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Ex① Consider the following LP model & solve it using Big M method.

Minimize $z = 2x_1 + 3x_2$

sub to.

$$x_1 + x_2 \geq 6$$
$$7x_1 + x_2 \geq 14$$
$$x_1 \text{ and } x_2 \geq 0$$

soln:-

step1 check the R.H.S. of L.P.P if it is Non -ve then ok.

step2. Remove inequality of constraints $\leq$, $\geq$ into equality $=$ for standard form

$\leq$ L.H.S + slack = R.H.S.

$\geq$ L.H.S. − surplus + Artificial = R.H.S.

OR $\geq$ L.H.S. + Artificial = R.H.S + surplus

MAX : −M
min : +M

coefficient of Artificial variable.

The standard form of the given problem is

minimize $z = 2x_1 + 3x_2 + 0 \cdot S_1 + 0 \cdot S_2 + MA_1 + MA_2$
subject to

$$x_1 + x_2 - S_1 + A_1 = 6$$
$$7x_1 + x_2 - S_2 + A_2 = 14$$

$$x_1, x_2, S_1, S_2, A_1 \,\&\, A_2 \geq 0$$

where S_1 and S_2 are called surplus variable which are introduced to balance the constraints.
Here A_1 & A_2 are artificial variables

optimality condition:-
For minimization problem, if all $c_j - z_j$ are greater than or equal to zero, then optimality reached, otherwise select the entering variable with the most negative value.

→ Now prepare initial basic feasible table.

For maximize select most +ve value

CB_i	C_j	2	3	0	0	M	M	Solution	min Ratio
	B.V.	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
M	A_1	1	1	-1	0	1	0	6	6/1=
M	A_2	$\boxed{7}$	1	0	-1	0	1	14	14/7=2 key Row
	$Z_j = \Sigma CB_i \cdot a_{ij}$	8M	2M	-M	-M	M	M	20M	
	$C_j - Z_j$	2-8M ↑ * most -ve key column	3-2M	M	M	0	0		

calculation of $Z_j = \Sigma CB_i \cdot a_{ij}$

$x_1 = 1 \cdot M + 7 \cdot M = 8M$

$x_2 = 1 \cdot M + 1 \cdot M = 2M$

$S_1 = M \times (-1) + M \times 0 = -M$

$S_2 = M \times 0 + M \times (-1) = -M$

$A_1 = M \times 1 + M \times 0 = M$

$A_2 = M \times 0 + M \times 1 = M$

$soln = M \times 6 + 14 \times M = 20M$

In the above table optimality condition for minimization problem for $C_j - Z_j$ are not greater than or equal to zero.

Hence optimality cannot reach Therefore the solution can be improved further since $c_j - z_j$ has the maximum negative value for the variable x_1 $\therefore$ x_1 is a Entering variable & such column is called as key column

Imp If $c_j - z_j$ is a function of M, then ignore the constant numeric terms in it while making comparison with another $c_j - z_j$

The ratio is the minimum for the second row A_2, Hence it is called as Key Row & A_2 is a Leaving variable.

The Intersection of key column & key Row is called pivot element i.e. Here is 7

$\therefore$ In next Iteration A_2 is replaced by x_1

Iteration 1

	c_j	2	3	0	0	m	m	soln	Ratio
CB_i	B.V.	x_1	x_2	S_1	S_2	A_1	A_2		
m	A_1	0	$\boxed{6/7}$	-1	$1/7$	1	$-1/7$	4	$4/6/7 = 4 \times \frac{7}{6}$ $\frac{28}{6} = \frac{14}{3}$ ← KR
2	x_1	1	$1/7$	0	$-1/7$	0	$1/7$	2	$\frac{2}{1/7} = 2 \times 7 = 14$
$z_j = CB_i \cdot a_{ij}$		2	$\frac{6}{7}m + \frac{2}{7}$	$-m$	$\frac{m}{7} - \frac{2}{7}$	$\boxed{m}$	$-\frac{m}{7} + \frac{2}{7}$	$4m + 4$	
$c_j - z_j$		0	$-\frac{6}{7}m + \frac{19}{7}$ ↑* key colum	m	$-\frac{m}{7} + \frac{2}{7}$	0	$\frac{8}{7}m - \frac{2}{7}$		

or $\frac{19}{7} - \frac{6}{7}m$ or $\frac{2}{7} - \frac{m}{7}$

$3 - \left(\frac{6}{7}m + \frac{2}{7}\right)$

$-\frac{6}{7}m + \frac{21 - 2}{7} = \frac{19}{7}$

$m - \left(-\frac{m}{7} + \frac{2}{7}\right)$

$\frac{7m + m}{7} \quad \frac{2}{7}$ $8m$

① $A_2 \to A_2/7$
$R_2 \to R_2/7$

OR. $A_{2(new)} \to A_{2(old)}/7$
$R_{2(New)} \to R_{2(old)}/7$

① $x_1 \to 7/7 = 1$

② $x_2 \to 1/7 = 1/7$

③ $S_1 \to 0/7 = 0$

④ $S_2 \to -1/7 = -1/7$

⑤ $A_1 \to 0/7 = 0$

⑥ $A_2 \to 1/7 = 1/7$

⑦ Soln or b $\to 14/7 = 2$

② $A_1 \to A_1 - A_2$
$R_1 \to R_1 - R_2$

OR

$R_{1(new)} \to R_{1(old)} - R_{2(New)}$

① $x_1 \to 1 - 1 = 0$

② $x_2 \to 1 - \frac{1}{7} = \frac{7-1}{7}$
$= \boxed{6/7}$

③ $S_1 \to -1 - 0 = -1$

④ $S_2 \to 0 - (-\frac{1}{7}) = 1/7$

⑤ $A_1 \to 1 - 0 = 1$

⑥ $A_2 \to 0 - (1/7) = -1/7$

⑦ Soln or b $\to 6 - 2 = 4$

Calculation of $Z_j = CB_i \cdot a_{ij}$

① $x_1 = M \times 0 + 2 \times 1 = 2$

② $x_2 = M \times \frac{6}{7} + 2 \times (\frac{1}{7}) = \frac{6}{7}M + \frac{2}{7}$

③ $S_1 = M \times (-1) + 2 \times 0 = -M$

④ $S_2 = M \times (\frac{1}{7}) + 2 \times (\frac{-1}{7}) = \frac{M}{7} - \frac{2}{7}$

⑤ $A_1 = M \times 1 + 2 \times 0 = M$

⑥ $A_2 = M \times (\frac{-1}{7}) + 2 \times \frac{1}{7} = -\frac{M}{7} + \frac{2}{7}$

⑦ Soln b $= M \times 4 + 2 \times 2 = 4M + 4$

In the above table, all the values of $c_j - z_j$ are not greater than or equal to zero, Hence optimality can't be reached

since, $c_j - z_j$ has the most negative value for the variable x_2. This is known as entering variable & corresponding column is called key column

The minimum ratio is getting in Row A_1 Hence, it is called key Row & pivot element is $6/7$ & the next iteration is

Iteration 2

C_{B_i}	C_j	2	3	0	0	M	M	soln	Ratio
	B.V.	x_1	x_2	S_1	S_2	A_1	A_2	b	
3	x_2	0	1	$-7/6$	$\boxed{1/6}$	$7/6$	$-1/6$	$14/3$	$\frac{14}{3} \times \frac{6}{1} = 28$ Key Row
2	x_1	1	0	$1/6$	$-1/6$	$-1/6$	$1/6$	$4/3$	—
	z_j	2	3	$-19/6$	$1/6$	$19/6$	$-1/6$	$50/3$	
$c_j - z_j$		0	0	$19/6$	$-1/6$	$M-19/6$	$M+1/6$		
					key col.				

(−ve Ratio is neglected No need to calculate it)

① $R_1 \to \frac{7}{6} R_1$ ② $R_2 \to R_2 - \frac{1}{7} R_1$

OR

or $R_{1\,(new)} \to \frac{7}{6} R_{1\,(old)}$ $R_{2\,(New)} \to R_{2\,(\cancel{old})\,new} - \frac{1}{7} R_{1\,(old)}$

$R_{2\,(New)} \to R_{2\,(old)} - \frac{1}{7} R_{1\,(New)}$

① $x_1 \to 7/6 \times 0 = 0$ ① $x_1 \to 1 - \frac{1}{7}(0) = 1$

② $x_2 \to 7/6 \times 6/7 = 1$ ② $x_2 \to \frac{1}{7} - \frac{1}{7} \times 1 = 0$

③ $s_1 \to 7/6 \times -1 = -7/6$ ③ $s_1 \to 0 - \frac{1}{7} \times \left(\frac{-7}{6}\right) = \frac{1}{6}$

④ $s_2 \to 7/6 \times \frac{1}{7} = 1/6$ ④ $s_2 \to \frac{-1}{7} - \left(\frac{1}{7} \times \frac{1}{6}\right) = -1/6$

⑤ $A_1 \to 7/6 \times 1 = 7/6$ ⑤ $A_1 \to 0 - \frac{1}{7} \times \frac{7}{6} = \frac{-1}{6}$

⑥ $A_2 \to 7/6 \times \frac{-1}{7} = -1/6$ ⑥ $A_2 \to +\frac{1}{7} - \left[\frac{1}{7} \times \left(\frac{-1}{6}\right)\right] = \frac{1}{6}$

⑦ soln $\to 7/6 \times 4 = \frac{28}{6} = \frac{14}{3}$ ⑦ soln $\to 2 - \frac{1}{7} \times \frac{14}{3} = \frac{4}{3}$

$\frac{6-2}{3} = \frac{4}{3}$

Calculation of $z_j = CB_i \cdot a_{ij}$

① $x_1 \to 3 \times 0 + 2 \times 1 = 2$

② $x_2 \to 3 \times 1 + 2 \times 0 = 3$

③ $s_1 \to 3 \times \frac{-7}{6} + 2 \times \frac{1}{6} = -19/6$

④ $s_2 \to 3 \times \frac{1}{6} + 2 \times \left(\frac{-1}{6}\right) = 1/6$

⑤ $A_1 \to 3 \times 7/6 + 2 \times \left(\frac{-1}{6}\right) = 19/6$

⑥ $A_2 \to 3 \times \left(\frac{-1}{6}\right) + 2 \times \left(\frac{1}{6}\right) = -1/6$

⑦ soln ~~3×28+2×~~ $= 3 \times \frac{14}{3} + 2 \times \frac{4}{3} = \frac{42}{3} + \frac{8}{3} = \frac{50}{3}$

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In the above table, all the values of $C_j - Z_j$ are not greater than or equal to zero, hence optimality cannot be reached since $C_j - Z_j$ has the most −ve value for variable S_2 & such column is called key column & the minimum ratio is in Row X_2 ∴ In next iteration X_2 is replaced by S_2 & next Iteration is

Iteration 3.

C_{B_i}	C_j	2	3	0	0	M	M	soln
	B.V	x_1	x_2	S_1	S_2	A_1	A_2	b
0	S_2	0	6	−7	1	7	−1	28
2	X_1	1	1	−1	0	1	0	6
Z_j		2	2	−2	0	2	0	12
$C_j - Z_j$		0	1	2	0	M−2	M	

In the above table all the values of $C_j - Z_j \geq 0$ Hence optimality is reached & the corresponding ~~column~~ & solution is

$x_1 = 6 \quad x_2 = 0$

$Z(\text{optimum}) = 12$

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① $R_1 \rightarrow 6R_1$

$R_{1(new)} \rightarrow 6R_{1(old)}$

② $R_2 \rightarrow R_2 + \frac{1}{6}R_1$

$R_{2(new)} \rightarrow R_{2(old)} + \frac{1}{6}R_{1(new)}$

① $x_1 \rightarrow 6 \times 0 = 0$

② $x_2 \rightarrow 6 \times 1 = 6$

③ $S_1 \rightarrow 6 \times \frac{-7}{6} = -7$

④ $S_2 \rightarrow 6 \times \frac{1}{6} = 1$

⑤ $A_1 \rightarrow 6 \times \frac{7}{6} = 7$

⑥ $A_2 \rightarrow 6 \times \frac{-1}{6} = -1$

⑦ soln $\rightarrow \cancel{6}^{2} \times \frac{14}{\cancel{3}} = 28$

① $x_1 \rightarrow 1 + \frac{1}{6}(0) = 1$

② $x_2 \rightarrow 0 + \frac{1}{6} \times 6 = 1$

③ $S_1 \rightarrow \frac{1}{6} + \frac{1}{6}(-7) = \cancel{-1}$

$\frac{1}{6} - \frac{7}{6} = \frac{-6}{6} = -1$

④ $S_2 \rightarrow \frac{-1}{6} + \frac{1}{6}(1) = 0$

⑤ $\cancel{S}A_1 \rightarrow -1/6 + \frac{1}{6} \times 7 = \frac{-1}{6} + \frac{7}{6}$

$= \frac{+6}{6} = +1$

⑥ $A_2 \rightarrow \frac{1}{6} + \frac{1}{6}(-1) = 0$

⑦ soln $\rightarrow \frac{4}{3} + \frac{1}{\cancel{6}_3} \times \cancel{28}\ 14$

$\frac{4+14}{3} = \frac{18}{3} = 6$

Z calculation
$C_{B_i} \cdot a_{ij}$

① $x_1 \rightarrow 0 \times 0 + 2 \times 1 = 2$

② $x_2 \rightarrow 0 \times 6 + 2 \times 1 = 2$

③ $S_1 \rightarrow 0 \times (-7) + 2 \times (-1) = -2$

④ $S_2 \rightarrow 0 \times (1) + 2 \times (0) = 0$

⑤ $A_1 \rightarrow 0 \times 7 + 2 \times 1 = 2$

⑥ $A_2 \rightarrow 0 \times (-1) + 2 \times 0 = 0$

⑦ soln $\rightarrow 0 \times 28 + 2 \times 6 = 12$

Ex(2) Solve the following LPP using Big M method.

minimize $Z = 10x_1 + 15x_2 + 20x_3$

subject to

$$2x_1 + 4x_2 + 6x_3 \geq 24$$

$$3x_1 + 9x_2 + 6x_3 \geq 30$$

$$x_1, x_2, x_3 \geq 0$$

Soln:- The standard form of the above problem is shown below.

In this form S_1, S_2 are called as surplus variables which are introduced to balance the constraints.