DOM5 Page No. 5. Hence from the otep 4 we can get the value of xt from egn KTXT=AT by backward substitution C: kt was upper a matrin Now do compute x from  $X^T$  we can again take transpose of  $X^T$  [:  $(X^T)^T = X$ ] and dry computing this we can get the value of Compute value of x in Eq. 1 X K= B cuhere

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## Answer-2:

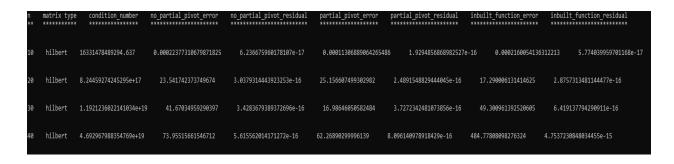
## Output:

n **	matrix ty		no_partial_pivot_error	no_partial_pivot_residual	partial_pivot_error ***************			inbuilt_function_residual
10	random	81.72862839178067	2.011312340136783e-15	2.6259540005203653e-16	2.6380356893428514e-15	1.6761063625462658e-16	1.6161278270545909e-1	9.226661858305929e-17
10	hilbert	16331478489294.637	0.00022377310679871825	6.236675960178107e-17	0.0001130688906426548	6 1.9294856868982527e-16	6 0.00021600541363122	5.774039959701168e-17
10	ones	22.360679774997898	0.0 0.0 0.0	0.0 0.0 0.0				
20	random	1570.6057563437437	3.1720248367765567e-13	1.5082096896723206e-15	7.204440719142018e-15	1.1073623911081181e-16	3.479210282559712e-1	2.4449899576579773e-16
20	hilbert	8.24459274245295e+17	23.541742373749674	3.0379314443923253e-16	25.156607499302982	2.4891548829444045e-16	17.290006131414625	2.8757313481144477e-16
20	ones	63.24555320336759	0.0 0.0 0.0	0.0 0.0 0.0				
30	random	2688.583475675636	1.949957343086635e-13	6.925126826085552e-16	1.0641923489529531e-13	2.2096234092041378e-16	9.67051096822761e-14	2.026504759812942e-16
30	hilbert	1.1921236022141034e+	19 41.67034959290397	3.4283679389372696e-16	16.98646050582484	3.7272342481073856e-16	49.300961392520605	6.419137794290911e-16
30	ones	116.1895003862225	9.0 9.0 9.0	0.0 0.0 0.0				
40	random	26036.945020093895	3.831352 <del>0</del> 93795588e-11	8.121527814798785e-15	8.578874543179429e-13	2.856945251387899e-16	7.463895681088446e-13	2.174732645517965e-16
40	hilbert	4.692967988354769e+1	9 73.95515661546712	5.615562014171272e-16	62.26890299996139	8.096140978918429e-16 48	84.77808098276324 4.7	7537230848034455e-15
40	ones	178.88543819998318	0.0 0.0 0.0	0.0 0.0 0.0				

1. **Hilbert Matrix:** The Value of the conditional number for the Hilbert matrix is large. Since the value is large the Hilbert matrix becomes highly ill conditioned and hence the value of error calculated becomes significantly large. On increasing the value of n from n=10 to 40 the value of conditional number increases and so is the value of the error Order of condition number is 10<sup>18</sup>

In the Hilbert matrix when pivoting is used the value of error is reduced because pivoting makes the algorithm backward stable and hence the error value decreases.

The following are the output of the Hilbert matrix for n = 10,20,30 and 40.



2. **Random Matrix**: For the random matrix the order of conditional number is 10<sup>1</sup> - 10<sup>3</sup> which is comparatively less than that of Hilbert matrix and hence is the magnitude of error for random matrix

For random matrices pivoting decreased the magnitude of error that was being produced by the no pivoting algorithm as the algorithm becomes backward stable.



3. **Ones Matrix:** For the ones matrix the order of magnitude 10<sup>1</sup> which is much less than that of hilbert matrix and random matrix and hence the problem is well conditioned. So the value of error is 0 in all cases. The following is the result for ones matrix for n = 10,20,30 and 40. For ones matrix the algorithm, Gaussian Elimination with partial pivoting and without partial pivoting produces 0 error and hence no difference is observed even if we use pivoting.

```
n matrix type condition number no partial pivot error no partial pivot residual partial pivot residual partial pivot residual inbuilt function error inbuilt function residual

10 ones 22.360679774997898 0.0 0.0 0.0 0.0 0.0 0.0

20 ones 63.24555320336759 0.0 0.0 0.0 0.0 0.0 0.0

30 ones 116.1895003862225 0.0 0.0 0.0 0.0 0.0 0.0

40 ones 178.88543819998318 0.0 0.0 0.0 0.0 0.0 0.0
```

NOTE: The values computed here are relative errors and residuals. Also the conditional number is relative

## Answer-3:

## **OUTPUT:**

```
Question-3
*****
Part-A
[[5.5555556e-01]
 [2.2222222e-01]
 [1.11111111e-10]]
Part-B
Row Equilibrated Matrices
(array([[ 5.e-10, 5.e-10, 1.e+00],
       [ 2.e-09, -1.e-09, 1.e+00],
       [ 5.e-01, 1.e+00, 0.e+00]]), array([[5.e-10],
       [1.e-09],
       [5.e-01]]))
Row Equilibrated and solved using Gaussian elimination without pivoting
[[5.5555556e-01]
 [2.222222e-01]
 [1.11111111e-10]]
```

The results of computation in both ways are the same. The result is the same because we are performing the scaling operations when we are row equilibrating the matrix A.

The scaling transformation is basically multiplying the matrix A with a matrix which has only diagonal entries(diagonal matrix). This is called the scaling matrix. By performing the scaling operation the solution of the equation Ax=b remains the same. Hence the result is the same while calculating using row equilibrated way and gaussian elimination with partial pivoting