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* LPP

Graphical Solution Methods

Graphi- Graph is the best tool to visualise any concept.

*→ when to use LPP of Graphical Method

If the number of variables in any Linear programming problem is only two
i.e. LPP involving 2 decision variable can ~~es~~ easily solved by Graphical method.

→ Graphical solution method.

The major steps in the solution of a Linear programming problem by Graphical method are summarised as follows:

step 1: Identify the problem - the decision variables, the objective and the restrictions.

step 2: set up the mathematical formulation of the problem

step 3: P

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step3: Plot a graph representing all the constraints of the problem & identify the feasible region (solution space)

The feasible region is the intersection of all the regions represented by the constraints of the problem and is restricted to the first quadrant only.

Imp.

step4: The feasible region obtained in step 3 may be bounded or unbounded. compute the coordinates of all the corner points of the feasible region

step5: Find out the value of the objective function at each corner (solution) point determined in step 4.

step6: select the corner point that optimizes (maximizes or minimizes) the value of the objective function.

It gives the optimum feasible solution

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Note: Identify the feasible region (or solution space) that satisfies all the constraints simultaneously.

For $\geq$ type constraints, the area on or above the constraint line i.e. away from the origin and

For $\leq$ type constraints the area on or below the constraint line i.e. towards origin will be considered.

The area common to all the constraints is called feasible region and is shown shaded

Any point on or within the shaded region represents a feasible solution to the given problem.
Though a number of infeasible points are eleminated, the feasible region still contains a large number of feasible points.

Feasible Region shown

Maxi $\leftrightarrow$ Inside
mini. $\leftrightarrow$ outside

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Ex ① Solve the following LPP using Graphical method.

maximize $Z = 6X_1 + 8X_2$

subject to -

$5X_1 + 10X_2 \leq 60$

$4X_1 + 4X_2 \leq 40$

X_1 and $X_2 \geq 0$

Soln:- In graphical method, the introduction of the non-negative constraints ($X_1 \geq 0$ and $X_2 \geq 0$) will eliminate the second, third and fourth quadrants of the $X_1 X_2$ plane

Now, we compute the coordinates on the X_1X_2 plane.

→ From the first constraint

→ $5X_1 + 10X_2 = 60$ —— ①

put $X_1 = 0$ then we get

$10X_2 = 60$

$X_2 = 60/10$

$\therefore \boxed{X_2 = 6}$

→ Now put $X_2 = 0$ in eqn ①, we get

$5X_1 = 60$

$\therefore X_1 = 60/5$

$\boxed{X_1 = 12}$

i.e.

x_1	x_2
0	6
12	0

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→ Now, consider the second constraint

$$4x_1 + 4x_2 = 40 \quad \text{--- ②}$$

→ Now put $x_1 = 0$

$\therefore 4x_2 = 40$

$\therefore x_2 = 40/4$

$\therefore \boxed{x_2 = 10}$

→ Now put $x_2 = 0$

$4x_1 = 40$

$\therefore x_1 = 40/4$

$\therefore \boxed{x_1 = 10}$

x_1	x_2
0	10
10	0

→ Now plot 1st & 2nd constraint as shown in figure

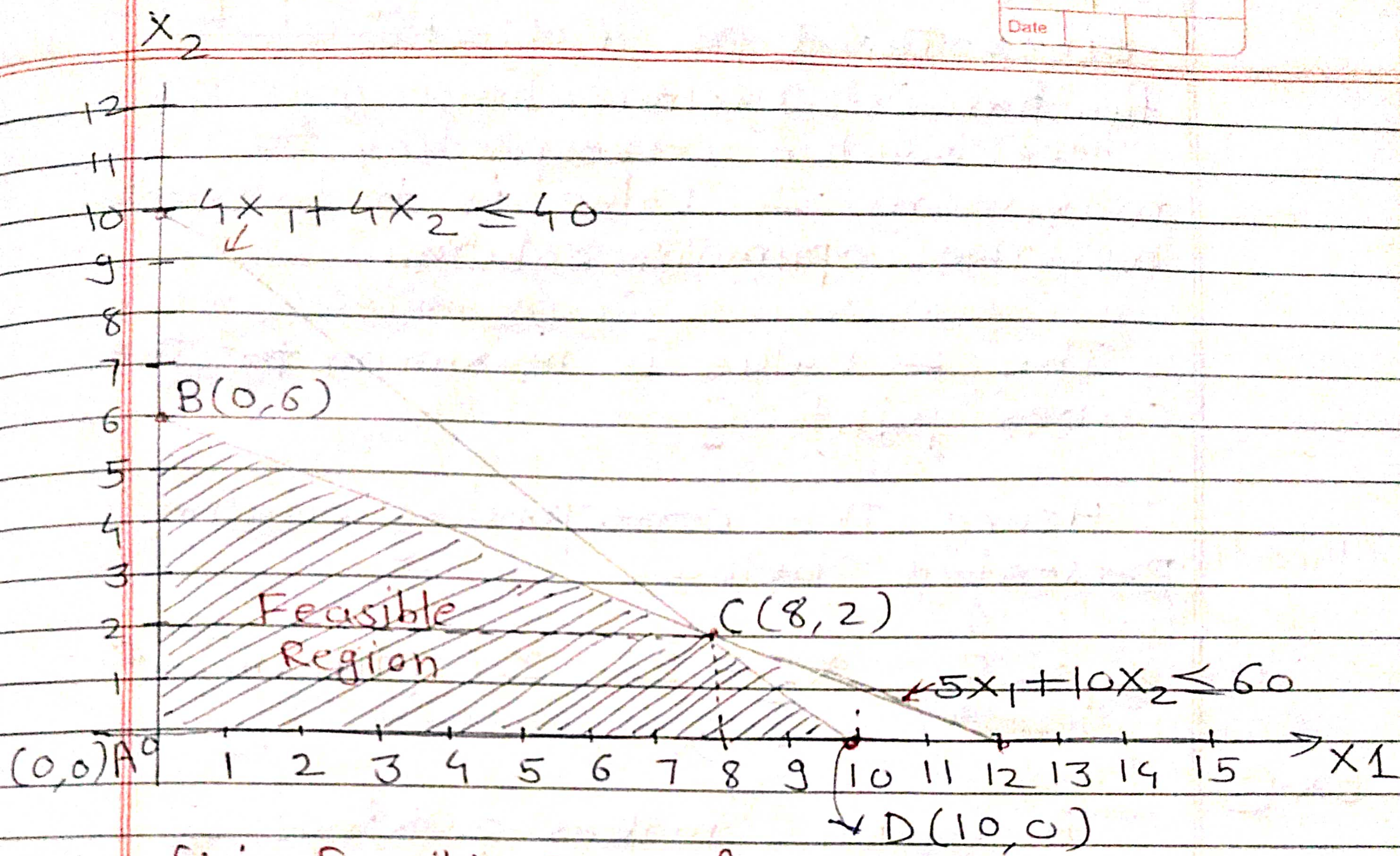

Fig:- Feasible Region of example

The closed polygon A-B-C-D is the feasible region. The objective function value at each of the corner points of the closed polygon is computed by substituting its co-ordinates in the objective function as

$$\text{Maximize } Z = 6X_1 + 8X_2$$

A(0,0) $\quad Z(A) = 6 \times 0 + 8 \times 0 = 0$

B(0,6) $\quad Z(B) = 6 \times 0 + 8 \times 6 = 48$

C(8,2) $\quad Z(C) = 6 \times 8 + 8 \times 2 = 48 + 16 = \underline{\underline{64}}$ max

D(10,0) $\quad Z(D) = 6 \times 10 + 8 \times 0 = 60$

Select MAX

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Since the type of objective function is maximization, the solution corresponding to maximum Z value is to be selected as the optimum solution.

The Z value is maximum for the corner point C

Hence the corresponding solution is presented below

$X_1 = 8$, $X_2 = 2$ $Z(\text{optimum}) = 64$

Ex. 2

A company makes 2 kinds of leather belts. Belt A is a high quality belt & belt B is of lower quality. The respective profits are Rs. 4.00 & Rs. 3.00 per belt. Each belt ~~requires~~ of type A requires twice as much time as a belt of type B & if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (Both A and B combined). Belt A requires a fancy buckle & only 400 buckles per day are available.

There are only 700 buckles a day available for belt B

Determine the optimal product mix.

→ The appropriate mathematical formulation of the given LPP is

Maximize $Z = 4x_1 + 3x_2$

subject to the constraints

$2x_1 + x_2 \leq 1000$ (Time constraint)

$x_1 + x_2 \leq 800$ (Availability of Leather)

$x_1 \leq 400$ & $x_2 \leq 700$ (Availability of buckles

$x_1 \geq 0$ & $x_2 \geq 0$

where x_1 = number of belts of type A

x_2 = number of belts of type B

Soln → Now we compute the co-ordinates on the X_1X_2 plane

→ From the 1st constraint

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Ex:- Solve the following LPP using graphical method.

~~Mini~~ Minimize $Z = 2x_1 + 3x_2$

subject to:- $x_1 + x_2 \geq 6$
$7x_1 + x_2 \geq 14$
x_1 & $x_2 \geq 0$

Soln:- The introduction of the non-negative constraints ($x_1 \geq 0$ and $x_2 \geq 0$) will eliminate the second, third & fourth quadrants of $x_1 x_2$ plane as shown in following figure.

→ Now we compute the co-ordinates to plot on the $X_1 X_2$ plane relating to different constraints.

→ From the 1st constraint

$$x_1 + x_2 = 6 \quad \text{---(1)}$$

put $x_1 = 0$ then we get

$$\boxed{x_2 = 6}$$

→ Now put $x_2 = 0$ in 1st constraint we get

$$\boxed{x_1 = 6}$$

x_1	x_2
0	6
6	0

* Now plot the 1st constraint in fig.

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→ Consider the 2nd constraint

$$7x_1 + x_2 = 14 \quad \text{---} (2)$$

→ Put $x_1 = 0$, then we get

$$\boxed{x_2 = 14}$$

→ Now put $x_2 = 0$ in 2nd constraint, then we get

$$7x_1 = 14$$

$$\therefore x_1 = 14/7$$

$$\therefore \boxed{x_1 = 2}$$

x_1	x_2
0	14
2	0

* Now, Plot the 2nd constraint in fig.

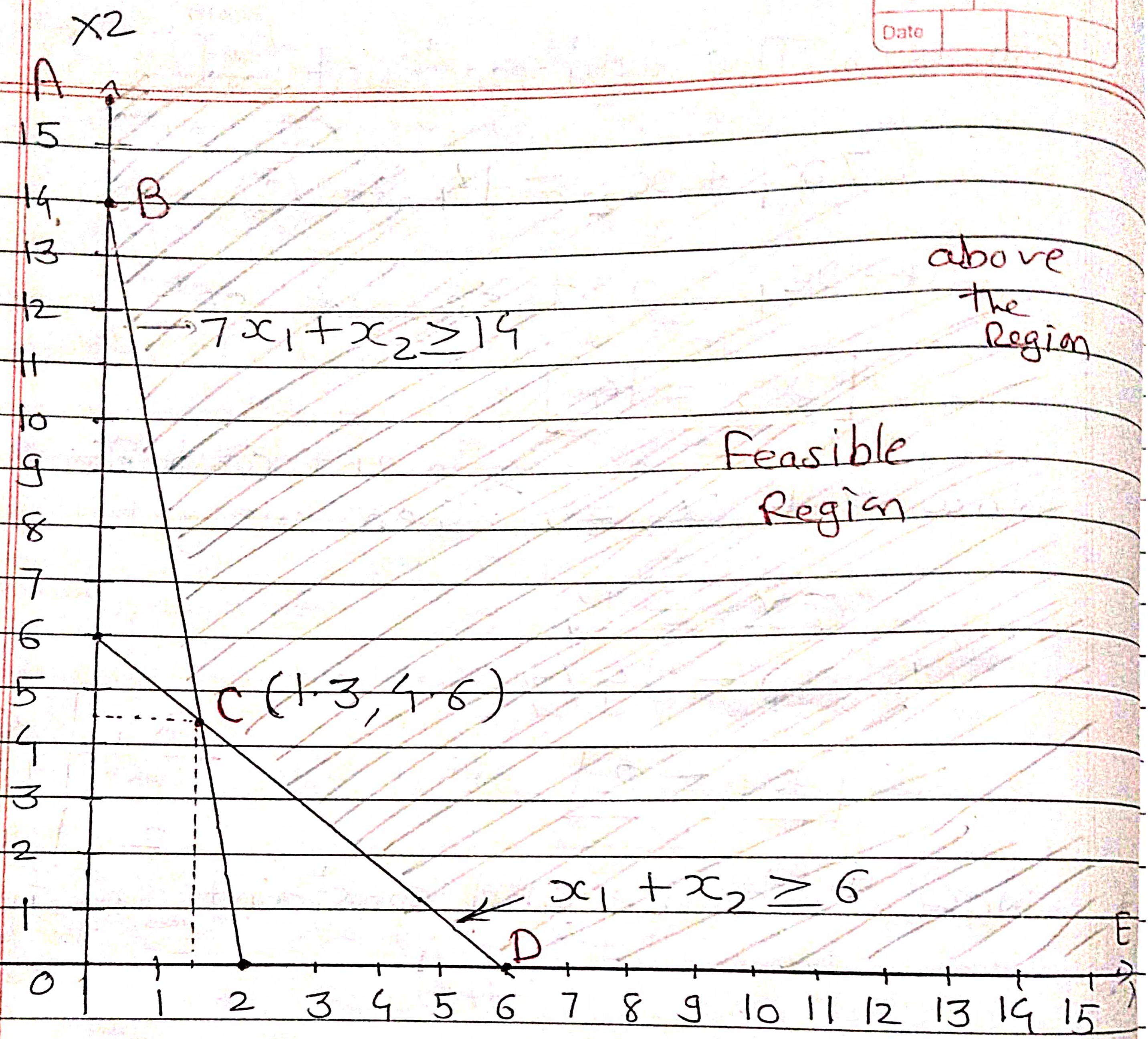

Fig. Feasible region of example

In fig. A-B-C-D-E is the feasible region

→ The optimum solution will be any one of the corner points B - C - D

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→ The objective function value at each of these corner points is computed as follows by substituting its co-ordinates in the objective function.

point A = (0, 0)
point B = (0, 14)
point C = (1.3, 4.6)
point D = (6, 0)
point E = (0, 0)

Here minimize $Z = 2x_1 + 3x_2$

$Z(A) = 2 \times 0 + 3 \times 0 = \underline{0}$

$Z(B) = 2 \times 0 + 3 \times 14 = \underline{42}$

$Z(C) = 2 \times 1.3 + 3 \times 4.6$
$= 2.6 + 13.8$
$= \underline{16.4}$

$Z(D) = 2 \times 6 + 3 \times 0$
$= \underline{12}$ → minimum

$Z(E) = 2 \times 0 + 3 \times 0$
$= 0$

Since, the type of the objective fun is minimization, the solution corresponding to the minimum Z value is to be selected as the optimum solution.

The Z value is minimum for the corner point D

Hence the corresponding optimum solution is -

$\boxed{X_1 = 6}$ & $\boxed{X_2 = 0}$

$$Z\,(\text{optimum}) = 2x_1 + 3x_2$$
$$= 2 \times 6 + 3 \times 0$$
$$= 12 + 0$$
$$= 12$$

$\therefore \boxed{Z\,(\text{optimum}) = 12}$