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Ex. (2) Solve following example by simplex method.

maximize $z = 5x_1 + 3x_2$

subject to
$x_1 + x_2 \le 2$
$5x_1 + 2x_2 \le 10$
$3x_1 + 8x_2 \le 12$
$x_1, x_2 \ge 0$

soln: (1) Convert to standard form
Introduce the slack variable
S_1 S_2 & S_3

maximize $z = 5x_1 + 3x_2 + 0.S_1 + 0.S_2 + 0.S_3$

subject to
$x_1 + x_2 + S_1 + 0.S_2 + 0.S_3 = 2$
$5x_1 + 2x_2 + 0.S_1 + S_2 + 0.S_3 = 10$
$3x_1 + 8x_2 + 0.S_1 + 0.S_2 + S_3 = 12$
x_1, x_2, x_3 S_1, S_2 & $S_3 \ge 0$

(2) Now
prepare initial simplex table.
of the above problem

C_{B_i}	Basic variable	x_1	x_2	s_1	s_2	s_3	b Soln	θ Ratio
	C_j	5	3	0	0	0		
0	s_1	$\boxed{1}$	1	1	0	0	2	$2/1=2$ ← min Key row
0	s_2	5	2	0	1	0	10	$10/5=2$
0	s_3	3	8	0	0	1	12	$12/3=4$
	$z_j=\Sigma C_{B_i} \cdot a_{ij}$	0	0	0	0	0	0	
	$\theta_j = c_j - z_j$	5	3	0	0	0	0	

max ↑ Key column

calculation of z_j

x_1: $0\times1 + 0\times5 + 0\times3 = 0$

x_2: $0\times1 + 0\times2 + 0\times8 = 0$

s_1: $0\times1 + 0\times0 + 0\times0 = 0$

s_2: $0\times0 + 0\times1 + 0\times0 = 0$

s_3: $0\times0 + 0\times0 + 0\times1 = 0$

b: $0\times2 + 0\times10 + 0\times12 = 0$

For maximize $\theta_j \leq c_j - z_j \leq 0$
For maximize perform iteration upto zero or −ve if we get zero or −ve

& For minimize $c_j - z_j \geq 0$ optimal soln.

→ Now find key column i.e. MAX +ve element from $c_j - z_j$ in the above case it is 5

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→ Now find key row i.e. minimum ratio i.e. Ratio values are divided by key column values.
in The above case we select 2/1 because the intersection element we get one.
If we select 10/5 = 2 Then we have to make intersection element by applying some formula condition
∴ I select 2/1

→ Now the intersection of key column & key row is 1 ∴ 1 is our key element or pivot element.

→ Find entering or Incoming variable above key element & Leaving variable besides key element.

Here Incoming or entering variable is x_1

& Leaving variable is S1

* In The above table all values of $c_j - z_j$ are not less than or equal to zero

(For answer values required ~~for~~ of $c_j - z_j$ is −ve or zero)

∴ The next Iteration is as follows

	C_j	5	3	0	0	0	Soln	Ratio θ
CB_j	Basic variable	x_1	x_2	S_1	S_2	S_3	b	
5	x_1	1	1	1	0	0	2	
0	S_2	0	−3	−5	1	0	0	
0	S_3	0	5	−3	0	1	6	
$z_j = \Sigma C_{Bi} \cdot a_{ij}$		5	5	5	0	0	10	
$C_j - z_j$		0	−2	−5	0	0		

→ Now make all corresponding key element value zero make 5 & 3 as zero

$\therefore \quad R_2 \rightarrow R_2 - 5R_1 \quad \& \quad R_3 \rightarrow R_3 - 3R_1$

or $\quad R_{2(New)} = R_{2(old)} - 5R_{1(new)} \qquad$ or $\quad R_{3(New)} = R_{3(old)} - 3R_{(new)}$

	$R_{2(New)}$		$R_{3(New)}$
x_1	$= 5 - 5(1) = 0$	x_1	$3 - 3(1) = 0$
x_2	$2 - 5(1) = -3$	x_2	$8 - 3(1) = 5$
S_1	$0 - 5(1) = -5$	S_1	$0 - 3(1) = -3$
S_2	$1 - 5(0) = 1$	S_2	$0 - 3(0) = 0$
S_3	$0 - 5(0) = 0$	S_3	$1 - 3(0) = 1$
b	$10 - 5(2) = 0$	b	$12 - 3(2) = 12 - 6 = 6$

Now calculate z_j

$z_j = \Sigma C_{Bi} \cdot a_{ij}$

$x_1 = 5\times1 + 0\times0 + 0\times0 = 5$

$x_2 = 5\times1 + 0\times-3 + 0\times5 = 5$

$s_1 = 5\times1 + 0\times-5 + 0\times-3 = 5$

$s_2 = 5\times0 + 0\times1 + 0\times0 = 0$

$s_3 = 5\times0 + 0\times0 + 0\times1 = 0$

$b = 5\times2 + 0\times0 + 0\times6 = 10$

Now in the above table all the $c_j - z_j$ values are less than or equal to zero

Hence optimality is reached & the corresponding optimal solution is

$X_1 = 2$ $\quad X_2 = 0$ $\quad$ in Basic variable No x_2 variable $\therefore x_2 = 0$

& z (optimum) = 10

Ex③ Solve the following LP problem using simplex method.

maximize $Z = 10X_1 + 15X_2 + 20X_3$
subject to

$$2x_1 + 4x_2 + 6x_3 \leq 24$$
$$3x_1 + 9x_2 + 6x_3 \leq 30$$

$$x_1, x_2 \ \& \ x_3 \geq 0$$

Soln:- The standard form of this problem is

maximize $Z = 10x_1 + 15x_2 + 20x_3 + 0.s_1 + 0.s_2$
subject to

$$2x_1 + 4x_2 + 6x_3 + s_1 = 24$$
$$3x_1 + 9x_2 + 6x_3 + s_2 = 30$$
$$x_1\ x_2\ x_3\ s_1 \ \& \ s_2 \geq 0$$

where s_1 & s_2 are slack variables which are introduce to balance the constraints.

The initial simplex table of the above problem is shown below.