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Transportation Problem

Introduction :-

Transportation problem is a special kind of linear programming problem in which goods are transported from a set of sources to a set of destinations,

subject to the supply & demand of the source and destination resp.

such that the total cost of transportation is minimized.

* Mathematical Model for Transportation Problem :-

In this section, a linear programming model for the transportation problem is presented.

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \cdot X_{ij}$$

* Generalized Format of the Transportation Problem.

$$
\begin{array}{c|ccccccc|c}
 & \multicolumn{7}{c|}{\text{Destination } (j)} & \\
 & 1 & 2 & \cdots & j & \cdots & n & & \text{supply} \\
\hline
1 & c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} & & a_1 \\
2 & c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{12} & & a_2 \\
\vdots & \vdots & \vdots & & \vdots & & & & \vdots \\
\text{source}(i)\ i & c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{in} & & a_i \\
\vdots & \vdots & \vdots & & \vdots & & & & \vdots \\
m & c_{m1} & c_{m2} & \cdots & c_{mj} & \cdots & c_{mn} & & a_m \\
\hline
\text{Demand} & b_1 & b_2 & \cdots & b_j & \cdots & b_n & &
\end{array}
$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i \,, \quad i = 1, 2, 3, \cdots m$$

&

$$\sum_{i=1}^{m} x_{ij} \geq b_j \,, \quad j = 1, 2, 3 \cdots n$$

where

$$x_{ij} \geq 0, \quad i = 1, 2, 3 \cdots m \ \& \ j = 1, 2, 3 \cdots n$$

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The objective fun minimizes the total cost of transportation (Z) between various sources & destinations.

The constraint i in the first set of constraints ensures that the total units transported from the source i is less than or equal to its supply.

The constraint j in the second set of constraints ensures that the total units transported to the destination j is greater than or equal to its demand.

* Types of Transportation Problem

The transportation problem can be classified in to 2 types

(i) Balanced Transportation Problem
(ii) Unbalanced Transportation Problem

(i) Balanced Transportation Problem:-

If the sum of the supplies of all the sources is equal to the sum of the demands of all the destinations, then the problem is termed as Balanced Transportation Problem

→ This may be represented by the relation:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

eg. Balanced T.P.

		Destination 1	2	3	Supply
Source	1	30	50	15	400
	2	35	22	75	100
	3	50	21	39	100
Demand		300	200	100	600/600

(ii) Unbalanced Transportation Problem

If the sum of the supplies of all the sources is not equal to the sum of the demands of all the destinations, then the problem is termed as unbalanced transportation problem.

→ For any unbalanced transportation problem we have

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

eg:

Destination

		1	2	3	Supply
	1	60	53	100	100
Source	2	50	38	25	600
	3	90	67	30	300
Demand		900	100	200	1200/1000

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Ex:- convert the transportation problem shown in following Table into a balanced transportation problem.

Table

Destination

Source	1	2	3	4	supply
1	5	12	6	10	300
2	7	8	10	3	400
3	9	4	9	2	300
Demand	200	300	450	250	1200/1000

soln:-

we have

$$\sum_{i=1}^{3} a_i = 1000 \quad \& \quad \sum_{j=1}^{4} b_j = 1200$$

since,

$$\sum_{j=1}^{4} b_j > \sum_{i=1}^{3} a_i$$

it is an unbalanced transportation problem. This is converted into a balanced transportation problem by including a dummy source a shown in following Table.

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Table :-

Balanced Transportation problem of example.

Destination

		1	2	3	4	Supply
	1	5	12	6	10	300
	2	7	8	10	3	400
Source	3	9	4	9	2	300
	4	0	0	0	0	200
Demand		200	300	450	250	1200