Minimization problem

① solve the following problem by simplex method.

mini $z = x_1 - 3x_2 + 2x_3$

sub. to

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

soln:- ① By introducing slack variables S_1, S_2, S_3 convert the problem in standard form

[In graphical method inequality directly convert into equality without slack variable but for simplex method we need slack variable.]

② Convert Min → MAX by multiplying objective fun with (−1)

$\therefore$ MAX $z' = -x_1 + 3x_2 - 2x_3 + 0.S_1 + 0S_2 + 0.S_3$

sub to. $3x_1 - x_2 + 2x_3 + S_1 = 7$

$$-2x_1 + 4x_2 + S_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$$

$$x_1\ x_2\ x_3\ S_1\ S_2\ S_3 \geq 0$$

The initial simplex table of above problem is shown below.

	C_j	-1	3	-2	0	0	0		
CB_i	B.V.	x_1	x_2	x_3	S_1	S_2	S_3	Soln b	Ratio
0	S_1	3	-1	2	1	0	0	7	$7/-1 = -7$
0	S_2	-2	4	0	0	1	0	12	$12/4 = 3 \leftarrow$ min key row
0	S_3	-4	3	8	0	0	1	10	$10/3 = 10/3$
$Z_j = \Sigma CB_i\, a_{ij}$		0	0	0	0	0	0	0	
$C_j - Z_j$		-1	3	-2	0	0	0		

↑ max most +ve
key row

In the above table all the values of $C_j - Z_j$ are not less than or equal to zero
Hence optimality is not reached

The variable x_2 is entering variable & S_2 is Leaving variable.

The corresponding key element is 4 or also called as pivot element

∴ In next Iteration S_2 is replaced by x_2

min ratio
b / keycolumn element
$x_1 \geq 0$
-ve value not calculated

Iteration 1

C_{B_i}	c_j / B.V.	-1 x_1	3 x_2	-2 x_3	0 s_1	0 s_2	0 s_3	soln b	Ratio
0	s_1	$5/2$	0	2	1	$1/4$	0	10	$\frac{10}{\frac{5}{2}} = \frac{10\times 2}{5} = 4$ ←
3	x_2	$-1/2$	1	0	0	$1/4$	0	3	- Not calculated
0	s_3	$-5/2$	0	8	0	$-3/4$	1	1	-
$z_j = \Sigma C_{B_i} a_{ij}$		$-3/2$	3	0	0	$3/4$	0	9	
$c_j - z_j$		$1/2$ ↑	0	-2	0	$-3/4$	0		

$-1-\left(\frac{-3}{2}\right)$ → keycolumn

$\frac{-2+3}{2} = 1/2$

① $R_2' \rightarrow R_2/4$ or $R_{2(New)} \rightarrow R_{2(old)}/4$

x_1	x_2	x_3	s_1	s_2	s_3	b
$\frac{-2}{4}$	$\frac{4}{4}$	$\frac{0}{4}$	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{0}{4}$	$\frac{12}{4}$
$= -1/2$	1	0	0	$1/4$	0	3

② $R_1' \rightarrow R_{1(\cancel{old})} + R_2'$ or $R_{1(new)} \rightarrow R_{1(old)} + R_{2(new)}$

	x_1	x_2	x_3	s_1	s_2	s_3	b
$R_1(old)$	3	-1	2	1	0	0	7
$R_2(new)$ +	$-1/2$	1	0	0	$\frac{1}{4}$	0	3
	$5/2$	0	2	1	$1/4$	0	10

$3+\left(\frac{-1}{2}\right)$

$\frac{6-1}{2}$

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(3) $R_3' \rightarrow R_3 - 3R_2'$ or $R_{3(new)} \rightarrow R_{3(old)} - 3R_{2(new)}$

	x_1	x_2	x_3	S_1	S_2	S_3	soln. b
	-4	3	8	0	0	1	10
multiply by 3	$3(\frac{-1}{2})$	$3(1)$	$3(0)$	$3(0)$	$3(\frac{1}{4})$	$3(0)$	$3(3)$
	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$\frac{-3}{2}$	3	0	0	$\frac{3}{4}$	0	9
$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
	$-5/2$	0	8	0	$-3/4$	1	1

$-4-(\frac{-3}{2})$

$-4+\frac{3}{2}$

$\frac{-8+3}{2} = \frac{-5}{2}$

Z_j calculation

$x_1 \Rightarrow 0 \times \frac{5}{2} + 3 \times (\frac{-1}{2}) + 0 \times (\frac{-5}{2}) = 3(\frac{-1}{2}) = \frac{-3}{2}$

$x_2 \Rightarrow 0 \times 0 + 3 \times 1 + 0 \times 0 = 3$

$x_3 \rightarrow 0 \times 2 + 3 \times 0 + 0 \times 8 = 0$

$S_1 \rightarrow 0 \times 1 + 3 \times 0 + 0 \times 0 = 0$

$S_2 \rightarrow 0 \times \frac{1}{4} + 3 \times \frac{1}{4} + 0 \times -\frac{3}{4} = \frac{3}{4}$

$S_3 \rightarrow 0 \times 0 + 3 \times 0 + 0 \times 1 = 0$

soln b $\rightarrow 0 \times 10 + 3 \times 3 + 0 \times 1 = 9$

In the above table all the values of $C_j - Z_j$ are not less than or equal to zero
$\therefore$ optimality not reached.

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The variable x_1 is entering variable & s_1 is Leaving variable

corresponding key element is $5/2$

In next Iteration s_1 is replaced by x_1

Second Iteration.

C_{B_i}	C_j / B.V	-1 x_1	3 x_2	-2 x_3	0 s_1	0 s_2	0 s_3	soln b
-1	x_1	1	0	4/5	2/5	1/10	0	4
3	x_2	0	1	2/5	1/5	3/10	0	5
0	s_3	0	0	10	1	-1/2	1	11
$z_j = \Sigma C_{B_i} a_{ij}$		-1	3	2/5	1/5	4/5	0	11
$c_j - z_j$		0	0	$c_j - z_j = -2 - \frac{2}{5} = \frac{-10-2}{5}$ -12/5	-1/5	-4/5	0	

① $R_1' \rightarrow R_1 * \frac{2}{5}$ or $R_{1(new)} \rightarrow R_{1(old)} * \frac{2}{5}$

x_1	x_2	x_3	s_1	s_2	s_3	b
$5/2 * \frac{2}{5}$	$0 \times \frac{2}{5}$	$2 * \frac{2}{5}$	1	$1/4 * \frac{2}{5}$	$0 \times \frac{2}{5}$	$10 \times \frac{2}{5}$
Ans. = 1	0	4/5	2/5	1/10	0	4

(2) $R_2' \to R_2 + \frac{1}{2} R_1'$ or $R_{2(new)} \to R_{2(old)} + \frac{1}{2} R_{1(new)}$

(1) $x_1 \to -1/2 + \frac{1}{2}(1) = 0$

(2) $x_2 \to 1 + \frac{1}{2}(0) = 1$

$x_3 \to 0 + \frac{1}{2} * \frac{4}{5} = \frac{2}{5}$

$S_1 \to 0 + 1/2 * \frac{2}{5} = 1/5$

$S_2 \to 1/4 + 1/2 * \frac{1}{10} = 3/10$

$S_3 \to 0 + \frac{1}{2} * (0) = 0$

$b \to 3 + \frac{1}{2}(4) = 5$

(3) $R_3' \to R_3 + \frac{5}{2} R_1'$ or $R_{2(new)} \to R_{3(old)} + \frac{5}{2} R_{1(new)}$

(1) $x_1 \to -\frac{5}{2} + \frac{5}{2}(1) = 0$

(2) $x_2 \to 0 + \frac{5}{2}(0) = 0$

(3) $x_3 \to 8 + \frac{5}{2} * \frac{4}{5} = 10$

(4) $S_1 \to 0 + \frac{5}{2} * \frac{2}{5} = 1$

(5) $S_2 \to \frac{-3}{4} + \frac{5}{2} * \frac{1}{10} = \frac{-3}{4} + \frac{1}{2} = \frac{-2}{4} = \frac{-1}{2}$

(6) $S_3 \to 1 + \frac{5}{2}(0) = 1$

(7) $b \to 1 + \frac{5 * (4)}{\cancel{2}} = 1 + 5 * 2 = 1 + 10 = 11$

Calculation of z_j

① $x_1 \rightarrow -1\times1 + 3\times0 + 0\times0 = \boxed{-1}$

② $x_2 \rightarrow -1\times0 + 3\times1 + 0\times0 = \boxed{3}$

③ $x_3 \rightarrow -1\times\frac{4}{5} + 3\times\frac{2}{5} + 0\times10 = -\frac{4}{5} + \frac{6}{5} = \boxed{\frac{2}{5}}$

④ $s_1 \rightarrow -1\times\frac{2}{5} + 3\times\frac{1}{5} + 0\times1$

$\rightarrow -\frac{2}{5} + \frac{3}{5} = \boxed{1/5}$

⑤ $s_2 \rightarrow -1\times\frac{1}{10} + 3\times\frac{3}{10} + 0\times(-\frac{1}{2})$

$\rightarrow -\frac{1}{10} + \frac{9}{10} = \frac{\cancel{8}4}{\cancel{10}5} = \boxed{\frac{4}{5}}$

⑥ $s_3 \rightarrow -1\times0 + 3\times0 + 0\times1 = \boxed{0}$

⑦ $b \rightarrow -1\times4 + 3\times5 + 0\times11$

$= -4 + 15 = \boxed{11}$

In the above table all the values of $c_j - z_j$ are less than or equal to zero

$\therefore$ optimality reached & the corresponding optimum solution is

$x_1 = 4$. $x_2 = 5$ $x_3 = 0$

$\therefore$ $Z_{optimum} = 11$

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$\because$ ~~min~~

$\because$ MAX $z' = -x_1 + 3x_2 - 2x_3$

$= -4 + 3 \times 5 - 2 \times 0$

$= -4 + 15$

$= 11$

$\therefore$ min $z = -$ MAX$(z') = -11$

min $z = x_1 - 3x_2 + 2x_3$

$= 4 - 3 \times 5 + 2 \times 0$

$= 4 - 15 + 0$

$\boxed{\text{min } z = -11}$