For minimize optimality condition $c_j - z_j \geq 0$

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Solve following LPP By simplex method.

⇒ Minimize $z = -40x_1 - 100x_2$

Sub to

$$10x_1 + 5x_2 \leq 250$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$

where $x_1, x_2 \geq 0$

step 1 check for standard form of LPP

2. Remove inequality constraints.

Minimize

$$z = -40x_1 - 100x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$$

Sub. to

$$10x_1 + 5x_1 + S_1 = 250$$
$$2x_1 + 5x_2 + S_2 = 100$$
$$2x_1 + 3x_2 + S_3 = 90$$

where $x_1\ x_2\ S_1\ S_2\ S_3 \geq 0$

Ratio = $\frac{\text{Soln}}{\text{Corresponding key column value}}$

Incoming variable

C_{Bj}	c_j	-40	-100	0	0	0	Soln	Ratio
	Basic variable	x_1	x_2	S_1	S_2	S_3	or b	Θ
0	S_1	10	5	1	0	0	250	250/5 = 50
0	S_2 (Leaving var.)	2	5 (Pivot ele.)	0	1	0	100	100/5 = 20 ** (tve Least Ratio, Key Row)
0	S_3	2	3	0	0	1	90	90/3 = 30
$z_j = \Sigma C_{Bj} \cdot a_{ij}$		0	0	0	0	0	0	
$c_j - z_j$		-40	-100 (↑ most -ve, key column)	0	0	0		

In the above table all the values of $c_j - z_j$ are not greater than or equal to zero

∴ Find Incoming variable $= x_2$
Leaving variable $= s_2$
key element or pivot element $= 5$

C_{B_i}	C_j	-40	-100	0	0	0	Soln
	B.V.	x_1	x_2	s_1	s_2	s_3	
0	s_1	8	0	1	-1	0	150
-100	x_2	2/5	1	0	1/5	0	20
0	s_3	4/5	0	0	-3/5	1	30
z_j		-40	-100	0	-20	0	-2000
$c_j - z_j$		0	0	0	20	0	

Now make pivot element 1 by following formula & corresponding key column value as zero

① $R_{2(New)} = \frac{R_2(old)}{\text{key element}}$

x_1 $2/5 = 2/5$
x_2 $5/5 = 1$
s_1 $0/5 = 0$
s_2 $1/5 = 1/5$
s_3 $0/5 = 0$

Soln $= b$ $\frac{100}{5} = 20$

Make zero

(2) $R_{1(New)} = R_{1(old)} - 5R_{2(New)}$

$x_2 = 5 - 5(1) = 0$

$x_1 = 10 - 5\left(\frac{2}{5}\right) = 10 - 2 = 8$

$S_1 = 1 - 5(0) = 1$

$S_2 = 0 - 5\left(\frac{1}{5}\right) = -1$

$S_3 = 0 - 5(0) = 0$

$Soln = 250 - 5(20) = 150$

(3) $R_{3(New)} = R_{3(old)} - R_{2(New)}$

$x_2 = 3 - 3(1) = 0$

$x_1 = 2 - 3\left(\frac{2}{5}\right) = 2 - \frac{6}{5} = \frac{4}{5}$

$S_1 = 0 - 3(0) = 0$

$S_2 = 0 - 3\left(\frac{1}{5}\right) = -3/5$

$S_3 = 1 - 3(0) = 1$

$Soln = 90 - 3(20) = 30$

Calculation for z_j

$x_1 = 0 \times 8 + (-100)\left(\frac{2}{5}\right) + 0 \times \frac{4}{5} = -40$

$x_2 = 0 \times 0 + (-100) \times 1 + 0 \times 0 = -100$

$S_1 = 0 \times 1 + (-100) \times 0 + 0 \times 0 = 0$

$S_2 = 0 \times (-1) + (-100) \times \frac{1}{5} + 0 \times \frac{-3}{5} = 20$

$S_3 = 0 \times 0 + (-100) \times 0 + 0 \times 1 = 0$

$Soln = 0 \times 150 + (-100) \times 20 + 0 \times 250 = -2000$

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Now check all the values of $c_j - z_j \geq 0$ ∴ optimality reached & we stop Here

$x_1 = 0 \quad x_2 = 20 \quad z = -2000$ (optimum)

Minimize $z = -40x_1 - 100x_2$

$= -40 \times 0 - 100 \times 20$

$= 0 - 2000$

$\therefore z_{(optimum)} = -2000$

(2) Solve the following L.P.P by using simplex method

Minimize $z = 2x_1 - 3x_2 + 6x_3$

Sub to

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 + 4x_2 \geq -12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2 \& x_3 \geq 0$$

In simplex method all R.H.S values of constraint must be positive

In this example (1) & (3) constraint R.H.S. value is positive but in second constraint R.H.S. value is -12

∴ you need to convert -ve value into positive value by multiplying (-1) to the entire constraint i.e eqn (2)

Soln-

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 - 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

} All +ve.

Now convert this into standard form by converting inequality to equality form by adding slack variable s_1, s_2, s_3

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$$\min Z = 2x_1 - 3x_2 + 6x_3 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3$$

sub to.

$$3x_1 - x_2 + 2x_3 + s_1 = 7$$

$$-2x_2 - 4x_2 \quad + s_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$

check optimality condition for minimization problem

For minimization problem

All the $c_j - z_j \geq 0$

For maximization problem

All the $c_j - z_j \leq 0$.

Ans $x_1 = 31/5$

$x_2 = \frac{58}{5}$ $x_3 = 0$

$Z = -112/5$

(select most -ve value for key column)

Initial table

always select least +ve Ratio

C_{B_i}	C_j	2	-3	6	0	0	0		⊙ Ratio
	B.V.	x_1	x_2	x_3	s_1	s_2	s_3	Soln b	
0	s_1	3	-1	2	1	0	0	7	7/-1 = -7
0	s_2	-2	-4	0	0	1	0	12	12/-4 = -3
0	s_3	-4	3	8	0	0	1	10	10/3 = 3.33 ←
z_j		0	0	0	0	0	0		** key Row
$c_j - z_j$		2	-3	6	0	0	0		

* key column ↑ most -ve