Circle Drawing Algorithms

Equation: $(X - Xc)^2 + (Y - Yc)^2 = R^2$

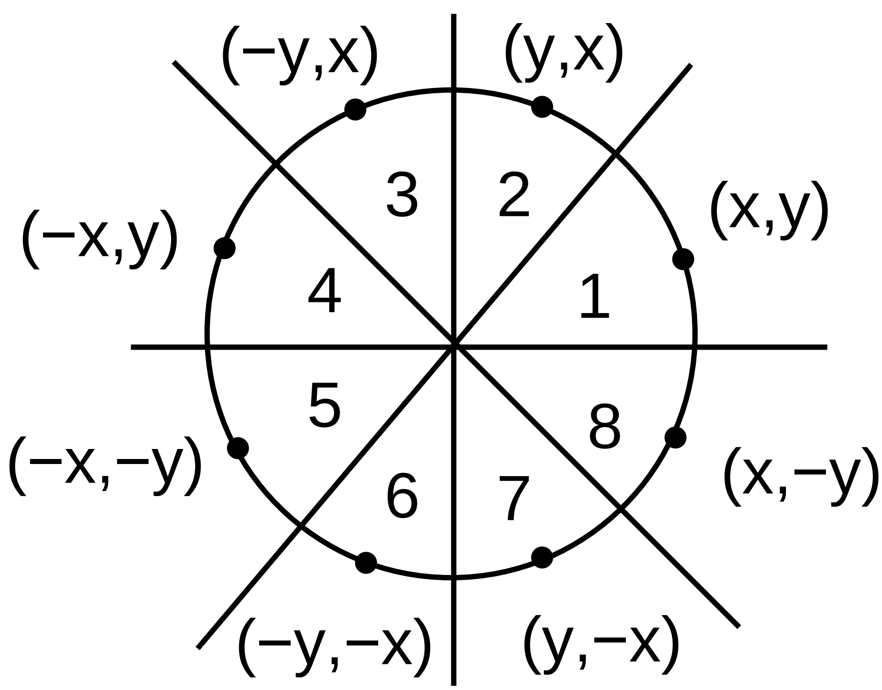
(Xc, Yc) : center   R: radius

- We only need to consider circles centered at the origin − apply translations to get non−origin centered circles

  So we have: $X^2 + Y^2 = R^2$

- Explicit equation: $Y = \pm \sqrt{R^2 - X^2}$

- Use Symmetry: only need to calculate one octant as we can get the points in other seven octants



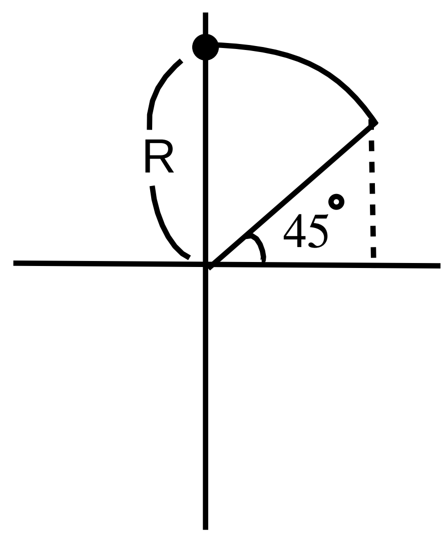
Circle Drawing Algorithms (cont'd)

(1) Direct solution: (use $Y = \pm\sqrt{R^2 - X^2}$)

− Draw the 2nd octant by incrementing X from 0 to $R / \sqrt{2}$

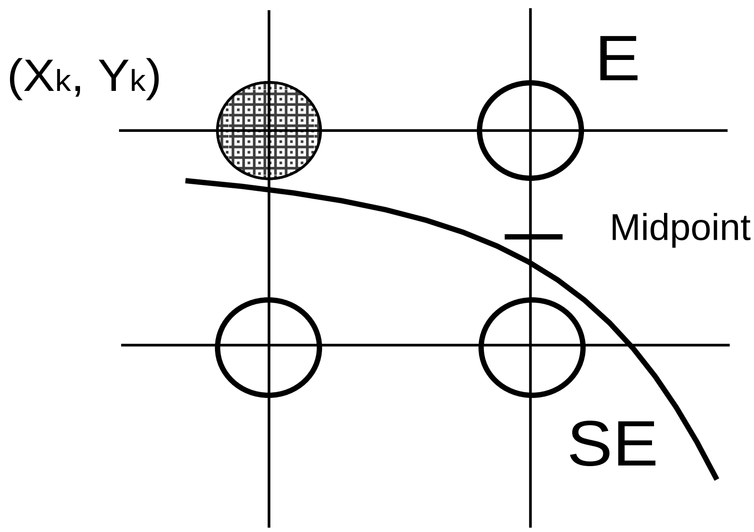
– At each step, solve $Y = \sqrt{R^2 - X^2}$

− Plot all the symmetry points



Problem: Slow !

Midpoint Circle Algorithm

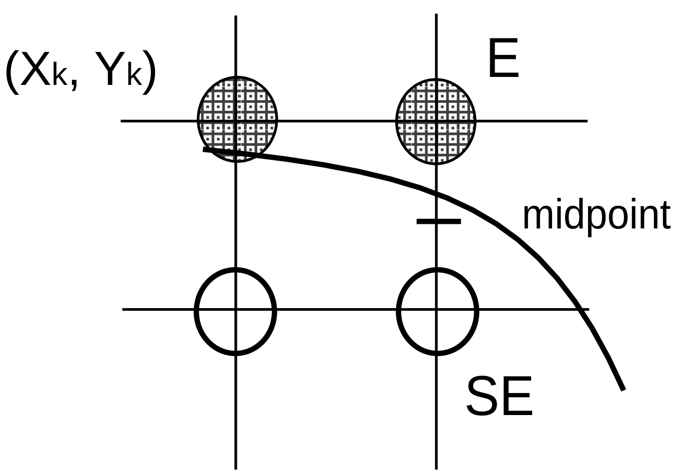


− We need to choose between

Pixel E (east) or Pixel SE (south east)

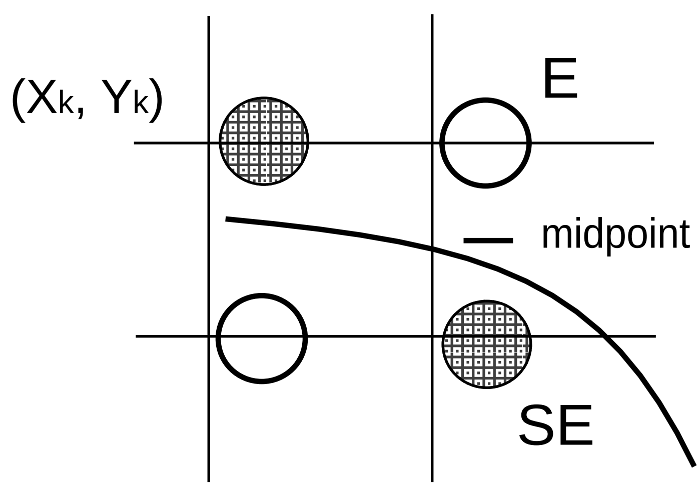
− Depend on if the midpoint is inside or outside the circle

If the midpoint is inside the circle ...



We choose Pixel E !!

If the midpoint is outside the circle ...



We choose Pixel SE !!

Midpoint Circle Algorithm (cont'd)

To determine a point is inside or outside

a circle:

$F(x,y) = x^2 + y^2 - R^2$

$F(x,y) < 0$: if point (x,y) is inside the circle

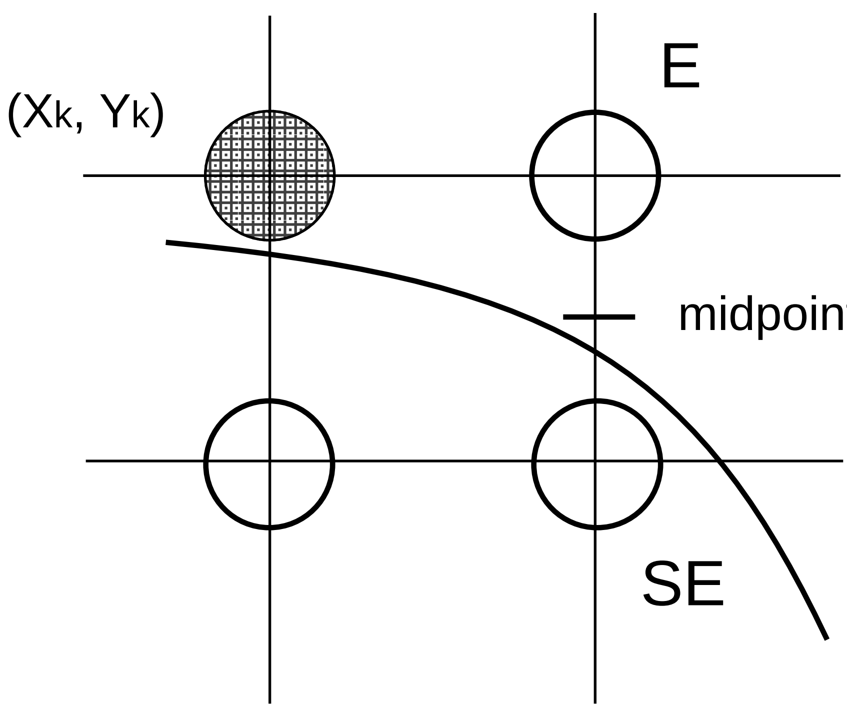
$F(x,y) > 0$: if point (x,y) is outside the circle

$F(x,y) = 0$: if point (x,y) is on the circle

$E = (X_k+1, Y_k)$

$SE = (X_k+1, Y_k-1)$

$\text{midpoint} = (X_k+1, Y_k - 1/2)$



Midpoint Circle Algorithm (cont'd)

− We have the decision parameter

$$
\boxed{Pk = F(X_k+1,\ Y_k - 1/2) = (X_k + 1)^2 + (Y_k - 1/2)^2 - R^2}
$$

$Pk < 0$: Choose Pixel E (X_k+1, Y_k)

$Pk >= 0$: Choose Pixel SE (X_k+1, Y_k-1)

− Can reduce the computation complexity using an incremental algorithm

$P_{k+1} = P_k +$??

Let's find out what the '??' is ...

Midpoint Circle Algorithm (cont'd)

$$P_{k+1} = F(X_{k+1} + 1,\ Y_{k+1} - 1/2)$$

$$= (X_{k+1} + 1)^2 + (Y_{k+1} - 1/2)^2 - R^2$$

$$= (X_k + 1 + 1)^2 + (Y_{k+1} - 1/2)^2 - R^2$$

$$= (X_k + 1)^2 + 2\,(X_k + 1) + 1 + Y_{k+1}^2$$

$$- Y_{k+1} + (1/2)^2 - R^2$$

$$P_{k+1} - P_k = \cancel{(X_k + 1)^2} + 2\,(X_k + 1) + 1 + Y_{k+1}^2$$

$$- Y_{k+1} + (1/2)^2 - \cancel{R^2}$$

$$- \cancel{(X_k + 1)^2} - (Y_k - 1/2)^2 + \cancel{R^2}$$

$$P_{k+1} = P_k + 2(X_k + 1) + (Y_{k+1}^2 - Y_k^2) - (Y_{k+1} - Y_k) + 1$$

Midpoint Circle Algorithm (cont'd)

$$P_{k+1} = P_k + 2(X_k + 1) + (Y_{k+1}^2 - Y_k^2) - (Y_{k+1} - Y_k) + 1$$

− If $P_k >= 0$, Pixel SE is chosen => $Y_{k+1} = Y_k - 1$

$$P_{k+1} = P_k + 2(X_k + 1) - 2(Y_k - 1) + 1$$

− If $P_k < 0$, Pixel E is chosen => $Y_{k+1} = Y_k$

$$P_{k+1} = P_k + 2(X_k + 1) + 1$$

$P0$?

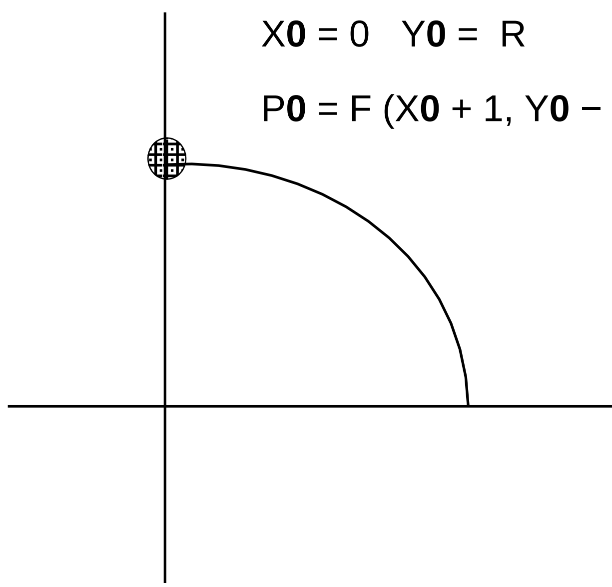
$P0 = F(1, R - 1/2)$

$= 1 + (R - 1/2)^2 - R^2$

$= 5/4 - R$

$X\mathbf{0} = 0 \quad Y\mathbf{0} = R$

$P\mathbf{0} = F(X\mathbf{0} + 1, Y\mathbf{0} - 1/2)$



Midpoint Circle Drawing Algorithm

1. input: radius R, center point (Xc, Yc)

   let $(X_0, Y_0) = (0, R)$

2. $P_0 = 5/4 - R$ $K = 0$

3. If $P_k < 0$

   $X_{k+1} = X_k + 1$
   $Y_{k+1} = Y_k$
   $P_{k+1} = P_k + 2(X_k + 1) + 1$

   else

   $X_{k+1} = X_k + 1$
   $Y_{k+1} = Y_k - 1$
   $P_{k+1} = P_k + 2(X_k + 1) - 2(Y_k - 1) + 1$

4. Determine symmetry points of (X_k, Y_k) in the other octants

5. For each calcuated point (X’,Y’), calculate the circle point (X,Y)

   $X = X' + Xc$ $Y = Y' + Yc$ draw (X,Y)

6. Increment K, go back to step 3 until $X_k > Y_k$