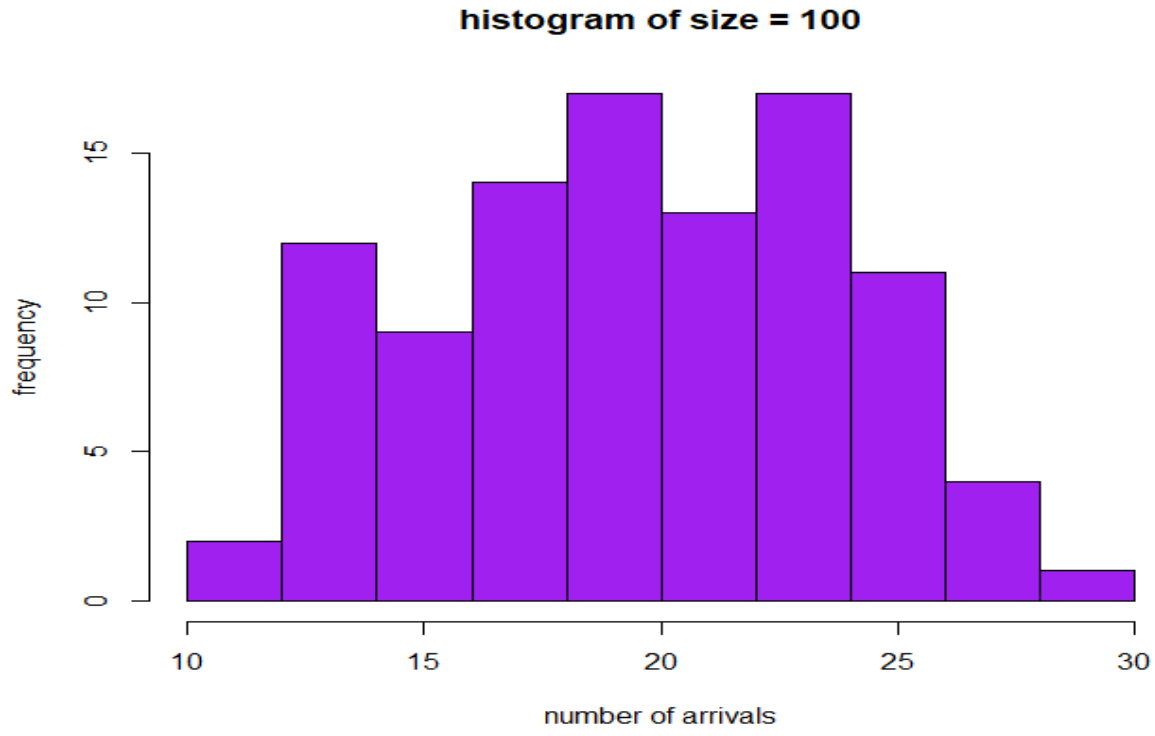

Scatter plot of size = 100
number of arrivals
number of observations
0
20
40
60
80
100
15
20
25
30

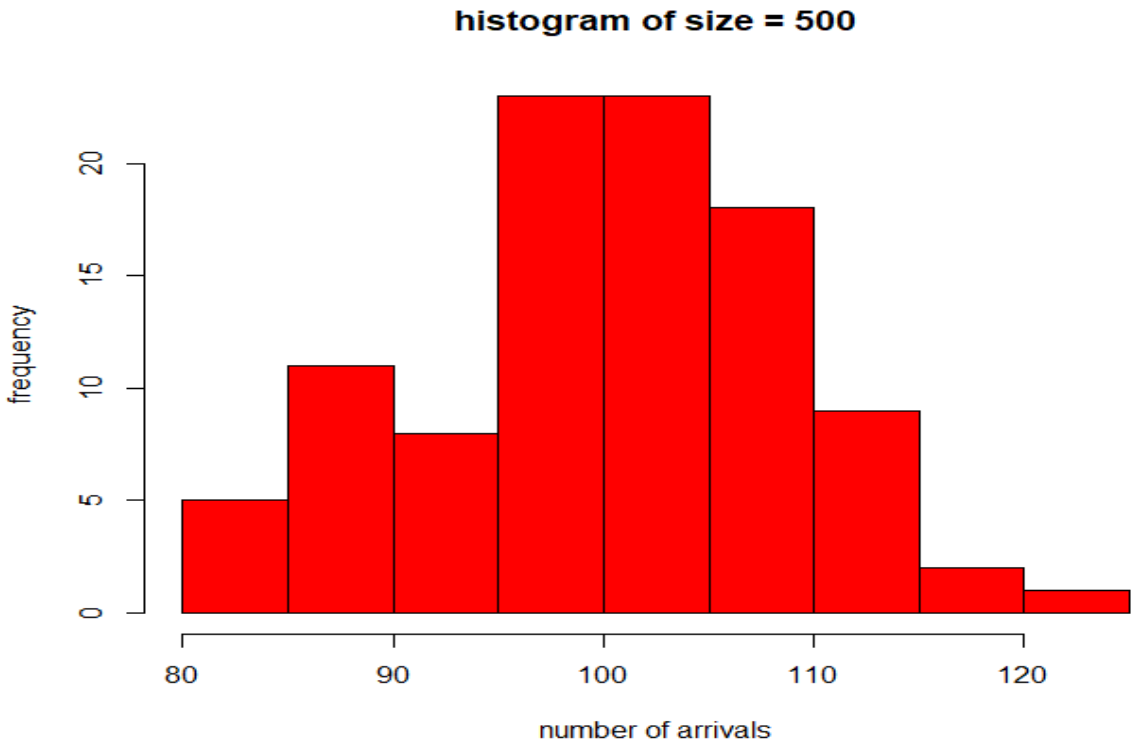


The histograms for the frequency vs number of arrivals are as shown below




histogram of size = 50
frequency
0
5
10
15
20
25
30
5
10
15
20
number of arrivals


histogram of size = 100
frequency
0
5
10
15
10
15
20
25
30
number of arrivals



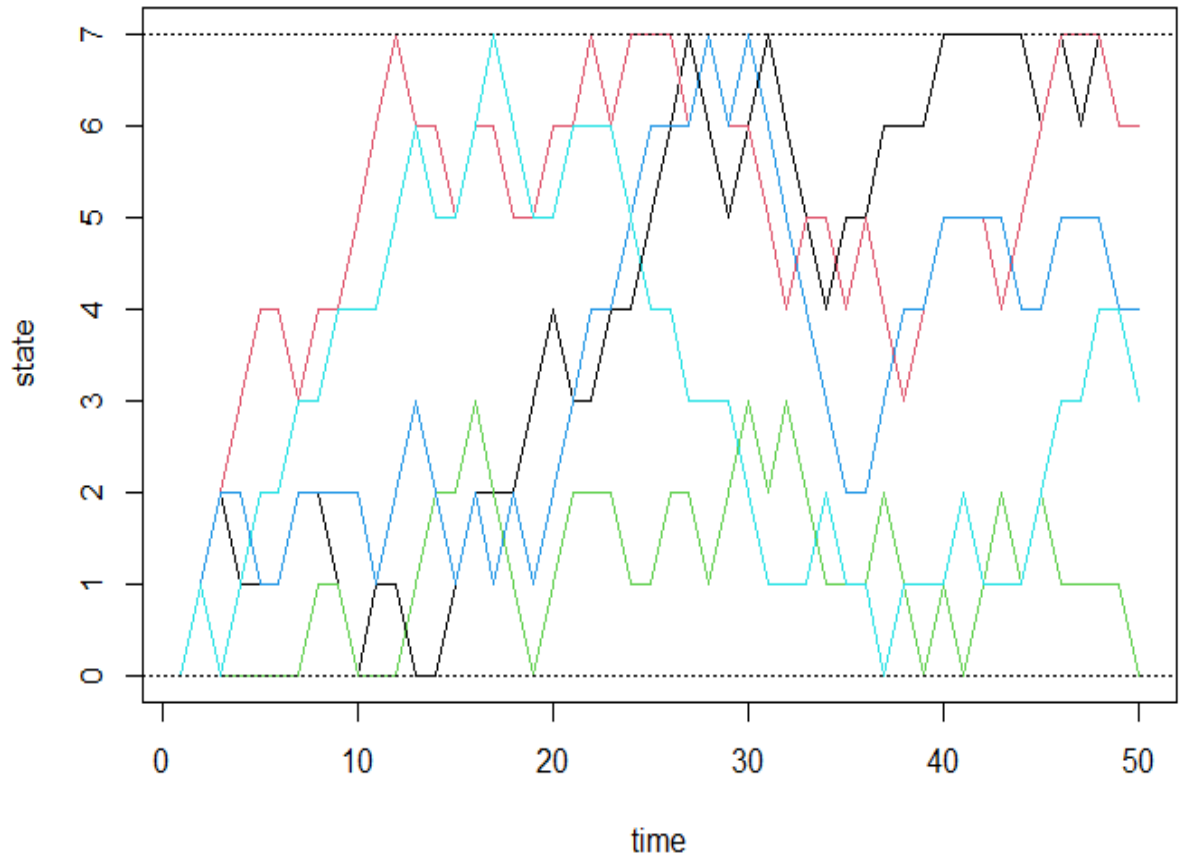
The mean values for the 4 values are:

```
> mean(ten_trials_binom)
[1] 1.81
> mean(fifty_trials_binom)
[1] 9.85
> mean(hundred_trials_binom)
[1] 20.1
> mean(five_hundred_trials_binom)
[1] 100.64
>
```

Answer-2

Given the matrix p denoting the one step probability of the markov chain. We need to simulate the Markov chain with 50 steps for 5 times.

The time to state graph is as follow:



The starting state is the 0th state.

The code is implemented using the two loops. The outer loop runs from 1 → 5 and the inner loop iterates from 2 → 50 where state 0 is set to 1 as it is the starting state. Finally the values are appended in a matrix. Of 50 x 5

For evaluating the the value of P10, P20, P30 we use matrix multiplication operation %*% in R. The following are the results:

```
[1] "P == 10"
           [,1]       [,2]       [,3]       [,4]       [,5]       [,6]       [,7]
[1,] 0.151619841 0.28281597 0.22913174 0.16054463 0.09656387 0.04951820 0.02269302
[2,] 0.141407983 0.26618571 0.22168030 0.16284780 0.10503141 0.05962844 0.03187184
[3,] 0.114565869 0.22168030 0.19990178 0.16616708 0.12591238 0.08738505 0.05962844
[4,] 0.080272316 0.16284780 0.16616708 0.16296635 0.14852072 0.12591238 0.10503141
[5,] 0.048281935 0.10503141 0.12591238 0.14852072 0.16296635 0.16616708 0.16284780
[6,] 0.024759098 0.05962844 0.08738505 0.12591238 0.16616708 0.19990178 0.22168030
[7,] 0.011346509 0.03187184 0.05962844 0.10503141 0.16284780 0.22168030 0.26618571
[8,] 0.007112737 0.02269302 0.04951820 0.09656387 0.16054463 0.22913174 0.28281597
           [,8]
```

```
[1] "P == 20"
           [,1]       [,2]       [,3]      [,4]      [,5]       [,6]       [,7]
[1,] 0.10831537 0.20908002 0.18810782 0.1582822 0.1258427 0.09721392 0.07773415
[2,] 0.10454001 0.20236928 0.18368113 0.1569752 0.1277481 0.10178841 0.08403078
[3,] 0.09405391 0.18368113 0.17123670 0.1531470 0.1329210 0.11456493 0.10178841
[4,] 0.07914112 0.15697524 0.15314697 0.1471824 0.1399638 0.13292098 0.12774808
[5,] 0.06292133 0.12774808 0.13292098 0.1399638 0.1471824 0.15314697 0.15697524
[6,] 0.04860696 0.10178841 0.11456493 0.1329210 0.1531470 0.17123670 0.18368113
[7,] 0.03886708 0.08403078 0.10178841 0.1277481 0.1569752 0.18368113 0.20236928
[8,] 0.03542381 0.07773415 0.09721392 0.1258427 0.1582822 0.18810782 0.20908002
```

```
[1] "P == 50"
           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]       [,8]
[1,] 0.07612503 0.1513198 0.1487134 0.1449471 0.1407669 0.1370008 0.1343947 0.06673227
[2,] 0.07565991 0.1504817 0.1481335 0.1447402 0.1409740 0.1375808 0.1352327 0.06719733
[3,] 0.07435670 0.1481335 0.1465085 0.1441603 0.1415540 0.1392058 0.1375808 0.06850041
[4,] 0.07247355 0.1447402 0.1441603 0.1433224 0.1423922 0.1415540 0.1409740 0.07038346
[5,] 0.07038346 0.1409740 0.1415540 0.1423922 0.1433224 0.1441603 0.1447402 0.07247355
[6,] 0.06850041 0.1375808 0.1392058 0.1415540 0.1441603 0.1465085 0.1481335 0.07435670
[7,] 0.06719733 0.1352327 0.1375808 0.1409740 0.1447402 0.1481335 0.1504817 0.07565991
[8,] 0.06673227 0.1343947 0.1370008 0.1407669 0.1449471 0.1487134 0.1513198 0.07612503
```

Values of PI

```
- -
          [,1]     [,2]      [,3]      [,4]       [,5]      [,6]       [,7]        [,8]
[1,] 0.1516198 0.282816 0.2291317 0.1605446 0.09656387 0.0495182 0.02269302 0.007112737
[1] "P == 20"
          [,1]    [,2]      [,3]      [,4]      [,5]       [,6]       [,7]       [,8]
[1,] 0.1083154 0.20908 0.1881078 0.1582822 0.1258427 0.09721392 0.07773415 0.03542381
[1] "P == 50"
           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]       [,8]
[1,] 0.07612503 0.1513198 0.1487134 0.1449471 0.1407669 0.1370008 0.1343947 0.06673227
```

Explanation:

The Markov chain is irreducible. This is so because all the states are communicating with each other and hence there will be one communicating class. All the states will be aperiodic since there is a self loop. Since the Markov chain is irreducible and aperiodic hence it will attain a limiting distribution. From the values of PI it is visible that the markov chain hasn't attained limiting distribution till the 50th step. But it will definitely attain a limiting distribution.