Page No.
Date

* Canonical and standard form of LPP

Canonical form

MAXI $Z = 3x_1 + 5x_2 + 7x_3$

subject to

$$6x_1 - 4x_2 \le 5$$
$$3x_1 + 2x_2 + 5x_3 \ge 11$$
$$4x_1 + 3x_3 \le 2$$

If there are 2 variables we solve it by Graphical method, but when variable are 3 or more then other methods can use i.e. simplex method.

But for simplex method, the constraints are always in standard form.

Main characteristics of standard form

1. The objective function may be of maximization or minimization type
2. All constraints are expressed as equations (=)
3. The Right hand side of each constraint is non-negative. (Positive)
4. All variables are non-negative.
5. If objective fun is minimization then multiply with (−1) & convert minimization to maximization.
6. If R.H.S. constraint value is −ve then make it positive by multiplying with (−) minus.

canonical form

$$z = 3x_1 + 5x_2 + 7x_3$$

sub. To

$$6x_1 - 4x_2 \leq 5$$
$$3x_1 + 2x_2 + 5x_3 \geq 11$$
$$4x_1 + \quad + 3x_3 \leq 2$$

standard form :- (convert constraint to =)

$$z = 3x_1 + 5x_2 + 7x_3$$

$$6x_1 - 4x_2 + s_1 = 5$$

$$3x_1 + 2x + 5x_3 - s_2 = 11$$

$$4x_1 + \quad 3x_3 + s_3 = 2$$

(After that apply method of simplex method)

* To convert inequality constraints to equality constraints

Two variables are used

called as slack variable &

surplus variablge

1. slack variables

let the constraints of a general L.P.P be

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad ai = 1, 2, \cdots k$$

Then, the non-negative variables x_{n+i} which satisfy

$$\sum_{j=1}^{n} a_{ij} x_j + x_{n+i} = b_i \quad (i=1,2 \cdots k)$$

are called slack variables.

2. surplus variables

let the constrainst of a general L.P.P be

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \qquad i = k+1, k+2, \cdots l$$

Then, the non-negative variables x_{n+i} which satisfy

$$\sum_{j=1}^{n} a_{ij} x_j - x_{n+i} = b_i \quad i = k+1, k+2, \cdots l$$

are called surplus variables.

→ The inequality constraints are changed to equality constraints by adding or subtracting a

non-negative variable from the L.H.S. of such constraints.

These variables are called slack variables or simply slacks.

They are added if the constraints are ($\leq$) called as slack variables.

& subtracted if the constraints are ($\geq$)

since in the case of ($\geq$) constraints the subtracted variable represents the surplus of L.H.S. over R.H.S.

it is called as surplus variable & is infact a negative slack.

Note:- The coefficients of slack and/or surplus variables in the objective function are always assumed to be zero, so that the conversion of the constraints to a system of simultaneous linear equations does not change the objective fun under consideration (The slack & surplus variable donot affect the objective fun anyway) ex:- MAX $Z = 3X + 5y$

sub. to. $X + 3y \le 60$

$3X + 4y \le 120$

$X \ge 10$

$X, y \ge 0$

convert it into standard form

For standard form convert ($\le$ $>$) inequalities into equalities by adding ($=$)

From the ex. since 1st &2nd constraint is $\le$ $\therefore$ we introduce slack variables & add it to L.H.S. of constraint

① $X + 3y + \underbrace{S_1}_{\text{(slack v.)}} = 60$

② $3X + 4y + \underbrace{S_2}_{\text{(slack v.)}} = 120$

③ $X \ge 10$

having $\ge$ sign $\therefore$ introduce surplus variable & substract it

③ $\therefore X - S_3 = 10$

→ surplus variable.

slack & surplus variables are always non-negative.

Page No.
Date

$\therefore$ MAX $Z = 3x + 5y + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

sub to:

$$x + 3y + S_1 = 60$$
$$3x + 4y + S_2 = 120$$
$$x - S_3 = 10$$

$$x, y, S_1, S_2, S_3 \geq 0$$

Ex ② min $Z = 8x + 7y$

sub to:

$$4x + 2y \geq 20$$
$$-6x + 4y \leq 6$$
$$x + y \geq 4$$
$$2x - y = 2$$
$$x, y \geq 0$$

convert it into standard form

Ans:- The standard form is

① min $Z = 8x + 7y \rightarrow$ min $8x + 7y + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

① $4x + 2y \geq 20 \longrightarrow 4x + 2y - S_1 = 20$

② $-6x + 4y \leq 6 \longrightarrow -6x + 4y + S_2 = 6$

③ $x + y \geq 4 \longrightarrow x + y - S_3 = 4$

④ $2x - y = 2 \longrightarrow 2x - y = 2$

↓ This constraint is already in standard form

$\therefore$ No slack or surplus variable is needed

$x, y, S_1, S_2, S_3 \geq 0$

(3) Express the following L.P. problem into standard form

maximize $Z = 7x_1 + 5x_2$

subject to $2x_1 + 3x_2 \leq 20$

$3x_1 + x_2 \geq 10$

$x_1, x_2 \geq 0$

Soln:- Introducing slack & surplus variables, the problem can be expressed in standard form as.

maximize $Z = 7x_1 + 5x_2$

subject to $2x_1 + 3x_2 + S_1 = 20$

$3x_1 + x_2 - S_2 = 10$

$x_1, x_2, S_1, S_2 \geq 0.$