To plot the line $Z = -4x_1 + 3x_2$, we assume $Z = 0$, giving $-4x_1 + 3x_2 = 0$ or $\frac{x_1}{x_2} = \frac{3}{4}$. The corresponding point (3, 4) is obtained, which when joined with origin, represents the dotted line $-4x_1 + 3x_2 = 0$. Lines are then drawn parallel to this line for increasing valueof Z. Clearly, Z can be made large arbitrarily and the problem has no finite maximum value of Z.

The problem, therefore, has an unbounded solution. Value of variable x_1 is limited to 4, while value of variable x_2 can be increased indefinitely.

EXAMPLE 2.10-5

$$\begin{aligned} \textit{Maximize } Z &= -x_1 + 4x_2, \\ \textit{subject to} \quad 3x_1 - x_2 &\geq -3, \\ -0.3x_1 + 1.2x_2 &\leq 3, \\ x_1, x_2 &\geq 0. \end{aligned}$$

Solution

The solution space satisfying the constraints $3x_1 - x_2 \geq -3$, $-0.3x_1 + 1.2x_2 \leq 3$ and meeting the non-negativity restrictions $x_1 \geq 0$, $x_2 \geq 0$ is shown shaded in Fig. 2.18.

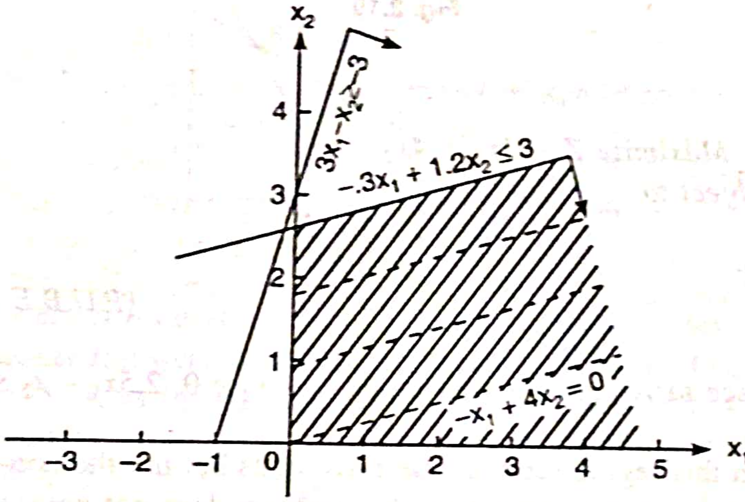

Fig. 2.18

Note that since the first constraint is $3x_1 - x_2 \geq -3$ (R.H.S. is negative), the direction of the arrowhead associated with this line is towards the origin.

For $Z = 0$, the objective function becomes $-x_1 + 4x_2 = 0$, which yields $x_1/x_2 = 4/1$. Thus the dotted line passing through origin O(0, 0) and the point (4, 1) represents $-x_1 + 4x_2 = 0$. The value of Z can be increased by drawing lines parallel to this line and the maximum value is limited by the upper edge of the shaded figure. Thus the optimum value of Z is 10. However, values of variables x_1, x_2 can be made arbitrarily large. Further, any point (x_1, x_2) lying on the upper edge of the region of feasible solutions, which extends to infinity, yields the same optimal value of Z = 10 for the objective function.

EXAMPLE 2.10-6

$$\begin{aligned} \textit{Maximize } Z &= 3x + 2y, \\ \textit{subject to} \quad -2x + 3y &\leq 9, \\ 3x - 2y &\leq -20, \\ x, y &\geq 0. \end{aligned}$$

[P.U.B.E. (Elect.)1996]

Solution

Fig. 2.19 indicates two shaded regions, one satisfying the constraint $-2x + 3y \leq 9$ and the other satisfying the constraint $3x - 2y \leq -20$. These two shaded regions in the first quadrant do not overlap with the result that there is no point (x, y) common to both the shaded regions. The problem cannot be solved graphically (or by any other method of solving L.P. problems) *i.e.*, the feasible solution to the problem does not exist.

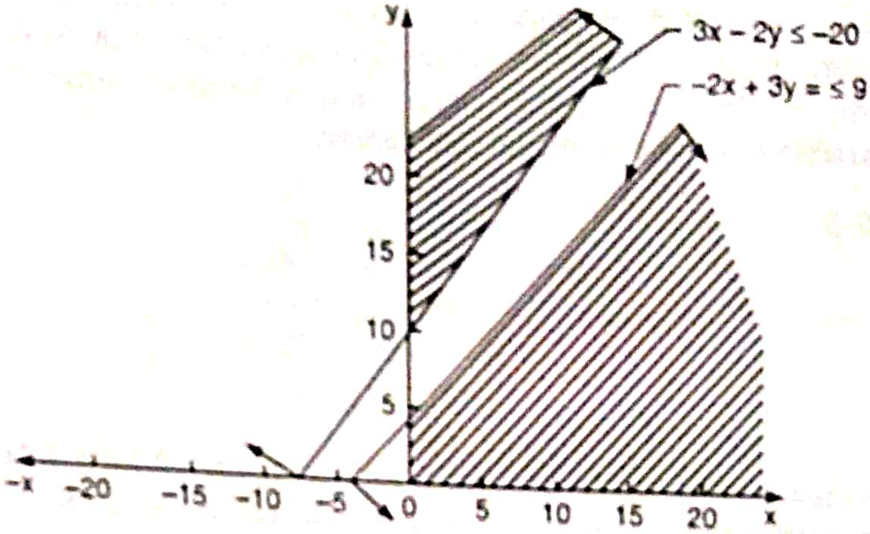

Fig. 2.19

EXAMPLE 2.10-7

Maximize $Z = 3x_1 + 4x_2$

subject to $x_1 - x_2 \geq 0,$

$2.5x_1 - x_2 \leq -3,$

$x_1, x_2 \geq 0.$

Solution [P.U.B.E. (Mech.)., Dec. 1982]

The solution space satisfying the constraints $x_1 - x_2 \geq 0$, $2.5x_1 - x_2 \leq -3$ is shown shaded in Fig. 2.20.

Any point within this region satisfies the constraints but not the non-negativity restrictions. Thus although the constraints are consistent, the problem does not possess a feasible solution.

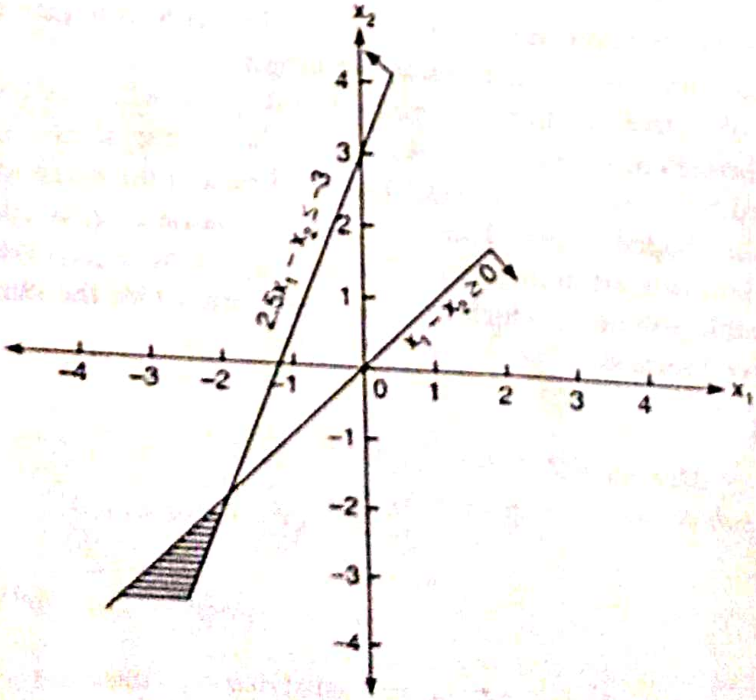

Fig. 2.20

EXAMPLE 2.10-8

$$\text{Maximize } Z = 5x_1 + 8x_2,$$
$$\text{subject to} \quad 3x_1 + 5x_2 = 18,$$
$$5x_1 + 3x_2 = 14,$$
$$x_1, x_2 \geq 0.$$

Solution

Fig. 2.21 shows the graphical solution. The feasible region reduces to the point A (1, 3). Thus the problem has just a single solution

$$x_1 = 1, x_2 = 3, Z = 5 + 24 = 29.$$

As there is nothing to be maximized, such a problem is not of much intrest from point of view of operations research.

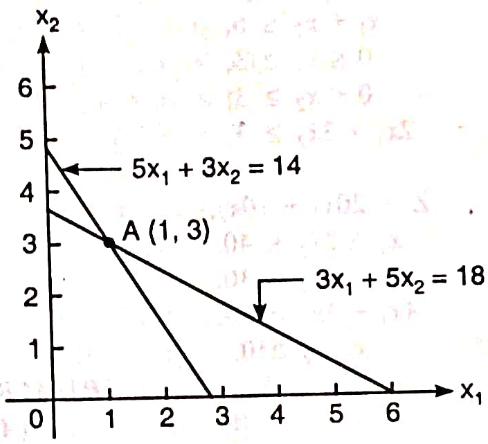

Fig. 2.21

Evidently, there were two variables x_1 and x_2 in the above examples and the problems were, therefore, two-dimensional and were simple to be represented (by the two axes lying in a plane) and solved graphically. Now, as the number of variables increases to 3, 4, ... we come across 3-dimensional, 4-dimensional, ... problems which become quite laborious to be solved by graphical methods. In such cases *simplex technique* helps us in

(*i*) starting with a feasible solution,

(*ii*) searching optimal solution in a systematic way.