Understanding Tensors and Their Dimensions

What is a Tensor?

- A tensor is a fundamental data structure in machine learning and deep learning. It is essentially a container for numbers, representing data in various dimensions. Tensors are a generalization of scalars, vectors, and matrices to higher dimensions.

Basic Data Structure: Tensor

- **0D Tensor (Scalar)**: A single number or value. It has no dimensions and is also known as a scalar.

    - **Example**: (x = 5) (Here, (x) is a 0D tensor or scalar.)
- **1D Tensor (Vector)**: An array or a sequence of numbers. It is a collection of scalars.

    - **Example**: (v = [1, 2, 3]) (Here, (v) is a 1D tensor or vector.)

Dimensions of a Tensor

- The dimension of a tensor refers to the number of indices required to access a specific element within it. This is also referred to as the **Rank** of the tensor.

- **Rank**: The number of dimensions or axes a tensor has.

- **Axes**: The individual dimensions along which the data is arranged.

- **Shape**: The size of each axis or dimension in the tensor.

- **Number of Axes = Rank = Number of Dimensions**

Vector and Its Types

- **Vector**: A vector is a collection of scalars. It is a 1D tensor.

    - **Example**: (v = [1, 2, 3]) (This is a vector with three elements.)
- **Matrix**: A matrix is a collection of vectors arranged in rows and columns. It is a 2D tensor.

    - **Example**: M =

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

(Here, (M) is a 2D tensor or matrix.)

ND Tensors (Higher-Dimensional Tensors)

- **1D Tensor**: A tensor with one dimension (vector).

- **Example**: (v = [1, 2, 3, 4])
- **2D Tensor**: A tensor with two dimensions (matrix).

  - **Example**:

    M =

    $$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- **3D Tensor**: A tensor with three dimensions. Used in applications like Natural Language Processing (NLP) and time series data.

  - **Example**: T =

    ```
    \begin{bmatrix}
    \begin{bmatrix}
    1 & 2 \\
    3 & 4
    \end{bmatrix}
    ```

    $$\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

    \end{bmatrix}

    (Here, (T) is a 3D tensor.)

- **4D Tensor**: A tensor with four dimensions. Commonly used for image data, where dimensions might represent the number of images, image height, image width, and color channels.

  - **Example**: Consider a batch of 5 color images, each of size 28x28 pixels.

    Shape = (5, 28, 28, 3)

    (Here, it's a 4D tensor representing a batch of images.)

- **5D Tensor**: A tensor with five dimensions. Typically used for video data, where dimensions might represent the number of videos, frames per video, image height, image width, and color channels.

  - **Example**: Consider a batch of 3 videos, each containing 10 frames, with frame size 64x64 pixels and 3 color channels.

    Shape = (3, 10, 64, 64, 3)

    (Here, it's a 5D tensor representing a batch of videos.)

Summary of Tensors and Their Examples:

- **0D Tensor**: Scalar (e.g., x = 5)
- **1D Tensor**: Vector (e.g., v = [1, 2, 3, 4])

- **2D Tensor**: Matrix (e.g., a 2x3 matrix)
- **3D Tensor**: NLP, Time series data (e.g., T = [matrix 1, matrix 2])
- **4D Tensor**: Images (e.g., a batch of color images)
- **5D Tensor**: Videos (e.g., a batch of videos with multiple frames)

Understanding tensors, their dimensions, and how they are used in different applications is crucial for working with machine learning models, especially in deep learning where high-dimensional tensors are common.