Assignment-7

Problem-1

Code:

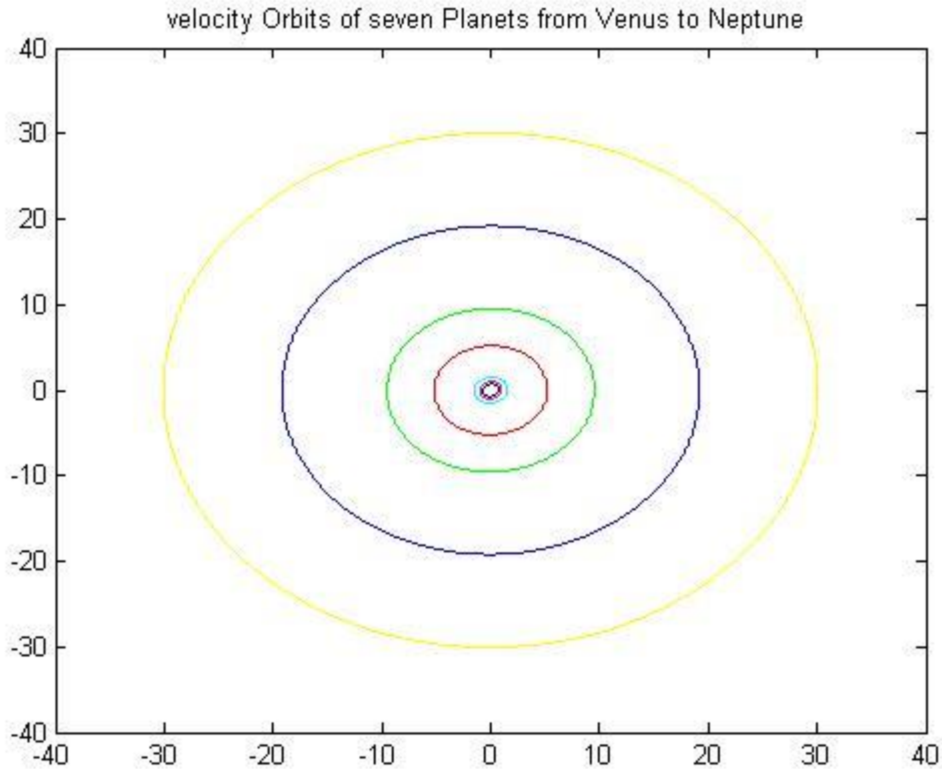
```
Clear;
 close all;
v_initial_inX=0;
G=4*pi*pi;
Ms=1;  % we normalized the mass of all the objects by the factor of (Mass of the sun)
R=[ 0.732, 1 , 1.524, 5.203, 9.537, 19.191,30.069 ]; % the distance is normalized by  1 astronomical unit
tinfinite=200;
Masses =[ 2.4478383e-6, 3.04043263333e-6, 0.3227151e-6, 954.79194e-6, 285.8860e-6, 43.66244e-6, 51.51389e-6 ] ; % normalized mass of all the planets.
timep=zeros(length(Masses),1);
kepler = zeros(length(Masses),1);    % counting kepler's constat (T^2)/(a^3)
for step1=1:1:7
   Me= Masses(step1);
   v_initial_inY=sqrt(G*Ms/R(step1));   % assuming initial velocity is only in y direction. (We can take it arbitrary ,but for simplicity we've taken it in y direction)
   tstart=0;
   dt=0.001;
   t=tstart:dt:tinfinite;
   C = {'k','m','c','r','g','b','y'};
   u=zeros(length(t),4);
   u(1,1)=v_initial_inX;
   u(1,2)=v_initial_inY;
   u(1,3)=R(step1);
   u(1,4)=0;
   pq=0;
   for step2=2:1:length(t)
      u(step2,1)=u(step2-1,1)+dt*(-G*Ms*u(step2-1,3)/(R(step1)*R(step1)*R(step1)));
      u(step2,2)=u(step2-1,2)+dt*(-G*Ms*u(step2-1,4)/(R(step1)*R(step1)*R(step1)));
      u(step2,3)=u(step2-1,3)+dt*u(step2,1);
      u(step2,4)=u(step2-1,4)+dt*u(step2,2);
      if t(step2)>0.5
         if u(step2-1,4)<0&&u(step2,4)>0
         timep(step1)=t(step2)
         pq=step2;
         kepler(step1)=(timep(step1)*timep(step1))/(R(step1)*R(step1)*R(step1));
         break;
      end
   end
```

```
end
vx=u(1:pq,1);
vy=u(1:pq,2);
x=u(1:pq,3);
y=u(1:pq,4);
plot(x,y,'color', C{step1} );
hold on;
title('Orbits of seven Planets from Venus to Neptune')

end
planets=1:7;
figure
plot(planets,kepler)
ylim ([0 2]);
set(gca, 'XTick',1:7, 'XTickLabel',{'venus' 'earth' 'mars' 'jupiter' 'saturn' 'uranus' 'neptune'})
```

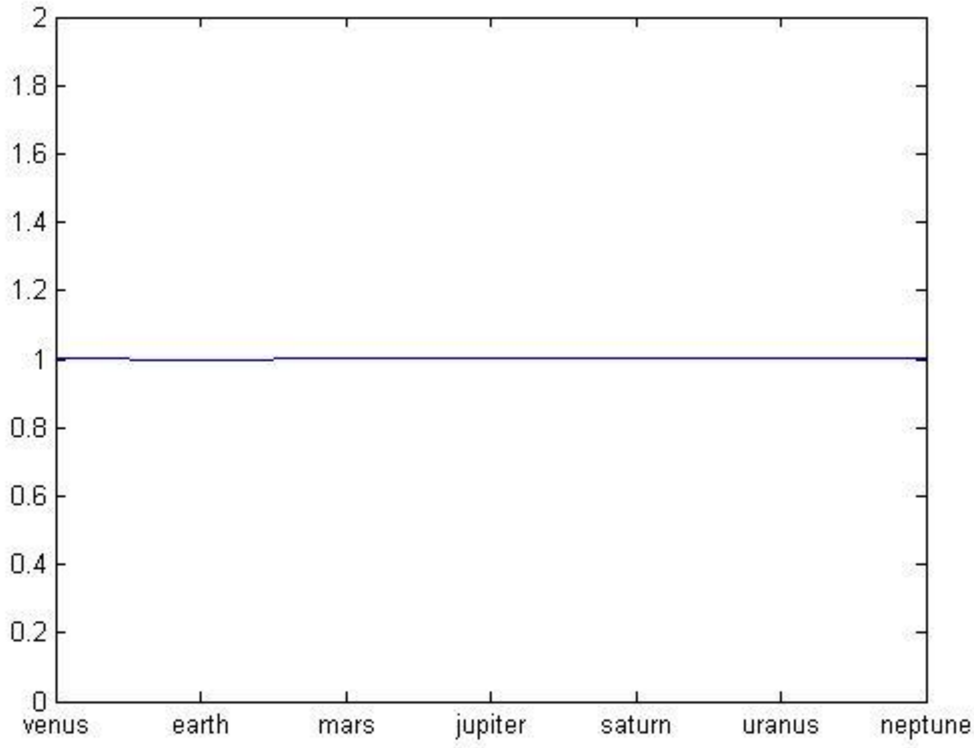
Table:

Planet	Mass(kg)	Radius(km)	time period(years)	eccentricity
Venus	4.87e24	108.2	.627	0
Earth	5.98e24	149.6	1.00	0
Mars	6.42e23	227.9	1.88	0
Jupiter	1.9e27	778.6	11.86	0
Saturn	5.69e26	1433.5	29.45	0
Uranus	8.68e25	2872.5	84.07	0
Neptune	1.02e26	4495.1	164.88	0

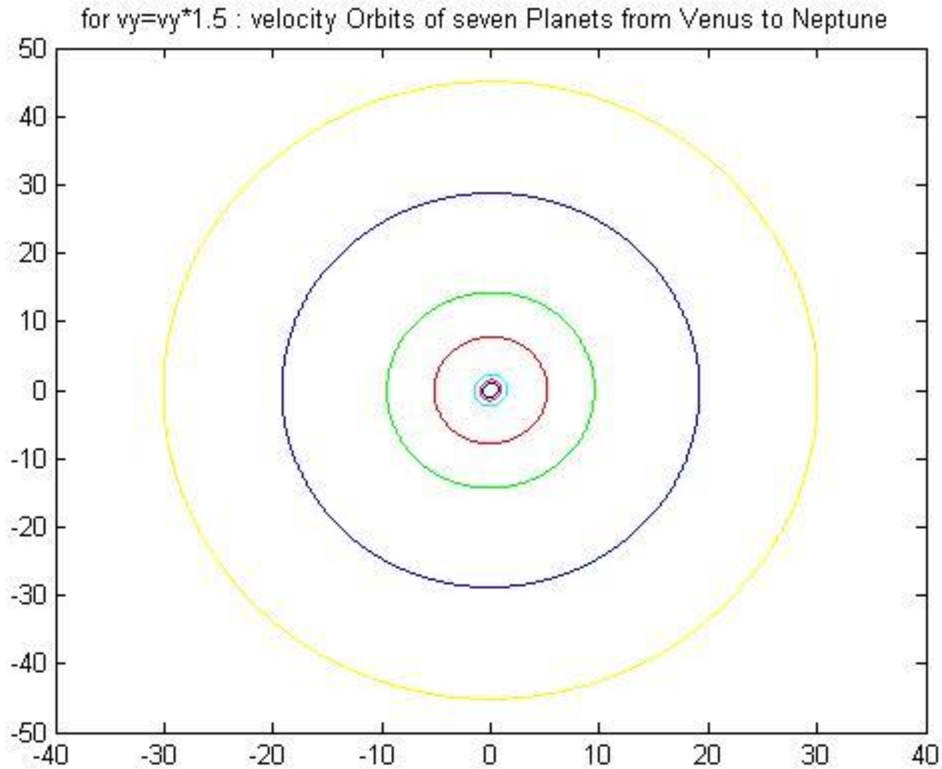
Circular Orbit taking velocity as sqrt(G* M / R)

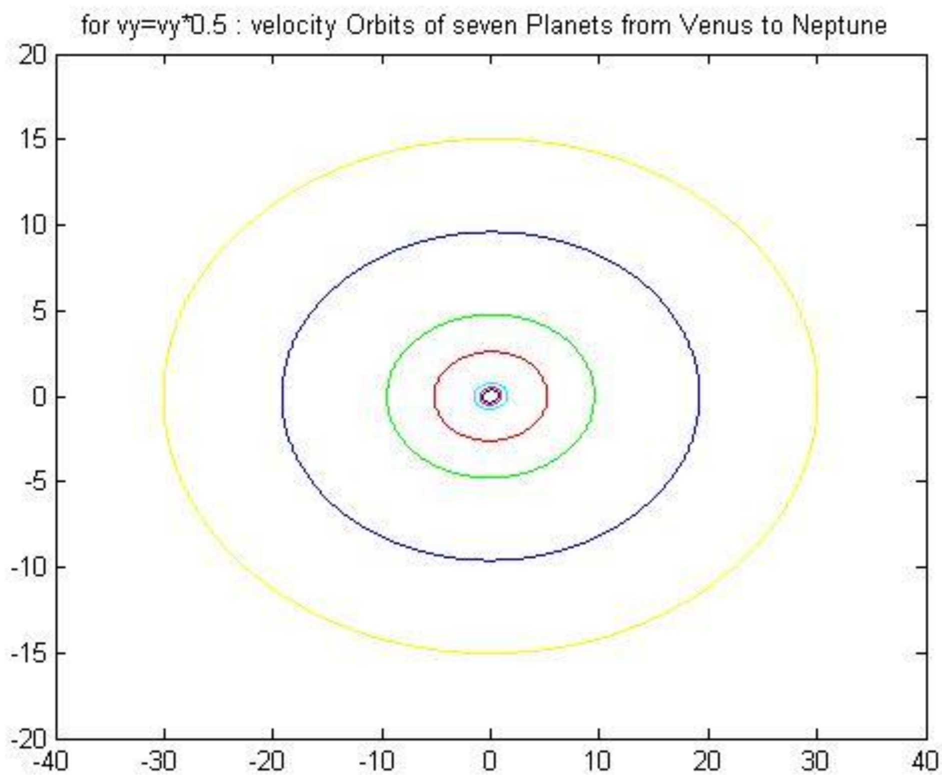


Kepler''s Constant for ll planets. Which Is nearly 1 (just because we normalized all the values.)



Graphs for different initial Velocities





In the above graphs, orbits of all 7 planets are plotted together. And first plot is showing circular path because we take velocity equal to centripetal velocity. The circular orbit exists for initial velocity vector pointing tangential to the line connecting the orbited and orbiting body with magnitude that balances 'centrifugal' acceleration (or acceleration of circular motion) with the acceleration caused by the mass of the orbited body. For these conditions to be met the initial velocity magnitude of the orbiting body has to be $v=(GM/R)^{1/2}$, where M is the orbited body, R is the distance between them, and G is the fundamental constant of gravity.

Plots when we increase the velocity and decrease the velocity are shown above. Both have elliptical orbits.

By changing velocity:
If we increase the velocity, orbit will become elliptical. Here, we start our motion from a point on x axis. So by increasing the velocity, highest dimension of ellipse would be in y direction. As it would consider the case where velocity is more -> point is away from the sun.
If we decrease the velocity, orbit will become elliptical, but here it will change in x direction. As it assumes that the point is near the sun, and velocity is less at that point.
Here if we increase or decrease the velocity, centripetal force changes, but force of attraction remains the same. Hence, if we want to balance them both, it has to change the path.

In each case, we calculated value of Kepler's constant (T^2/a^3) (as shown in the graph) and is approximately 1 in every case. Even if we increase or decrease the initial velocity, it will remain constant.

Problem-2

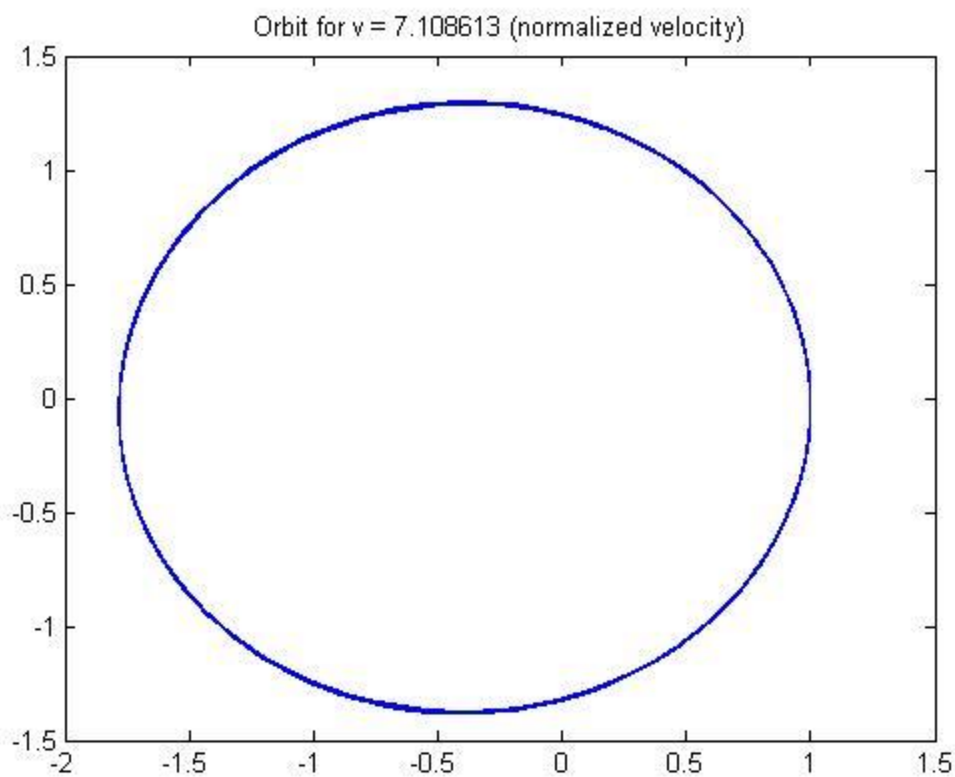
Code:

```
clear; close all;
v_initial_inY = 0;
G=4*pi*pi;
Ms=1; % normalized mass
tfinal=20;
Me=3.04043263333e-6;
R=1;  % 1 AU
v_initial_inX=.8 * sqrt(2*G*Ms/R); % less than parabolic motion
tstart=0;
dt=0.01;
t=tstart:dt:tfinal;
u=zeros(length(t),4);
u(1,1)=R;
u(1,2)=0;
u(1,3)=v_initial_inY;
u(1,4)=v_initial_inX;
ax=zeros(length(t),1);
ay=zeros(length(t),1);
ax(1)=-G*Ms/(R*R);
area=zeros(length(t),1);
ay(1)=0;
flag=0;
for step=2:1:length(t)
    R=sqrt((u(step-1,1)^2)+(u(step-1,2)^2));
    u(step,3)=u(step-1,3)+dt*(-G*Ms*u(step-1,1)/(R*R*R));
    u(step,4)=u(step-1,4)+dt*(-G*Ms*u(step-1,2)/(R*R*R));
    u(step,1)=u(step-1,1)+dt*u(step,3);
    u(step,2)=u(step-1,2)+dt*u(step,4);
    a=-(G*Ms/(R*R));
    dx=abs(u(step,1)-u(step-1,1));
    dy=abs(u(step,2)-u(step-1,2));
    area(step)=dx*dy/2;
    dr=sqrt(dx^2+dy^2);
    area(step)=R*dr/2;

      ax(step)=a*(u(step,1)/R);
      ay(step)=a*(u(step,2)/R);

    if u(step-1,2)<0&&u(step,2)>0&& flag==0
        tp=(step);
```

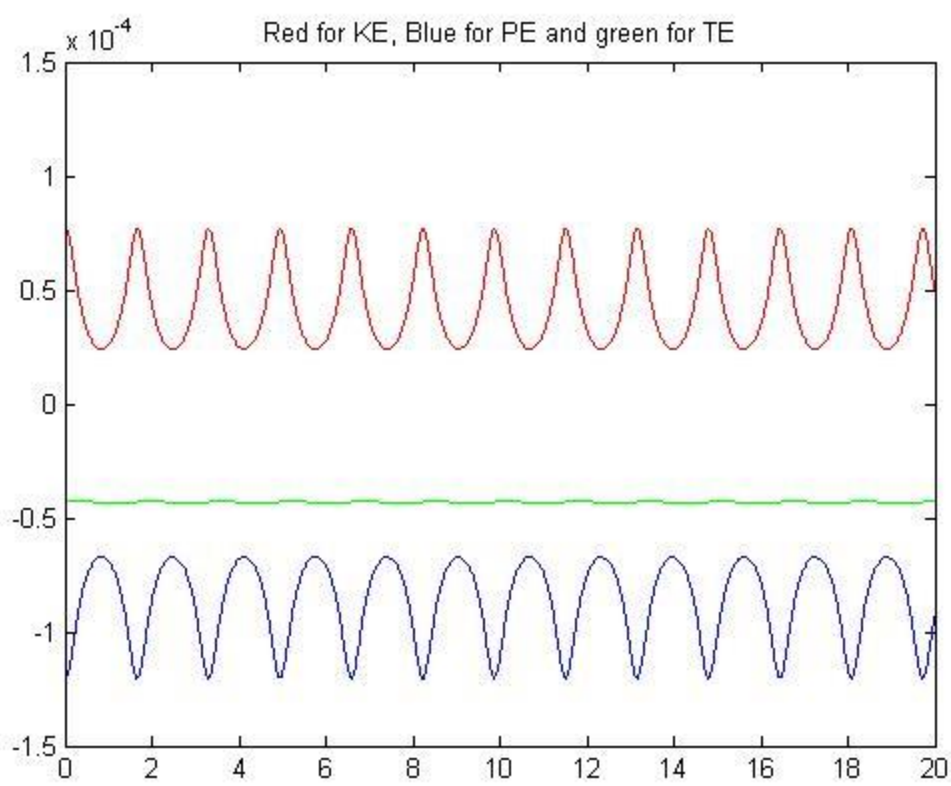
```
            flag=1;

      end
end

x=u(:,1);      % getting x
y=u(:,2);      % getting y
vx=u(:,3);     % getting velocity in x
vy=u(:,4);     % getting vy
plot(x,y); % simple path for earth
title_for_path = sprintf('Orbit for v = %f (normalized velocity)', v_initial_inX );
title(title_for_path)
figure;


quiver(x(1:tp),y(1:tp),vx(1:tp),vy(1:tp))
title('Velocity quiver to show velocity vector at evry point in path ')
figure;
tmp=zeros(tp,1);
quiver(x(1:tp),y(1:tp),vx(1:tp),tmp)
hold on
quiver(x(1:tp),y(1:tp),tmp,vy(1:tp))

title('Velocity quiver: in x and in y')


KE=Me.*(vx.^2+vy.^2)/2;

PE=-G*Me./sqrt(x.^2+y.^2);
figure;
plot(t,KE,'r')
hold on;
plot(t,PE,'b')
hold on;
plot(t,(KE+PE),'g')
title('Red for KE, Blue for PE and green for TE')
figure;
Ax=ax(1:tp);
Ay=ay(1:tp);
quiver(x(1:tp),y(1:tp),Ax,Ay);
title('Acceleration quiver')
figure;
```

```
temp=zeros(length(Ax),1);
quiver(x(1:tp),y(1:tp),Ax,temp)
hold on
quiver(x(1:tp),y(1:tp),temp,Ay)
title('Acceleration quiver : in x and in y')
figure;
tm=1:1:round(length(t)/tp);
final_area=zeros(length(tm),1);
for stp=1:1:round(length(t)/tp)
  for step=round((stp-1)*tp+1):1:round(stp*tp)
    final_area(stp)=final_area(stp)+area(step);
  end
end
bar(tm,final_area)
title('Calculated area  every year')
ylim([0 20])
```

Orbit for Earth for velocity different then centripetal velocity



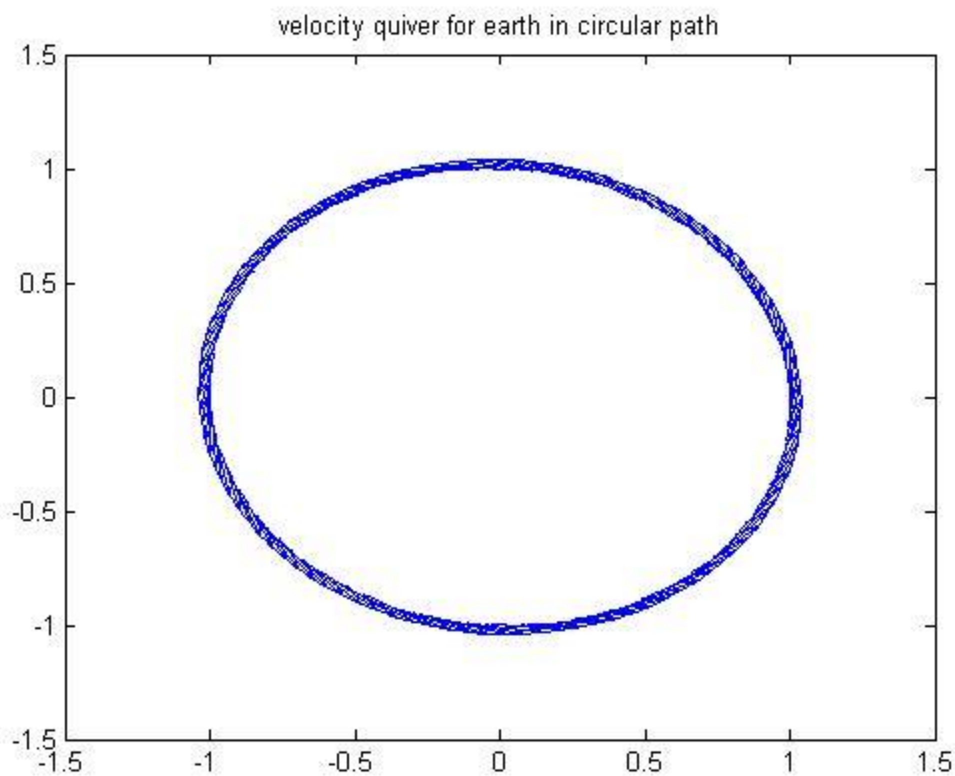
Energy:



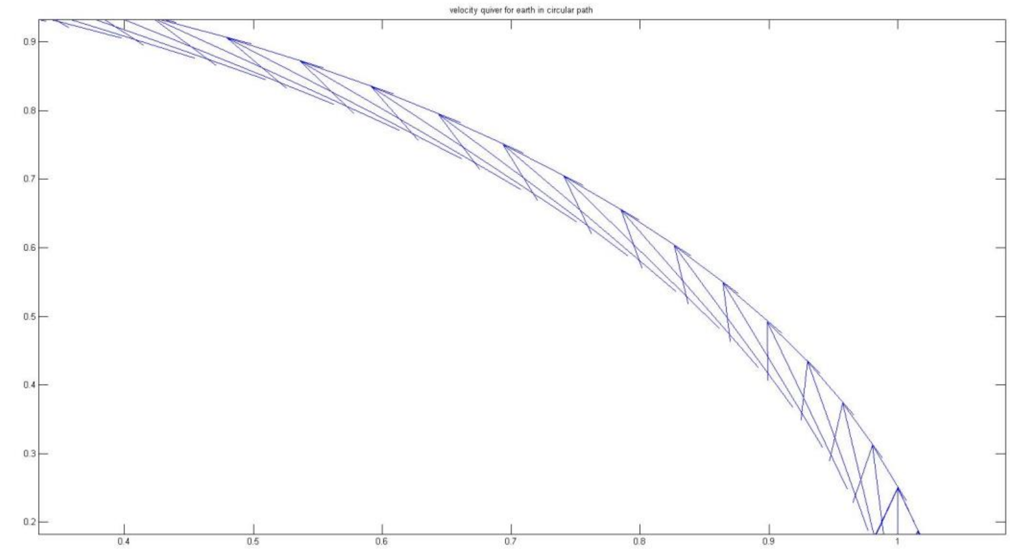
Red for KE, Blue for PE and green for TE
x 10^-4
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10
12
14
16
18
20

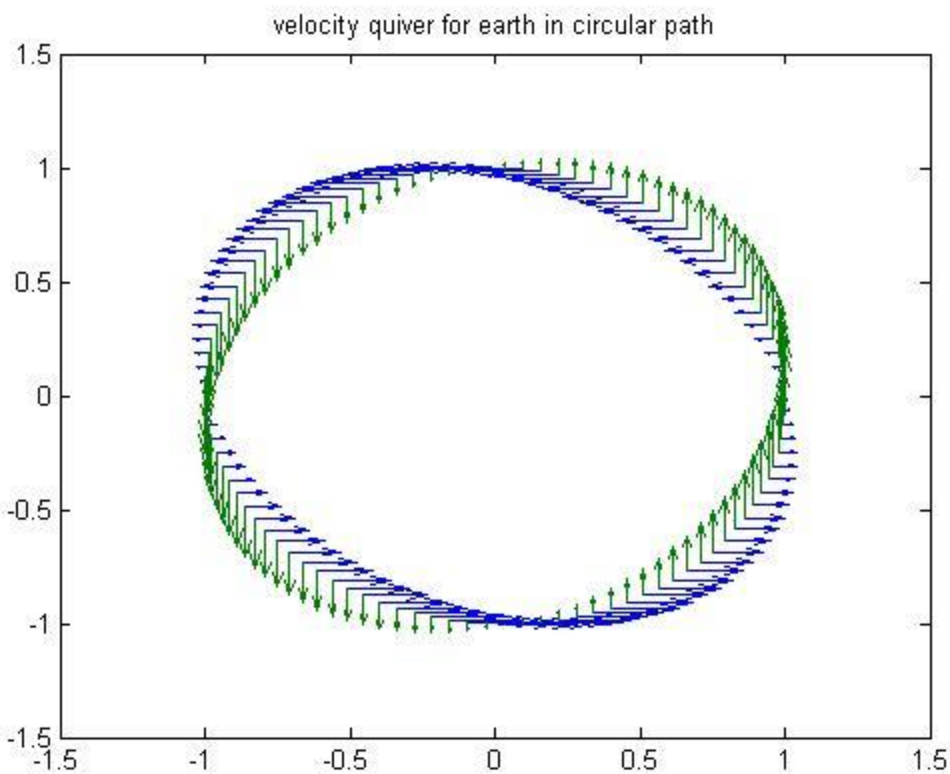
Investigate the direction of velocity and acceleration in both the case (circular and elliptical) as well as magnitude. (You can use the radial and tangential directions to describe the motion of the planet)

For circular

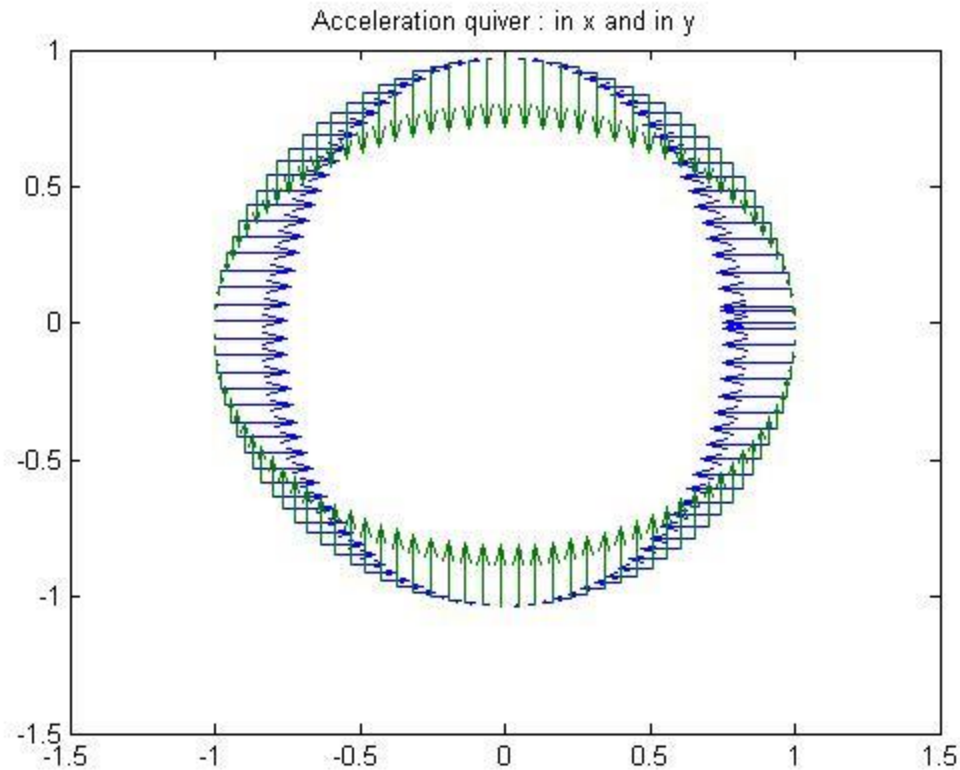


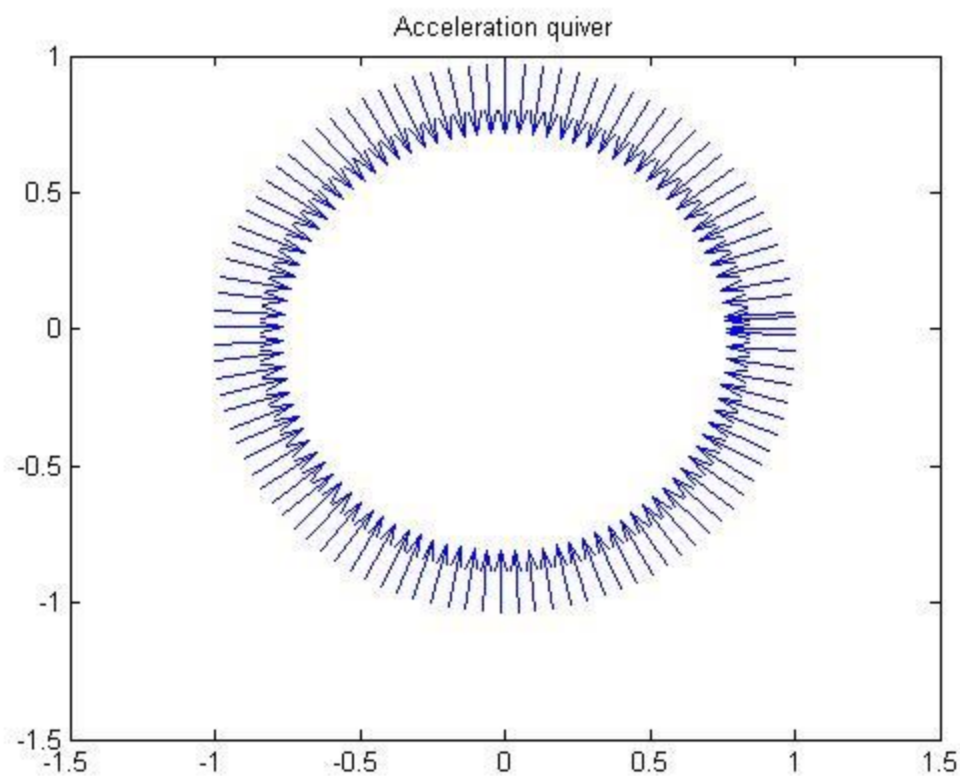
Zoomed in version:





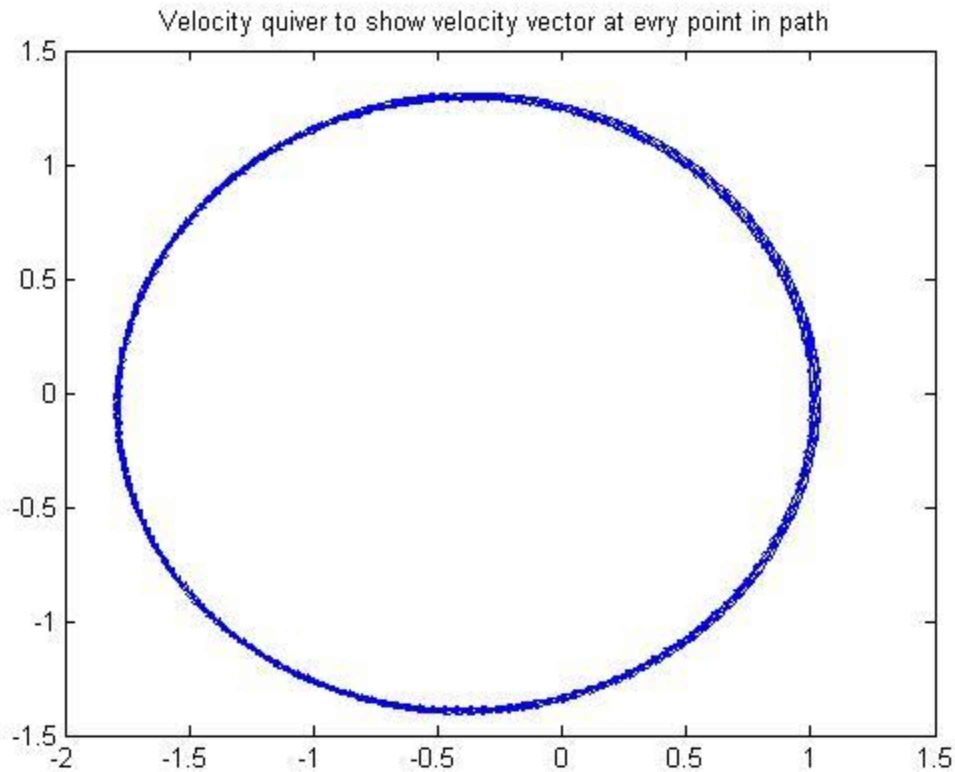
For circular path acceleration

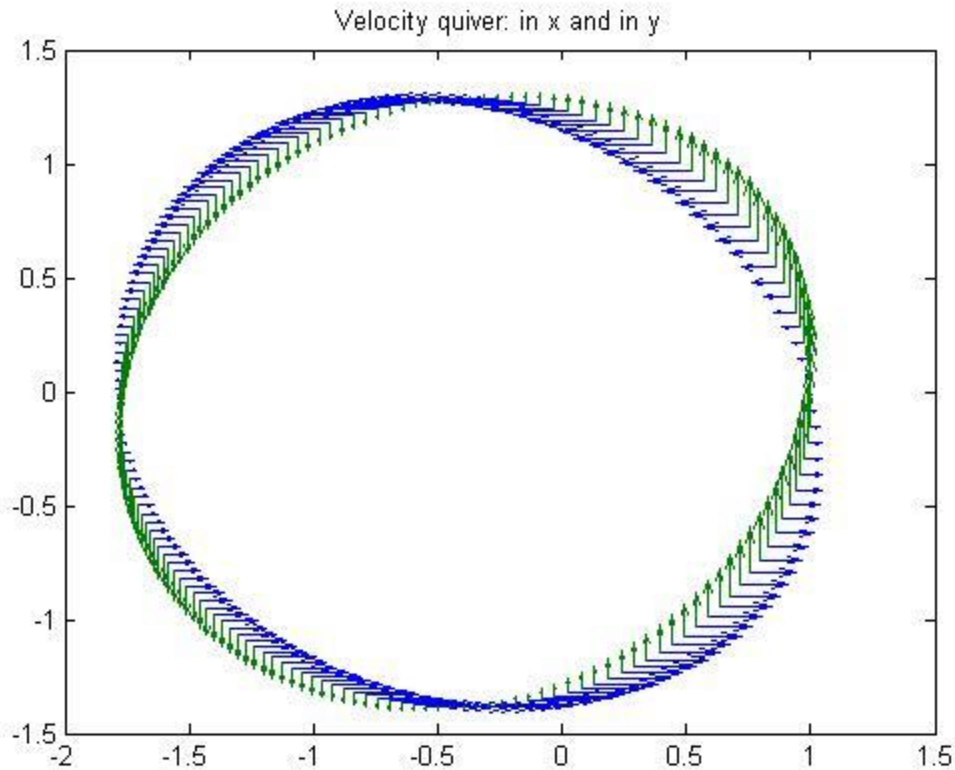

Acceleration quiver : in x and in y
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5


Acceleration quiver
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5

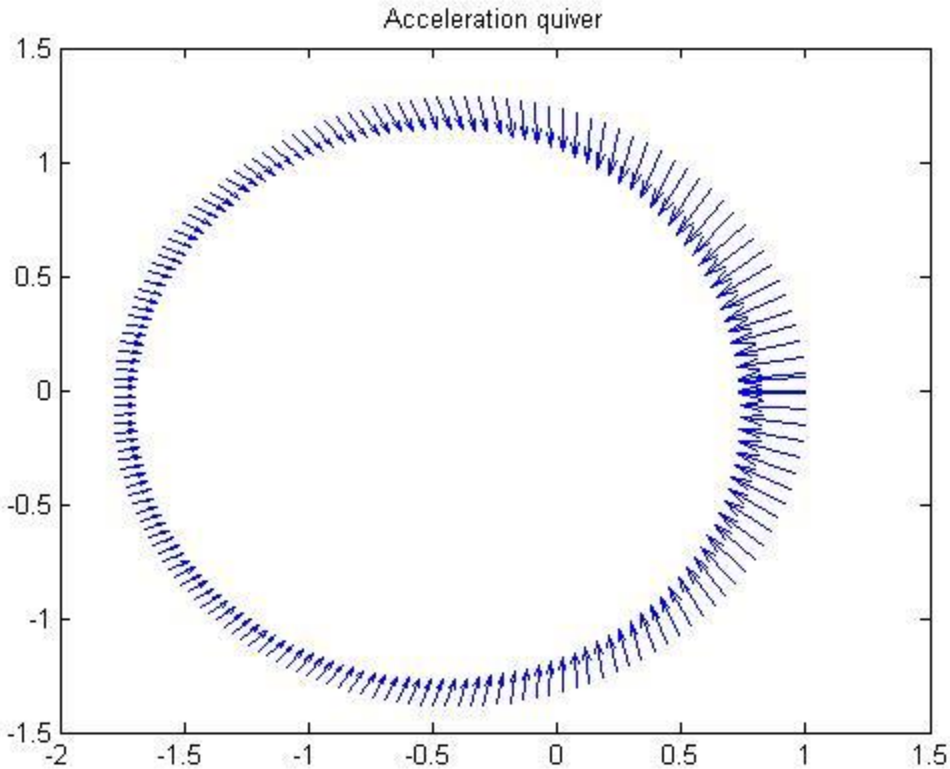
For Elliptical Path:

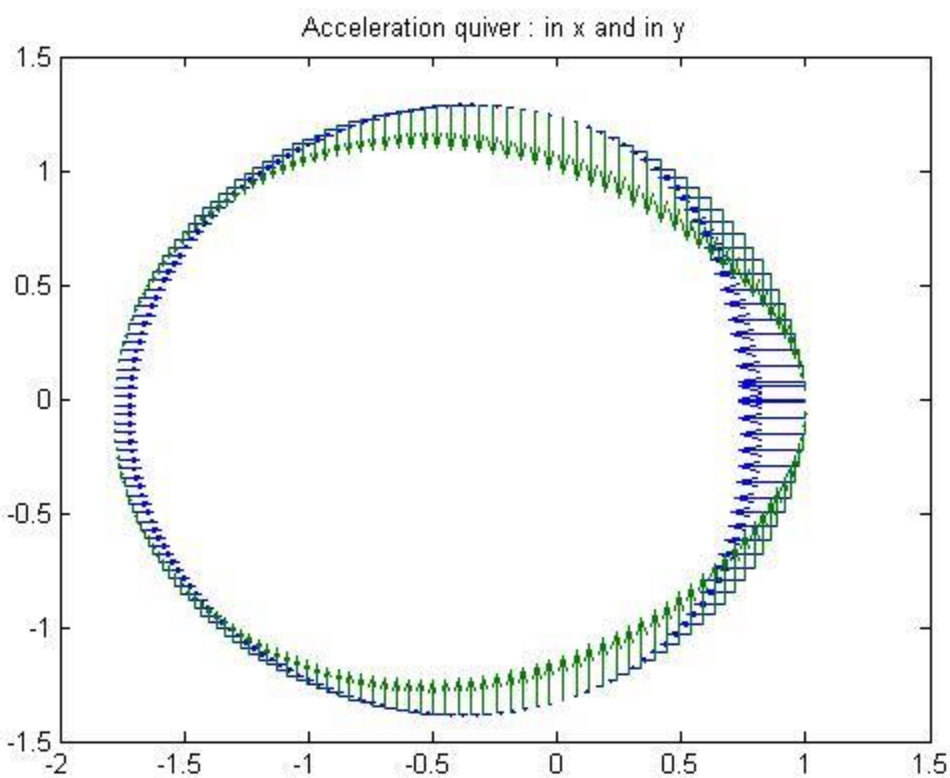
Velocity is very high near the sun and at a larger distance it is comparatively less.




Velocity quiver: in x and in y
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5

Acceleration:





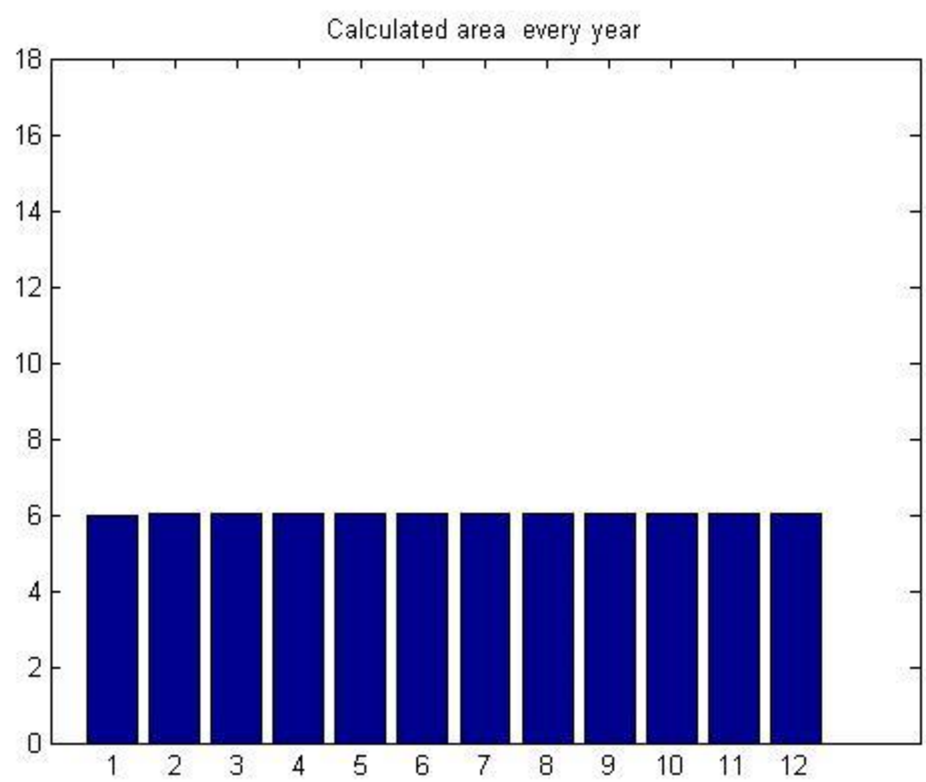
Notice that between some points (along the orbit) the planet is speeding up, and between some other points the planet is slowing down. Report on it.

It speeds up and slows down. It speeds up when it is near the sun. And speeds down when it is at a distance.

Planets follow the second law of Kepler (A line joining a planet and the Sun sweeps out equal areas during equal intervals of time) so that means they have to move faster.

Report about conservation of angular momentum

As we can see that here the only force acting is gravitational force. And it cannot create torque because It passes through the centre of the sun. And torque is r*F. and here r and F both are in same direction.



Here by seeing above plot we can tell that It sweeps out equal area in every time interval.

This can be explained by the newton's second law of motion. Here net torque acting on body is zero. So change in angular momentum zero. i.e Angular momentum is conserved.

Problem-3

A) What happens to the orbit when x gets really small?

Simple Solution is to increase the velocity because if the velocity is small it will collide with the Sun. When x gets very small, the force of attraction increases. Centripetal force has to increase to balance it. X is small, it would be nearer to sun. So if it can't move that fast it will collide with sun.

A) What happens to the orbit when x gets really large?

Simple Solution is to decrease the velocity because if the velocity is very high it will go away from sun and will never come back. The orbit will be open. And path becomes hyperbolic in nature. When x gets very large, the force of attraction decreases. Centripetal force has to decrease to balance it.

C)Now, as you vary the initial velocities of the planets, how do the orbital trajectories change?

When we vary initial velocities, their centripetal force changes but attraction force i.e. gravitational force remains constant. Or alternately we can say that its kinetic energy increases, but we need to keep the total energy constant, hence the path cannot be circle. Hence it takes the shape of ellipse to reach that point again and move around the sun. It conserves energy by coming near and going far from sun i.e. changing potential energy. But we need to conserve energy so kinetic energy will vary accordingly.

E)What happens to the orbit when v gets really small?

When v is really small, its kinetic energy would decrease, but for conservation of energy its potential energy should increase, so it would be far from sun.It will gradually come near the sun with increasing velocity i.e. kinetic energy and decreasing potential energy and it will again go far from sun, revolving around the sun in elliptical orbit.

f) What happens to the orbit when v gets really large?

When v is really small, its kinetic energy would decrease, but for conservation of energy its potential energy should increase, so it would be nearer to sun.It will gradually go away from sun with decreasing velocity i.e. kinetic energy and decreasing potential energy and it will again come near to the sun, revolving around the sun in elliptical orbit.

G) How do the values for total energy and angular momentum change when the type of orbit is changed?

For Circular Path KE is half of the magnitude of the PE. And total energy is negative and equal to half of the PE.

For Elliptical Path relation between KE and PE changes. As we know it speeds up and slows down We can say that the difference between their magnitude is maximum at farthest point .and minimum at nearest point. Total energy is again constant but it depends upon initial velocity of the Object.

h)For different initial velocities:

For velocity less than circular path it gives ellipse. For velocity greater than circular path it again gives ellipse.

- If the Total energy is positive, the body's kinetic energy is greater than its potential energy: The orbit is thus open, following a path of hyperbola with focus at the sun.
- For the zero-energy case, the body's kinetic energy is exactly equal to its potential energy: the orbit is thus a parabola with its focus at the sun.
- If the energy is negative, the body's potential energy is greater than its kinetic energy: The orbit is thus closed. The motion is on an ellipse or in a very special case where a and b of ellipse becomes equal and it becomes circle.