Page No.
Date

Linear programming method.

Simplex Method.

The graphical method cannot be applied when the number of variables involved in the L. P. Problem is more than two,

since even with 3 variables the graphical method of solution becomes tedious as it involves intersection of planes in 3 dimensions.

The simplex method developed by Prof. George B. Dantzig can be used to solve any L. P. Problem (for which the solution exists) involving any number of variables and constraints (100 or even 1000s)

Simplex method is the basic building block for all the other methods.

This method is devised based on the concept of solving simultaneous equations.

It is demonstrated using a suitable numerical problem.

Ex:- Consider the linear programming model & solve it using simplex method.

maximize $Z = 6X_1 + 8X_2$

subject to :

$$5X_1 + 10X_2 \le 60$$
$$4X_1 + 4X_2 \le 40$$
$$X_1 \text{ and } X_2 \ge 0$$

soln:- The given problem is said to be expressed in standard form

i.e. 1) if Decision variables are non - negative

2) R.H.S. of the constraints are non-negative

3) & constraints are expressed in equations

∵ First 2 conditions met with in the problem

Now Non-negative slack variable S1 & S2 are added to the L.H.S. of the 1st & 2nd constraints resp. to convert them into equations (slack variables are introduced to balance the constraints)

→ The standard form of given L.P.P is

Maximize $Z = 6X_1 + 8X_2 + 0S_1 + 0S_2$

Subject to

$$5X_1 + 10X_2 + S_1 = 60$$
$$4X_1 + 4X_2 + S_2 = 40$$
$$X_1, X_2 \; S_1 \text{ and } S_2 \ge 0.$$

Definition of Basic variable

A variable is said to be a basic variable if it has unit coefficient in one of the constraints & zero coefficient in the remaining constraints

If all the constraints are '≤' type then, the standard form is to be treated as the Canonical form

⇒ The s

Now prepare initial simplex table

⇒ The initial simplex table of the above problem is given below.

Initial simple Table

CB_j	C_j	6	8	0	0		θ
	Basic variable	X_1	X_2	S_1	S_2	Solution or b	Ratio
0	S_1	5	$\boxed{10}$	1	0	60	60/10=6 **
0	S_2	4	4	0	1	40	40/4=10
	$Z_j = \Sigma CB_j \cdot a_{ij}$	0	0	0	0	0	
	$C_j - Z_j$	6	8	0	0		
			MAX * Keycolumn				

* Key column MAX

** Key Row MIN ratio element

Key element Pivot element

Here c_j is the coefficient of the objective function &

CB_i is the coefficient of Basic variable.

The value at the intersection of the key row & the key column is called key element or pivot element

The value of Z_j is computed by using the formula.

$$\boxed{Z_j = \sum_{i=1}^{2} (CB_i) \cdot (a_{ij})}$$

where a_{ij} is the technological coefficient for the ith row & jth column of the table.

$c_j - z_j \longrightarrow$ is the relative contribution. In this c_j is the objective fun coefficient for the jth variable.

The value of z_j against the solution column is the value of the objective fun & in this iteration it is zero.

Optimality Condition

For maximization problem, if all $C_j - Z_j$ are less than or equal to zero, then optimality is reached,

otherwise select the variable with maximum $C_j - Z_j$ value as the entering variable.

& (For minimization problem, if all $C_j - Z_j$ are greater than or equal to zero, the optimality is reached. otherwise select the variable with the most -ve value as the entering variable)

In the above table all the values of $C_j - Z_j$ are not less than or equal to zero, Hence the solution can be improve further

$C_j - Z_j$ is maximum for the variable X_2

So X_2 enters the basis. This is known as entering variable & corresponding column is called Key column.

& In the above table the leaving variable is S_1 & the row 1 is the key row

And the Pivot element is 10.

The next iteration is as follows.

* In the next table, the basic variable S1 of the previous table is replaced by X_2

Imp. → In the new table make key element i.e. pivot element as 1

& the corresponding value of pivot element as zero

⇒ ① $R_1 \rightarrow R_1 / 10$

or $R_{1(new)} \rightarrow R_{1(old)}$ / Pivot element

① $x_1 = 5/10 = 1/2$
② x_2 $10/10 = 1$
③ S_1 $1/10 = 1/10$
④ S_2 $0/10 = 0$
⑤ b $60/10 = 6$

② $R_2 \rightarrow R_2 - 4R_1$

or $R_{2(new)} \rightarrow R_{2(old)} - 4R_{1(new)}$

	C_j	6	8	0	0		
CB_i	Basic variable	x_1	x_2	s_1	s_2	b Soln	Ratio
8	x_2	1/2	1	1/10	0	6	$6/(1/2)=12$
0	s_2	$\boxed{2}$	0	$-2/5$	1	16	$16/2=8$ **
	$Z_j = \Sigma CB_i \cdot a_{ij}$	4	8	4/5	0	48	
	$C_j - Z_j$	2 *	0	$-4/5$	0		

Key column

(2) $R_2 \rightarrow R_2 - 4R_1$

or

$R_{2(new)} \rightarrow R_{2(old)} - 4R_{1(new)}$

(1) $x_1 \rightarrow \quad 4 - 4\left(\frac{1}{2}\right) = 4 - 2 = 2$

(2) $x_2 \rightarrow \quad 4 - 4(1) = 0$

(3) $s_1 \rightarrow \quad 0 - 4\left(\frac{1}{10}\right) = -\frac{4}{10} = -\frac{2}{5}$

(4) $s_2 \rightarrow \quad 1 - 4(0) = 1$

(5) $b \rightarrow \quad 40 - 4(6) = 40 - 24 = 16$

calculation of z_j

$x_1 = 8 \times \frac{1}{2} + 0 \times 2 = \frac{8}{2} + 0 = 4$

$x_2 = 8 \times 1 + 0 \times 0 = 8$

$s_1 = 8 \times \frac{1}{10} + 0 \times -\frac{2}{5} = \frac{8}{10} = \frac{4}{5}$

$s_2 = 8 \times 0 + 0 \times 1 = 0$

b or soln $\Rightarrow$ $8 \times 6 + 0 \times 16 = 48$

→ Now 1st find key column by selecting maximum ~~element~~ +ve element from $C_j - Z_j$ i.e we get 2 $\therefore$ x1 is key column

→ & then find key row by dividing the solution value or b by respective key column element and assign it as Ratio

& select the minimum ratio value & select that row as a key row

The minimum ratio is 8 $\therefore$ s2 is key row

The intersection of key column & key row is 2 $\therefore$ 2 is a pivot ele.

In the above table,
all the values for $C_j - Z_j$ are
not less than or equal to zero
Hence optimality cannot be reach
$\therefore$ perform next Iteration

→ select the variable with maximum $C_j - Z_j$ value as the entering variable

In the above table the $C_j - Z_j$ maximum value is for variable X_1

Hence X_1 is called entering variable
& The key row element S_2 is called Leaving variable.

$\therefore$ In next iteration S_2 is replaced by X_1 & the next iteration is

	C_j	6	8	0	0	b
CB_i	Basic variable	x_1	x_2	S_1	S_2	Solution
8	x_2	0	1	1/5	-1/4	2
6	x_1	1	0	$-\frac{1}{5}$	$\frac{1}{2}$	8
	$Z_j = \Sigma CB_i \cdot a_{ij}$	6	8	2/5	1	64
	$C_j - Z_j$	0	0	-2/5	-1	

① $R_2 \rightarrow R_2 / 2$

or $R_2(\text{new}) \rightarrow \dfrac{R_2(\text{old})}{\text{Pivot element}}$

② $R_1 \rightarrow R_1 - \frac{1}{2} R_2$

or $R_1(\text{new}) \rightarrow R_1(\text{old}) - \frac{1}{2} R_2(\text{new})$

① $x_1 \rightarrow 2/2 = 1$
② $x_2 \rightarrow 0/2 = 0$
③ $S_1 \rightarrow -2/5/2 = -1/5$
④ $S_2 \rightarrow 1/2 = 1/2$
⑤ b or soln $\rightarrow 16/2 = 8$

① $x_1 = \frac{1}{2} - \frac{1}{2}(1) = 0$
② $x_2 = 1 - \frac{1}{2}(0) = 1$
③ $S_1 = 1/10 - \frac{1}{2}\left(-\frac{1}{5}\right) = +\frac{1}{5}$
④ $S_2 = 0 - 1/2\left(\frac{1}{2}\right) = -1/4$
⑤ $b = 6 - 1/2(8)$
$6 - \frac{8\,4}{\cancel{2}} = 2$

Now Calculate Z_j

$x_1 \rightarrow 8 \times 0 + 6 \times 1 = 6$

$x_2 \rightarrow 8 \times 1 + 6 \times 0 = 8$

$S_1 \rightarrow 8 \times \frac{1}{5} + 6 \times \left(-\frac{1}{5}\right) = \frac{8}{5} - \frac{6}{5} = \frac{2}{5}$

$S_2 \rightarrow 8 \times \left(-\frac{1}{4}\right) + 6 \times \left(\frac{1}{2}\right) = -\frac{8}{4} + \frac{6}{2} = -2 + 3 = 1$

$b \rightarrow 8 \times 2 + 6 \times 8 = 16 + 48 = 64$

Page No.
Date

In the above table all the $c_j - z_j$ values are less than or equal to zero. Hence optimality is Reached & the corresponding optimal solution is

$x_1 = 8$ units $\quad x_2 = 2$ units

& Z (optimum) $= 64$

Ex. (2) Solve following example by simplex method.

Maximize $Z = 5x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

Soln: (1) Convert to standard form

Introduce the slack variable

S_1 S_2 & S_3

Maximize $Z = 5x_1 + 3x_2 + 0.S_1 + 0.S_2 + 0.S_3$

Subject to

$x_1 + x_2 + S_1 + 0.S_2 + 0.S_3 = 2$

$5x_1 + 2x_2 + 0.S_1 + S_2 + 0.S_3 = 10$

$3x_1 + 8x_2 + 0.S_1 + 0.S_2 + S_3 = 12$

x_1, x_2, x_3, S_1, S_2 & $S_3 \geq 0$

(2) Now

prepare initial simplex table.

of the above problem