UNIFORM DIST $y \in \{0, 1\}$

$0 \times 0.5 + 1 \times 0.5 = 0.5$

$y \in \{1, 2, 3, 4\}$

$1 \times 0.25 + 2 \times 0.25 + 3 \times 0.25 + 4 \times 0.25$

$= 2.5$

$$\frac{1+2+3+4}{4} = 2.5 .$$

Expected value for uniform dist = average.

EXPECTED VALUE OF BERNOULLI DIST

$$1 * \mu + 0 * (1-\mu) = \mu.$$

VARIANCE $\sum_x (x - E_p[x])^2 p(x)$

$$(1-\mu)^2 \cdot \mu + (0-\mu)^2 \cdot (1-\mu)$$

$$= (1+\mu^2 - 2\mu) \cdot \mu + \mu^2 (1-\mu)$$

$$= \mu + \mu^3 - 2\mu^2 + \mu^2 - \mu^3$$

$$= \mu + \cancel{\mu^3} - 2\mu^2 + \mu^2 - \cancel{\mu^3}$$

$$= \mu - \mu^2 = \mu(1-\mu)$$

$$P(H=3) = HHH \rightarrow 1/8 = \binom{3}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$$

$$P(H=2) = \left.\begin{matrix} HHT \\ HTH \\ THH \end{matrix}\right\} \frac{3}{8} = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$$

$$P(H=1) = \left.\begin{matrix} HTT \\ THT \\ TTH \end{matrix}\right\} \frac{3}{8} = \binom{3}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2$$

$$P(H=0) = TTT \longrightarrow \frac{1}{8} = \binom{3}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3$$

BINOMIAL DIST

$$\text{BINOMIAL}(m \mid N, \mu) = \binom{N}{m} \mu^{m} (1-\mu)^{N-m}$$

$$E[m] = \sum_{m=0}^{N} m \;\; Bin(m \mid N, \mu)$$

\# of heads

P(heads = m)

$$= \sum_{m=1}^{N} m \;\; Bin(m \mid N, \mu)$$

$$= \sum_{m=1}^{N} m \cdot \frac{N!}{m!\,(N-m)!} (\mu^{m}) (1-\mu)^{N-m}$$

$$= \sum_{m=1}^{N} \frac{N \cdot (N-1)!}{(m-1)!\,[N-1-(m-1)]!} \cdot \mu \cdot \mu^{m-1} \cdot (1-\mu)^{N-1-(m-1)}$$

$$= \mu N \underbrace{\sum_{j=0}^{M} \binom{M}{j} \cdot \mu^{j} (1-\mu)^{N-j}}_{\text{Sums to 1}}$$

$$= \mu N$$