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since the type of objective function is maximization, the solution corresponding to maximum z value is to be selected as the optimum solution.

The z value is maximum for the corner point C

Hence the corresponding solution is presented below

$X_1 = 8$, $X_2 = 2$ $\quad z(\text{optimum}) = 64$

Ex. 2

A company makes 2 kinds of leather belts. Belt A is a high quality belt & belt B is of lower quality. The respective profits are Rs. 4.00 & Rs. 3.00 per belt. Each belt ~~requires~~ of type A requires twice as much time as a belt of type B & if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (Both A and B combined). Belt A requires a fancy buckle & only 400 buckles per day are available.

These are only 700 buckles a day available for belt B
Determine the optimal product mix.

→ The appropriate mathematical formulation of the given LPP is

Maximize $Z = 4x_1 + 3x_2$
subject to the constraints

$2x_1 + x_2 \leq 1000$ (Time constraint)
$x_1 + x_2 \leq 800$ (Availability of Leather)
$x_1 \leq 400$ & $x_2 \leq 700$ (Availability of buckles
$x_1 \geq 0$ & $x_2 \geq 0$

where x_1 = number of belts of type A
x_2 = number of belts of type B

soln:
→ Now we compute the co-ordinates on the X_1X_2 plane
→ From the 1st constraint

→ $2x_1 + x_2 = 1000$ — ①
put $x_1 = 0$ then we get,

$\therefore \boxed{X_2 = 1000}$

→ Now put $X_2 = 0$ in eqn ① we get
$2x_1 = 1000$
$x_1 = 1000/2$ $\therefore \boxed{x_1 = 500}$

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From eqn ①

we get $x_1 = 500$

$x_2 = 1000$

Now, Plot the 1st constraint as shown in following figure ①

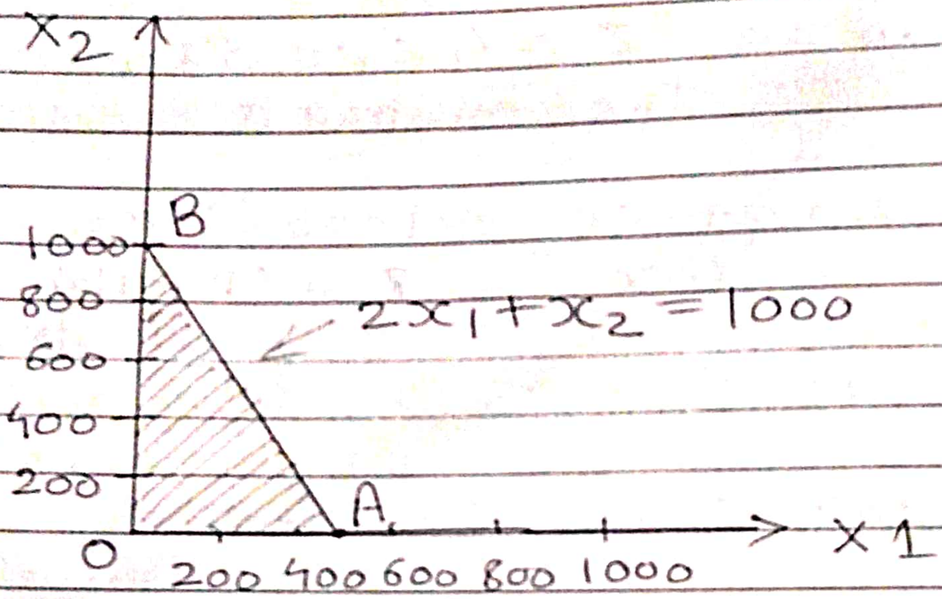

fig ①

→ Now consider 2nd constraint

$$x_1 + x_2 = 800 \quad \text{——} \quad (2)$$

Now put $x_1 = 0$ we get

$$\boxed{x_2 = 800}$$

Now put $x_2 = 0$ we get

$$\boxed{x_1 = 800}$$

Now, Plot the 2nd constraint in fig ①

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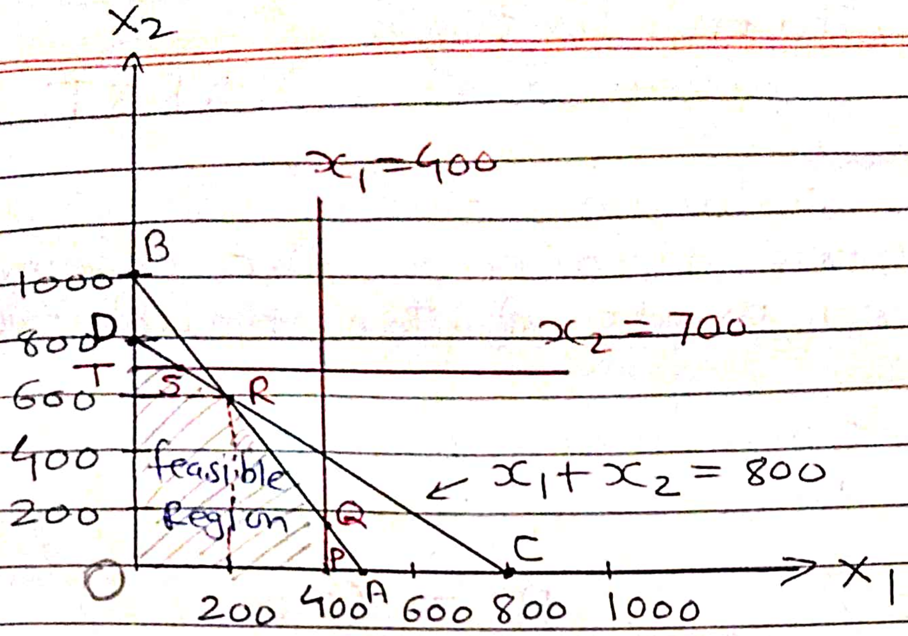

fig. ②

The line CD in fig② represents the equation $x_1 + x_2 = 800$. The region OCD, formed by the two axes and this line represents the area in which any point would satisfy this constraint of Leather ~~activity~~. availability.

→ Further, the constraints $x_1 \leq 400$ and $x_2 \leq 700$ are also plotted on the graph which represents area between the two axes and the lines $x_1 = 400$ & $x_2 = 700$ as shown in fig. ②

→ Now all the constraints have been graphed.

The area bounded by all these constraints, called Feasible Region ~~or~~ or solution space is as shown in fig ② by the shaded area OPQRST

→ The optimum value of objective fun occurs at one of the extreme (corner) points of the feasible region

The co-ordinates of the extreme points are —

O = (0, 0) , R = (200, 600)
P = (400, 0) , S = (100, 700)
Q = (400, 200) , T = (0, 700)

→ Now, we compute the Z-value corresponding to the extreme points

Extreme Points	(x_1, x_2)	$Z = 4x_1 + 3x_2$
O	(0, 0)	0
P	(400, 0)	$4 \times 400 + 0 = 1600$
Q	(400, 200)	$4 \times 400 + 3 \times 200 = 2200$
R	(200, 600)	$4 \times 200 + 3 \times 600 = 2600$ ← Max
S	(100, 700)	$4 \times 100 + 3 \times 700 = 2500$
T	(0, 700)	$4 \times 0 + 3 \times 700 = 2100$

The optimum solution is that extreme point for which the objective fun has the largest value.

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Thus, the optimum solution occurs at the point R.
i.e. $x_1 = 200$ & $x_2 = 600$

with the objective function value of Rs. 2600.

→ Hence, to maximize profit the company should produce 200 belts of Type A & 600 belts of type B per day.