Modeling and Simulation - Lab Assignment 2

Vaibhav Amit Patel (201401222)* and Tanmay Patel (201401409)*

E-mail: vaibhav290797@gmail.com; tanmaypatel273@gmail.com

Assignment 1

Problem 4

In this problem we attempt to look at a simple blood alcohol levels in humans. There are many factors that influence the body's ability to absorb alcohol. These include person's weight, percentage of body fat and age. An important quantity is the blood alcohol level (BAL), more body weight implies that a person has more water in his body and hence alcohol can be absorbed quickly. Older people absorb alcohol at a slower rate due to their decreased metabolic activity. The presence of food in the GI-tract can also effect the BAL. Soon after the alcohol enters the blood it enters the liver for elimination, which eliminates most of the alcohol. The equation for such a simple model is usually written as:

$$\frac{dx}{dt} = I - k_1 x$$

$$\frac{dy}{dt} = k_2 x - \frac{k_3 y}{y + M}$$

$x(t)$ is the alcohol level in the GI tract and $y(t)$ represents the alcohol level in the bloodstream. The typical units are g/100mL. The first equation has two terms, I represents the alcohol intake and the second term represents the diffusion from GI tract into the bloodstream. The y equation has the inflow and outflow to the liver term. The outflow is most aptly represented by what is called a Michaelis-Menton function. For y $>>$ M it equals a constant k3. M is a constant with value 0.005. Some other information that is relevant is the total volume of body fluid in liters (C) is 0.67w for males and 0.82w for females, w being the body weight in kilograms. I = 14n/10C where n is the number of drinks (we assume the glass size to be the same in all cases), and k3 = 8/10C. Both I and K3 have units g/100mL while k1 and k2 are dimensionless. Also, for empty stomach drinking we assume k1 = k2 and for drinking after a substantial meal we assume k2 = k1/2.

Part A:

Assuming $k1 = 6$ and the person takes 3 drinks initially explore the alcohol concentrations in bloodstream and GI tract with time. If BAL above 0.05 leads to dizziness and below it leads to happy feeling, which scenario above would lead to dizziness.

Before meal BAL is above 0.05 (dizziness) for both male and female of weight 75 kg while it is less than 0.05 (happy feeling) after meal for both because value of k2 is higher in case of no meal so diffusion rate in blood is also high. BAL depends on gender, weight, number of drinks and time of taking alcohol.

As the weight increases we see that value of C increases thus initial intake decreases and also value of k3 decreases. Hence initially value of BAL is less for more weight male but the decrease in BAL is slower.

For same weight male and female value of C is lower in male. So initial intake is higher in male and also value of k3 is higher in male. Hence initially value of BAL is more for male compared to female but the decrease is faster.

Code:

```
close all;

clear all;

total = 10;

dt = 0.01;
```

```
iter = total / dt + 1;
w = 75;
Cs = [0.67 * w, 0.82 * w];
n = 3;
k1 = 6;
k2s = [k1, k1 / 2];
M = 0.005;
for ci = 1 : 2
    C = Cs(ci);
    k3 = 8 / (10 * C);
    I = (14 * n) / (10 * C);
    for ki = 1 : 2
        k2 = k2s(ki);
        x = zeros(iter, 1);
        y = zeros(iter, 1);
        t = zeros(iter, 1);
        x(1) = I;
        for i = 2 : iter
            x(i) = x(i - 1) - dt * (k1 * x(i - 1));
            y(i) = y(i - 1) + dt * (k2 * x(i - 1) - (k3 * y(i - 1)) / (y(i - 1) + M));
            t(i) = t(i - 1) + dt;
        end;
        figure;
        plot(t, x);
        hold on;
        plot(t, y);
        if(ci == 1 && ki == 1)
```

```
            title('Male before meal')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        if(ci == 1 && ki == 2)
            title('Male after meal')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        if(ci == 2 && ki == 1)
            title('Female before meal')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        if(ci == 2 && ki == 2)
            title('Female after meal')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        legend('GI Tract','Bloodstream')
    end;
end;
```

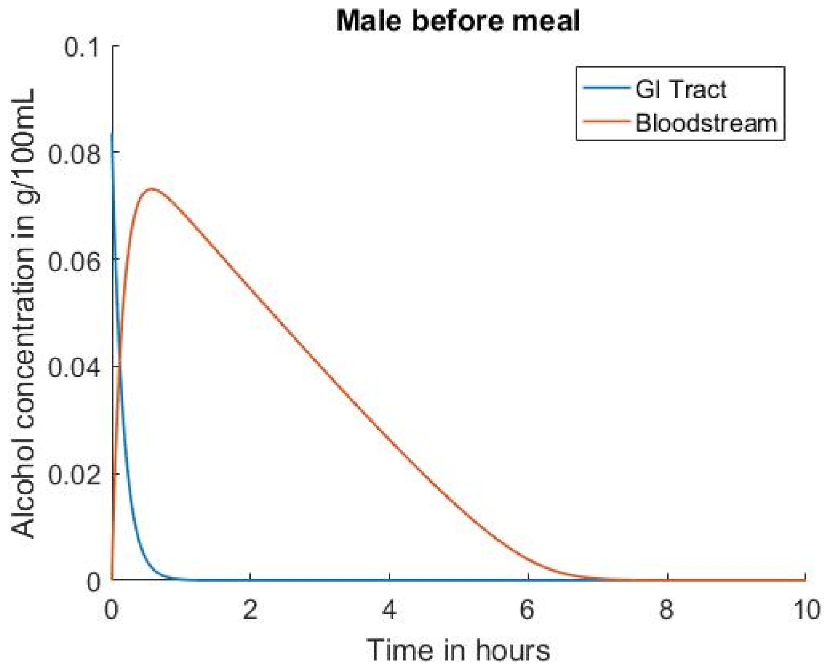

Figure 1

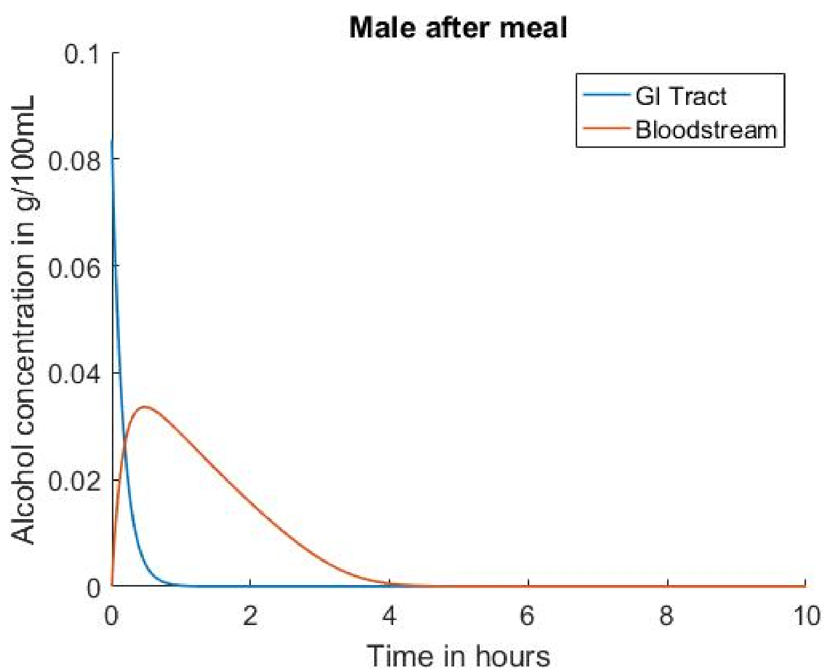

Figure 2

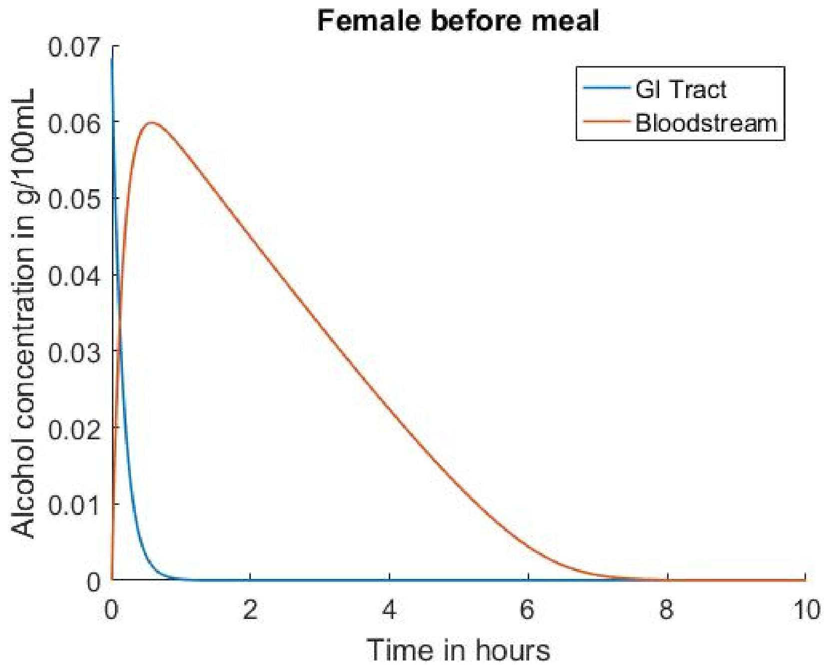

Figure 3

Figure 4

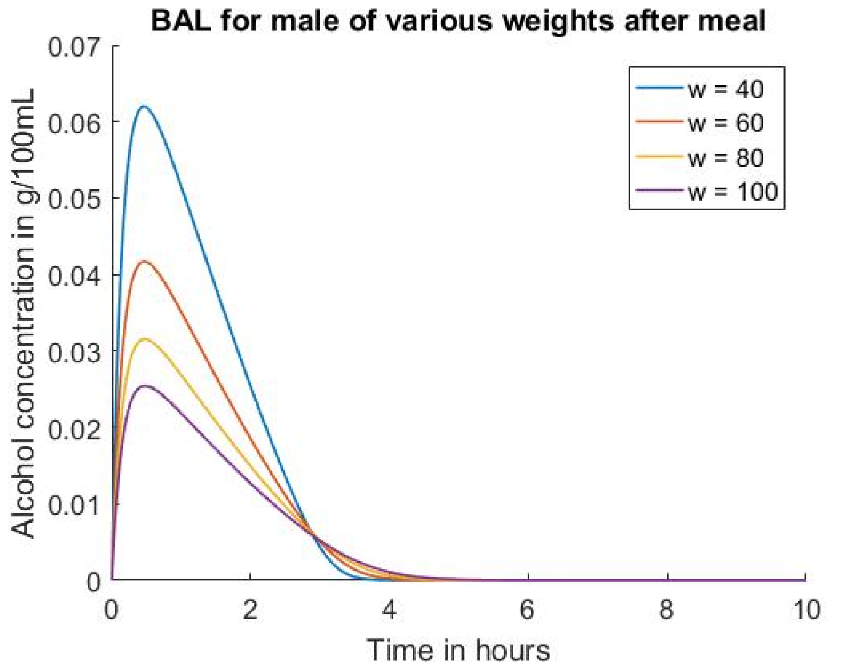

Figure 5

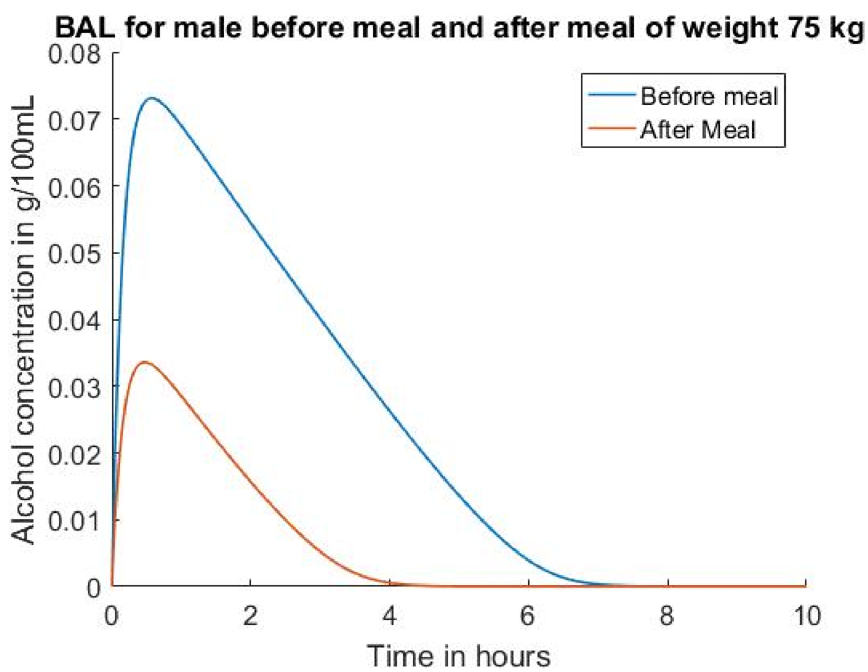

Figure 6

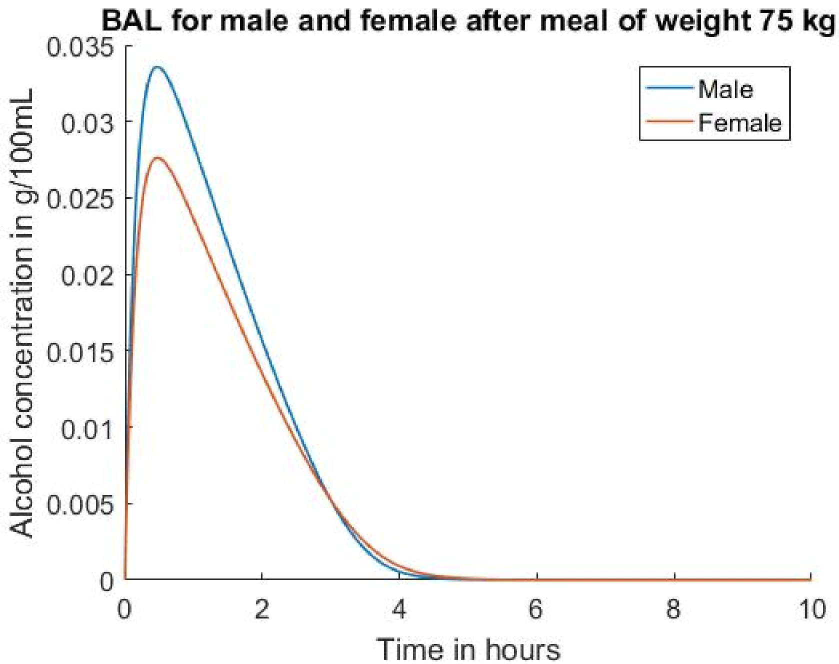

Figure 7

Part B:

Assuming continuous drinking explore what happens in (a)

Increase in BAL before meal is faster than increase in BAL after meal as k2 (diffusion in blood rate) is higher before meal.

Increase in BAL for person of less weight is faster compared to person to more weight as value of C is less hence value of I and k3 is higher.

Also increase in BAL for male is faster compared to female of same weight because of higher values of I and k3.

Code:

```
close all;

clear all;

total = 10;

dt = 0.01;
```

```
iter = total / dt + 1;

w = 75;

Cs = [0.67 * w, 0.82 * w];

n = 3;

k1 = 6;

k2s = [k1, k1 / 2];

M = 0.005;

for ci = 1 : 2

    C = Cs(ci);

    k3 = 8 / (10 * C);

    I = (14 * n) / (10 * C);

    for ki = 1 : 2

        k2 = k2s(ki);

        x = zeros(iter, 1);

        y = zeros(iter, 1);

        t = zeros(iter, 1);

        x(1) = I;

        for i = 2 : iter

            x(i) = x(i - 1) + dt * (I - k1 * x(i - 1));

            y(i) = y(i - 1) + dt * (k2 * x(i - 1) - (k3 * y(i - 1)) / (y(i - 1) + M));

            t(i) = t(i - 1) + dt;

        end;

        figure;set(gca,'fontsize',13)

hold on

        plot(t, x,'lineWidth',1.2)

        hold on;

        plot(t, y,'lineWidth',1.2)
```

```
        if(ci == 1 && ki == 1)
            title('Male before meal (75 kg)')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        if(ci == 1 && ki == 2)
            title('Male after meal (75 kg)')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        if(ci == 2 && ki == 1)
            title('Female before meal (75 kg)')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        if(ci == 2 && ki == 2)
            title('Female after meal (75 kg)')
            ylabel('Alcohol concentration in g/100mL')
            xlabel('Time in hours')
        end
        legend('GI Tract','Bloodstream (75 kg)')
    end;
end;
```

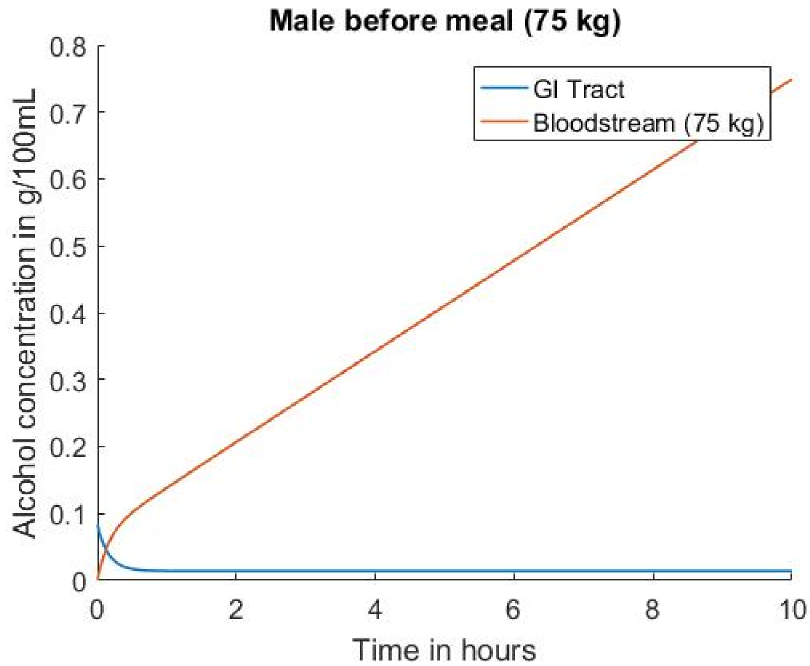

Figure 8

Figure 9

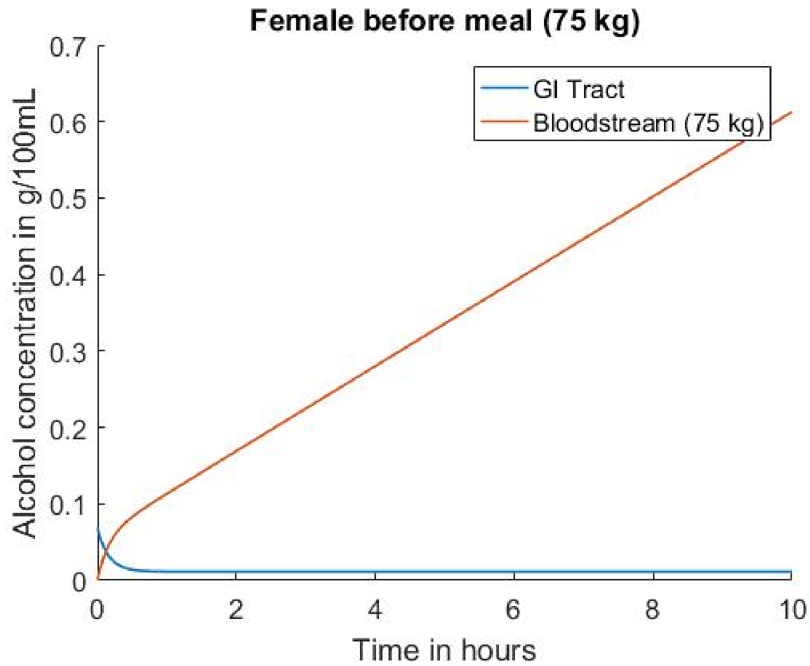

Figure 10

Figure 11

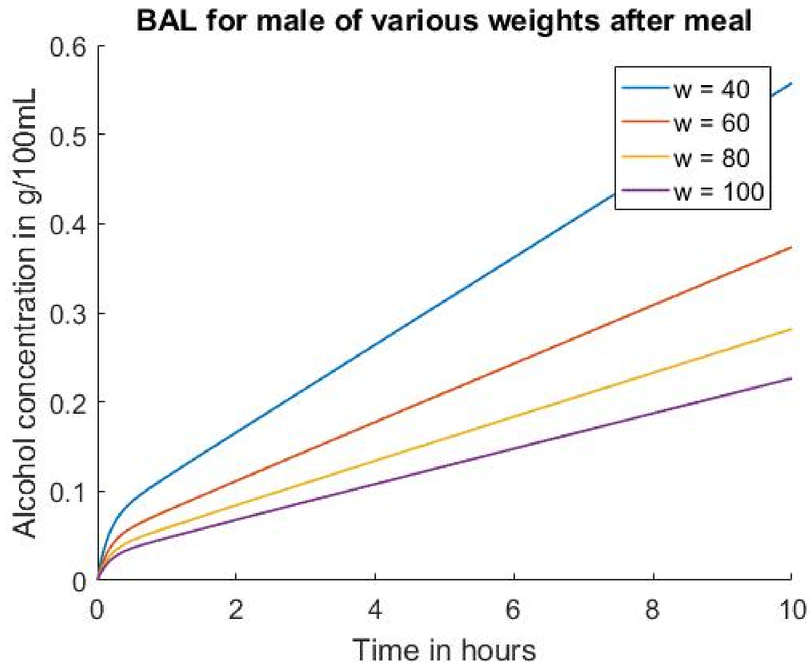

Figure 12

Figure 13

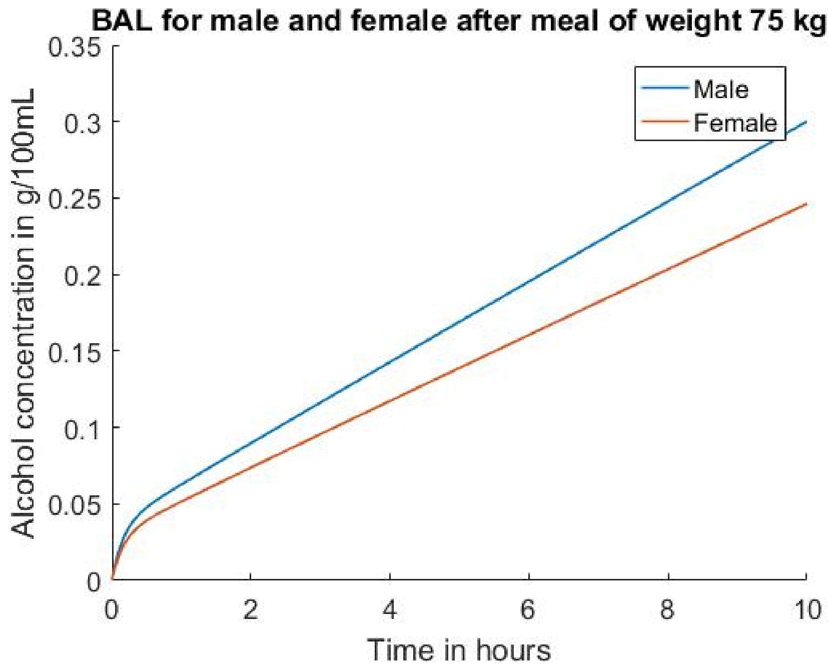

Figure 14

Assignment 2

Problem 1:

Models of innovation diffusion:

Part A:

First consider the external influence model and study the effect of variation of p. Plot similar to the figure in the assignment. both the market share and groups of consumers adopting the product.

As the value of p increases the market-share saturates faster because rate of change in number of people using technology depends on p. Here influence of people already using technology is not considered only influence due to advertisements and such sources(external influence) are considered.

Code:

```
% As value of p increases, rate of change in number of people adopting the product incre
close all;
clear all;
total = 200;
dt = 0.5;
set(gca,'fontsize',13)
hold on
iter = total / dt + 1;
p = [0.02, 0.04, 0.06, 0.08, 0.1];
C = 100000;
y = zeros(5, iter);
for j = 1 : 5
    N = zeros(iter, 1);
    x = zeros(iter, 1);
    t = zeros(iter, 1);
    for i = 2 : iter
        N(i) = N(i - 1) + dt * p(j) * (C - N(i - 1));
        x(i) = (N(i) / C) * 100;
        y(j, i-1) = ((N(i) - N(i - 1)) / (C*dt )) ;
        t(i) = t(i - 1) + dt;
    end;
    y(j, iter) = y(j,iter-1);
    plot(t, x,'lineWidth',1.2)
    hold on;
```

```
end;

legend('p = 0.02', 'p = 0.04', 'p = 0.06', 'p = 0.08', 'p = 0.1');
title('Market share (External influence model)');
xlabel('Time (years)');
ylabel('Market share');
ylim([0 120])



figure;
set(gca,'fontsize',13)
hold on
for j = 1 : 5
    plot(t, y(j, :),'lineWidth',1.2)
    hold on;
end;
legend('p = 0.02', 'p = 0.04', 'p = 0.06', 'p = 0.08', 'p = 0.1');
%ylim([0 10]);
title('New customers (External influence model)');
xlabel('Time (years)');
ylabel('New customers (fraction)');
```

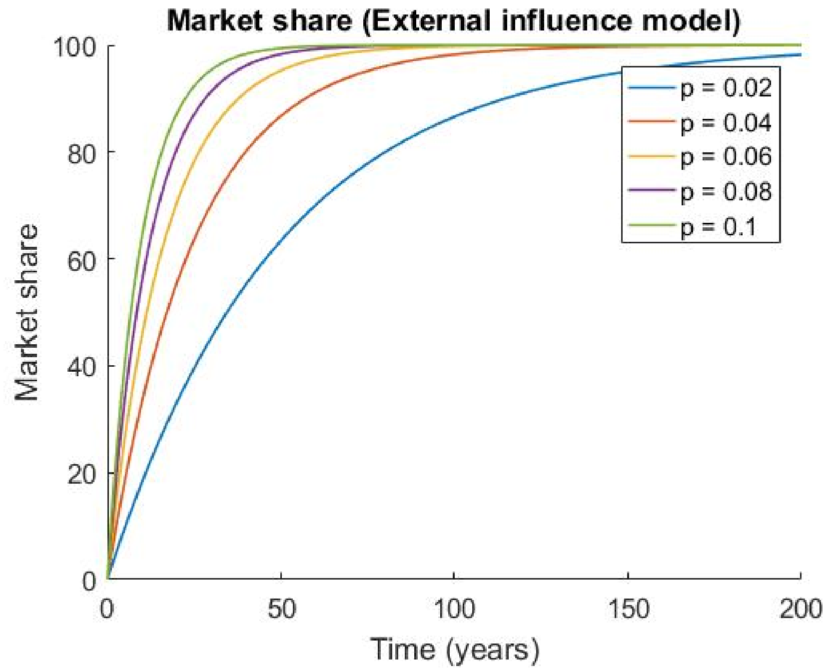

Figure 15

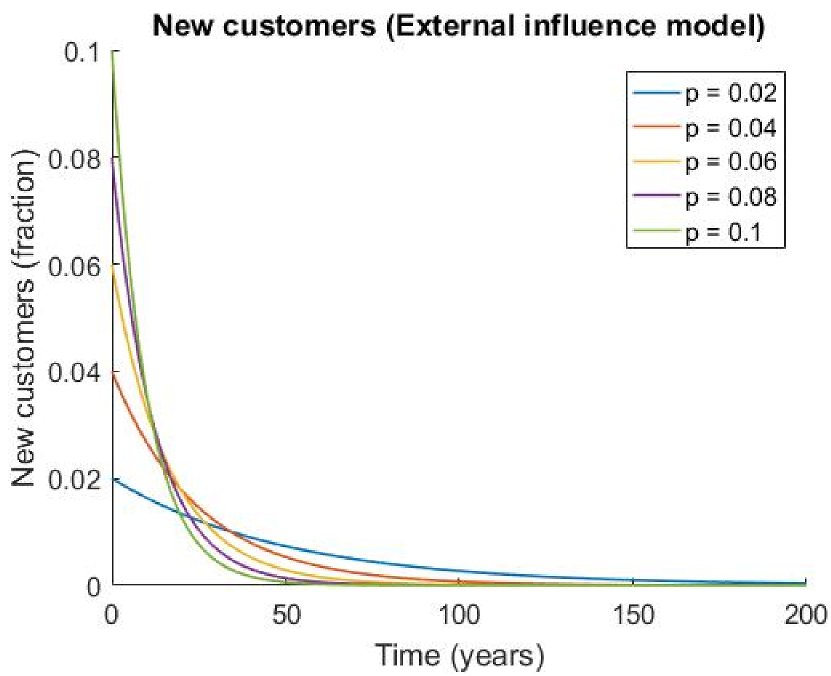

Figure 16

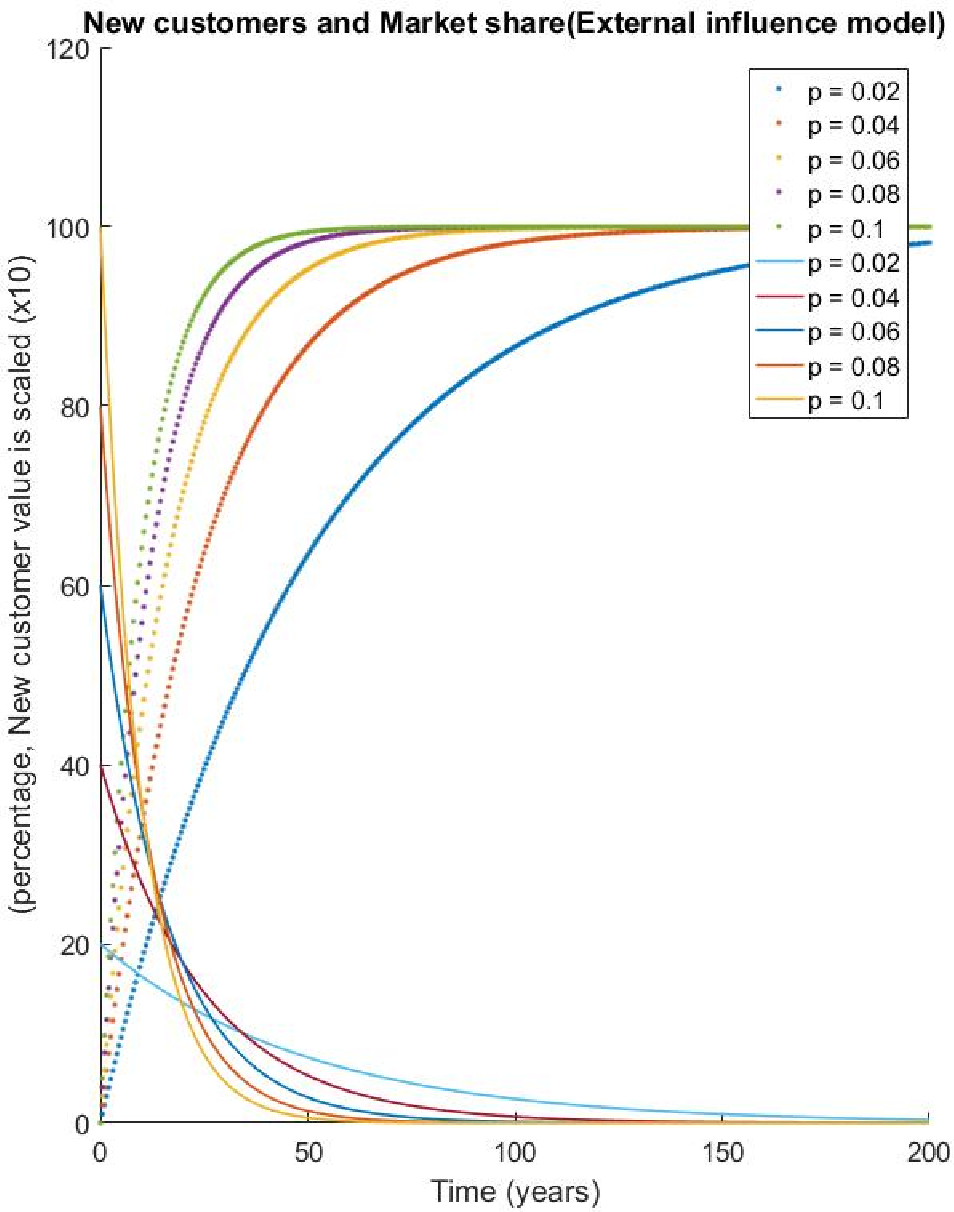

Figure 17

Part B:

Similar to (a) but now consider the internal influence model. Study the effect of varying q. Can you comment on the aspect of the problem which the logistic equation can not capture.

As value of q increases the market-share saturates faster because rate of change in number of people using technology depends on q.

g(t) depends on number of people already using technology. It considers how people of the population who has not adopted the technology are influenced by the people who are using it already(internal influence), but the external influences like advertisements are not considered here.

Code:

```
close all;
clear all;
total = 80;
dt = 0.5;set(gca,'fontsize',13)
hold on
iter = total / dt + 1;
q = [0.2, 0.4, 0.6, 0.8, 1];
C = 100000;
y = zeros(5, iter);
for j = 1 : 5
    N = zeros(iter, 1);
    x = zeros(iter, 1);
    t = zeros(iter, 1);
    N(1) = 0.025 * C;
    x(1) = (N(1) / C) * 100;
    %y(j, 1) = (N(1) / (C * dt)) * 100;
    for i = 2 : iter
        a = (q(j) * N(i - 1)) / C;
        N(i) = N(i - 1) + dt * a * (C - N(i - 1));
        x(i) = (N(i) / C) * 100;
        y(j, i-1) = ((N(i) - N(i - 1)) / (C * dt)) ;
```

```
        t(i) = t(i - 1) + dt;
    end;
    y(j, iter) = y(j,iter-1);
    plot(t, x,'lineWidth',1.2)
    hold on;
end;
title('Market share (internal influence model)');
xlabel('Time (years)');
ylabel('Market share');
ylim([0 120])


legend('q = 0.2', 'q = 0.4', 'q = 0.6', 'q = 0.8', 'q = 1');
figure;set(gca,'fontsize',13)
hold on
for j = 1 : 5
    plot(t, y(j, :),'lineWidth',1.2)
    hold on;
end;
legend('q = 0.2', 'q = 0.4', 'q = 0.6', 'q = 0.8', 'q = 1');
%ylim([0 25]);
title('New customers (internal influence model)');
xlabel('Time (years)');
ylabel('New customers (fraction)');
```

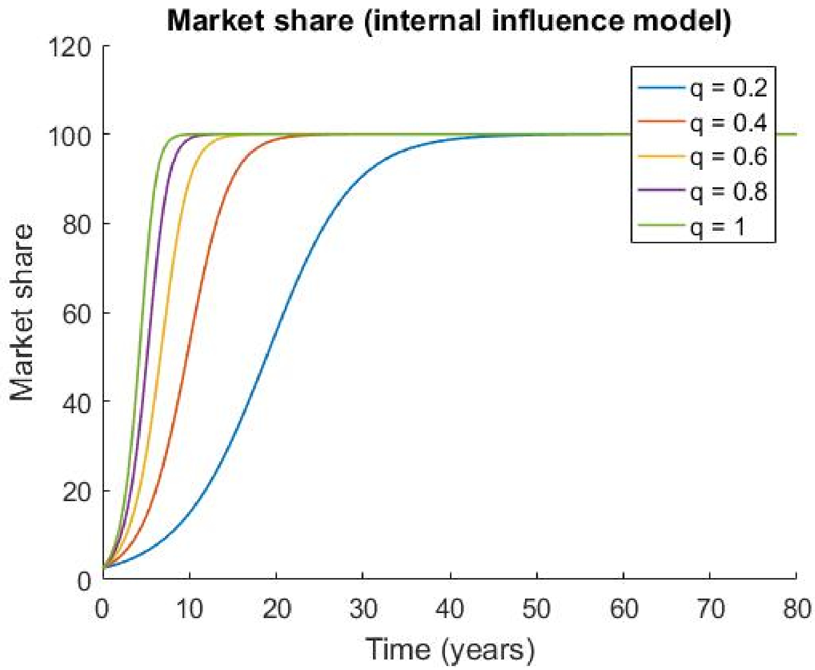

Figure 18

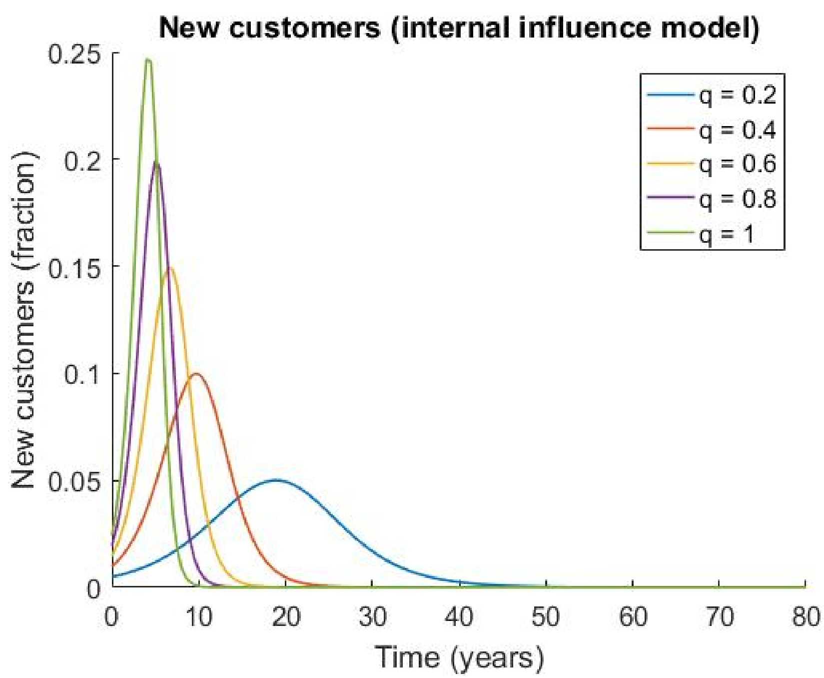

Figure 19

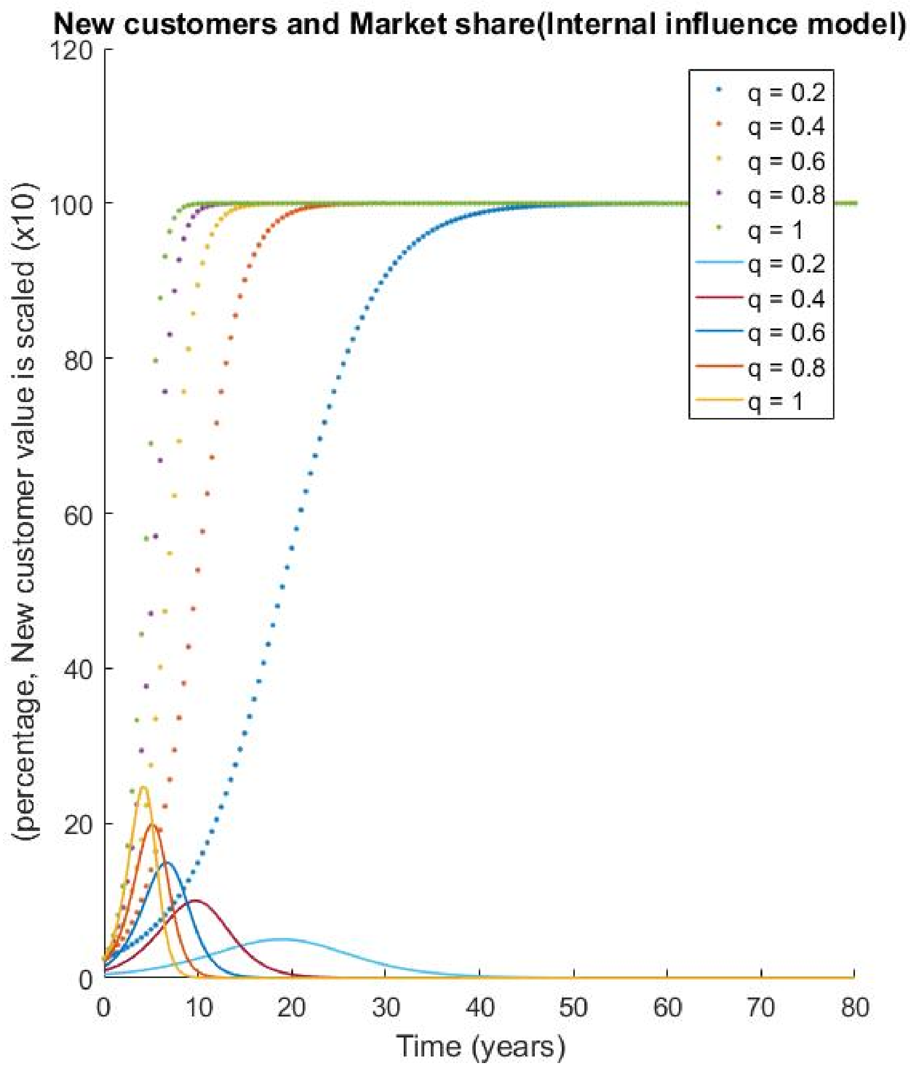

Figure 20

Part C:

Similar to (a) but consider the mixed influence model. Can you identify how the missing aspect in (b) is already accounted for in the mixed model. What processes according to you would be captured by the constants p and q. Comment on the behavior observed with a rational choice of parameters.

The constants p and q considers both the external and internal influence on the population using the technology respectively. It is a mixed model of the models in part a and b. Model in (b) failed to account for the external influences like advertisements on the people not using the technology which are captured in this model by the term $p(C-N(t))$. Keeping p constant and increasing q, the effect due to people already using technology(internal influence) increases and market-share saturates faster.

Keeping q constant and increasing p, the influence of external factors also increases hence a steep increase in number of people adopting technology can be seen and also market-share saturates faster.

Code:

```
close all;
clear all;
total = 40;
dt = 0.5;set(gca,'fontsize',13)
hold on
iter = total / dt + 1;
p = [0.02, 0.04, 0.06, 0.08, 0.1];
q = [0.2, 0.4, 0.6, 0.8, 1];
C = 100000;
y = zeros(5, iter);
for j = 1 : 5
    N = zeros(iter, 1);
    x = zeros(iter, 1);
    t = zeros(iter, 1);
    N(1) = 0;
    for i = 2 : iter
        a = p(2) + (q(j) * N(i - 1)) / C;
        N(i) = N(i - 1) + dt * a * (C - N(i - 1));
```

```
        x(i) = (N(i) / C) * 100;
        y(j, i-1) = ((N(i) - N(i - 1)) / (C * dt)) ;
        t(i) = t(i - 1) + dt;
    end;
    y(j, iter) = y(j,iter-1);
    plot(t, x,'lineWidth',1.2)
    hold on;
end;
legend('q = 0.2', 'q = 0.4', 'q = 0.6', 'q = 0.8', 'q = 1');
title('Market share, constant p,  (mixed influence model)');
xlabel('Time (years)');
ylabel('Market share');
figure;set(gca,'fontsize',13)
hold on
for j = 1 : 5
    plot(t, y(j, :),'lineWidth',1.2)
    hold on;
end;
legend('q = 0.2', 'q = 0.4', 'q = 0.6', 'q = 0.8', 'q = 1');
title('New customers, constant p,  (internal influence model)');
xlabel('Time (years)');
ylabel('New customers (fraction)');


figure;set(gca,'fontsize',13)
hold on
y = zeros(5, iter);
```

```
for j = 1 : 5
    N = zeros(iter, 1);
    x = zeros(iter, 1);
    t = zeros(iter, 1);
    N(1) = 0;
    x(1) = (N(1) / C) * 100;
    for i = 2 : iter
        a = p(j) + (q(2) * N(i - 1)) / C;
        N(i) = N(i - 1) + dt * a * (C - N(i - 1));
        x(i) = (N(i) / C) * 100;
        y(j, i-1) = ((N(i) - N(i - 1)) / (C * dt)) ;
        t(i) = t(i - 1) + dt;
    end;
    y(j, iter) = y(j,iter-1);

    plot(t, x,'lineWidth',1.2)
    hold on;
end;
legend('p = 0.02', 'p = 0.04', 'p = 0.06', 'p = 0.08', 'p = 0.1');
title('Market share, constant q, (mixed influence model)');
xlabel('Time (years)');
ylabel('Market share');
figure;set(gca,'fontsize',13)
hold on
for j = 1 : 5
    plot(t, y(j, :),'lineWidth',1.2)
    hold on;
```

```
end;
legend('p = 0.02', 'p = 0.04', 'p = 0.06', 'p = 0.08', 'p = 0.1');
title('New customers , constant q, (internal influence model)');
xlabel('Time (years)');
ylabel('New customers (fraction)');
```

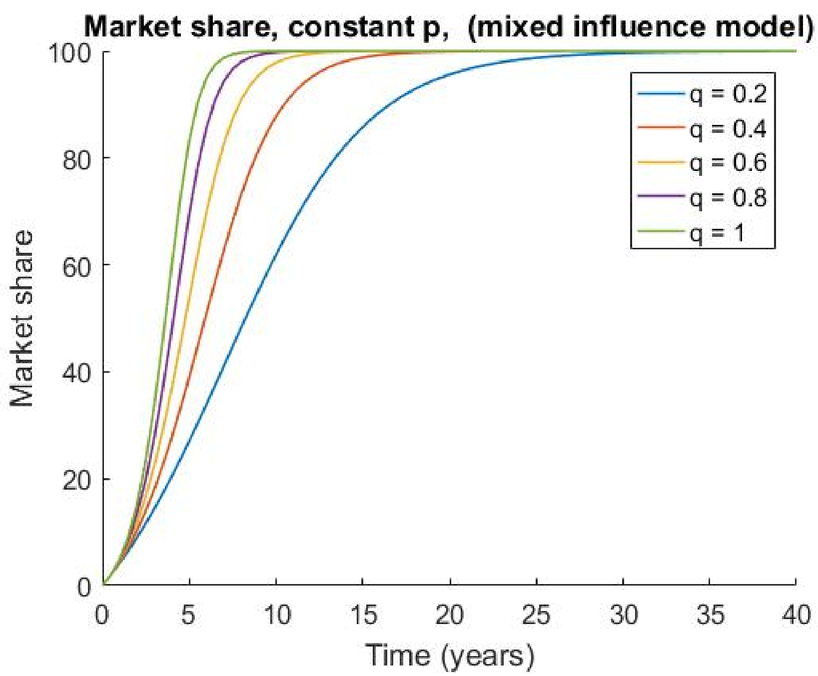

Figure 21

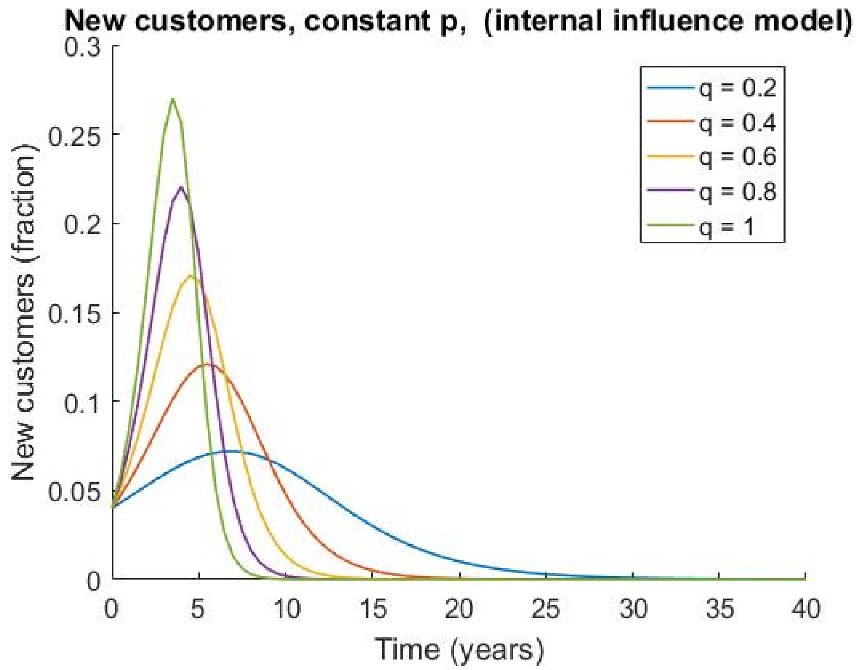

Figure 22

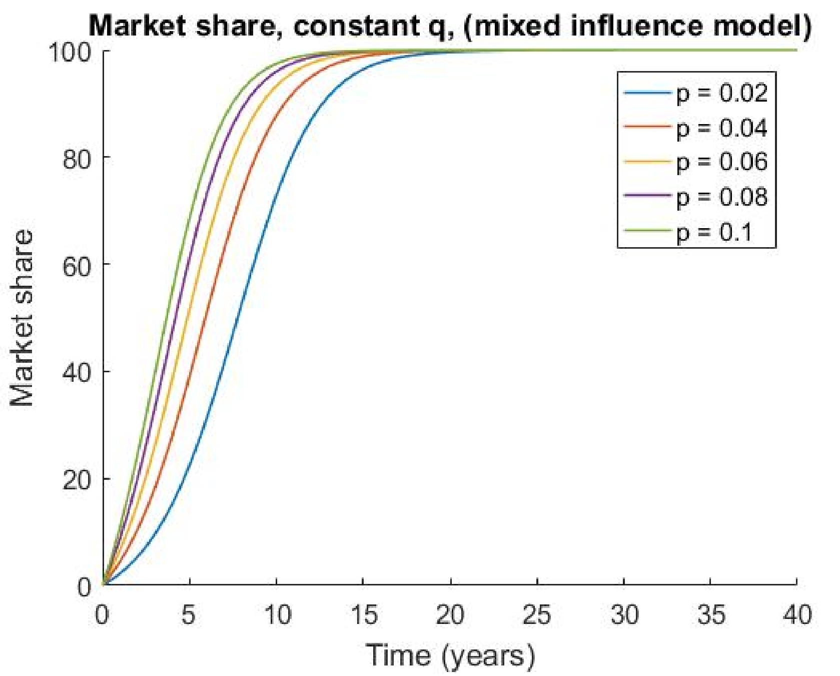

Figure 23

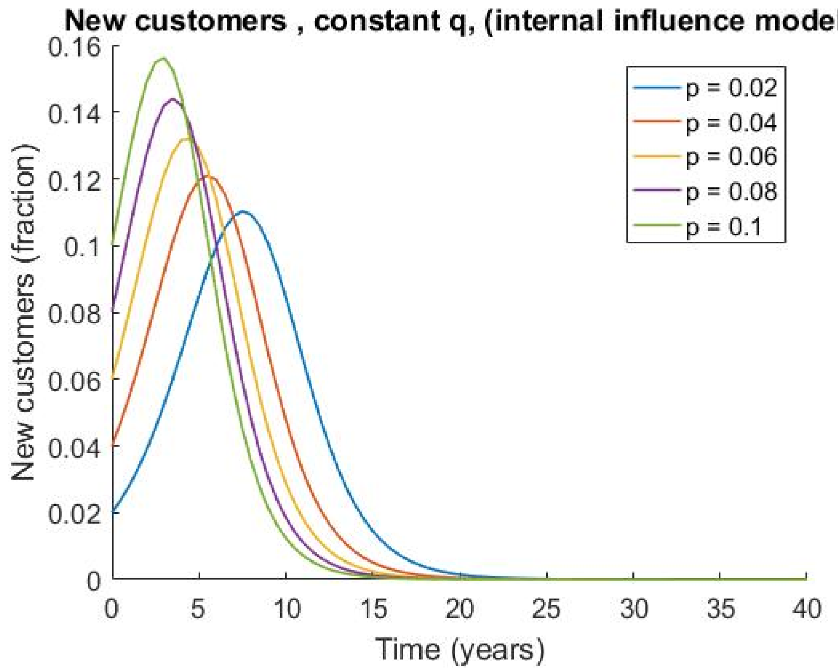

Figure 24

Problem 2:

Models of innovation diffusion:

Part A:

Model the dynamics of the carrying capacity as shown in figure given in assignment. through a differential equation. Explain the parameters used.

$$\frac{dK}{dt} = \alpha(K - k1)(1 - \frac{K}{k1 + k2})$$

where c and alpha are constants.

Alpha shows how fast the value of capacity will change. It decides how long it will take to the value of capacity to have a head start and leave the surface at K1. The change or increase (slope - gradient) in the value of K depends on K itself. As the value of K is very small, K*(1-K) is very small which results in small change in its value. Smaller we take the value of K, more time it will take to have a significant value of increase which can result to increase the value of K such that the difference is seen. We have taken alpha = 0.07 and K(0) = 2e-7 to match the figure in question.

Part B:

Which of the parameter(s) you have used in your expression when changed would change the time the carrying capacity spends at the lower value K1. Now attempt to produce a figure similar to figure. What is the initial condition and why? As alpha increases, time spent on lower value k1 decreases. **Code:**

```
close all;
clear all;
dt = 0.1;
total = 600;
iter = total / dt + 1;
t = zeros(iter, 1);
x = 0.07;set(gca,'fontsize',13)
hold on
k1 = 20;
k2 = 50;
k = zeros(iter, 1);
k(1) = 0.0000002;
for i = 2 : iter
    k(i) = k(i - 1) + x * k(i - 1) * (1 - k(i - 1) / k2) * dt;
    t(i) = t(i - 1) + dt;
end;
k = k + k1;
plot(t, k,'lineWidth',1.2)
```

```
ylim([0 100]);
xlim([0 600]);
title('time varying K');
xlabel('Time ');
ylabel('value of K');
```

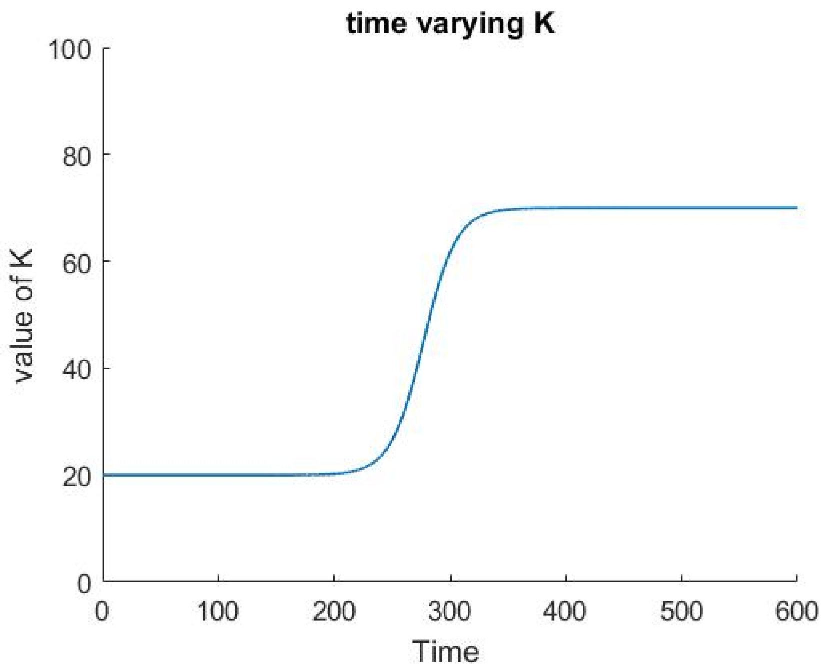

Figure 25

Part C:

Now solve the equation

$$\frac{dP}{dt} = \alpha P(1 - \frac{P}{K(t)})$$

Using reasonable values from part (b) above, what according to you should be a reasonable value of α. If you think that α can take any positive value then taking a range of values comment on the different behaviors observed. As value of alpha increases, the increase in population is faster which means it reaches carrying capacity faster. For value near to 1, it almost coincides the carrying capacity. Value of 0.05 looks reasonable here as it does not completely coincide with carrying capacity and also increase in population is not slower.

Code:

```
close all;

clear all;

dt = 0.1;

total = 700;

iter = total / dt + 1;

t = zeros(iter, 1);

x = 0.07;

k1 = 20;

k2 = 50;set(gca,'fontsize',13)

hold on

k = zeros(iter, 1);

p = zeros(iter, 1);

alpha = [0.005, 0.01, 0.05, 0.1, 0.5, 1];

k(1) = 0.0000002;

for i = 2 : iter

    k(i) = k(i - 1) + x * k(i - 1) * (1 - k(i - 1) / k2) * dt;

    t(i) = t(i - 1) + dt;

end;

k = k + k1;

for j = 1 : 6

    p(1) = 10;

    for i = 2 : iter

        p(i) = p(i - 1) + dt * alpha(j) * p(i - 1) * (1 - p(i - 1) / k(i));

    end;
```

```
    plot(t, p,'lineWidth',1.2)
    hold on;
end;
ylim([0, 100]);
xlim([0, 700]);
legend('alpha = 0.005', 'alpha = 0.01', 'alpha = 0.05', 'alpha = 0.1', 'alpha = 0.5', 'a
title('Population (time varying K)');
xlabel('Time (years)');
ylabel('no. of people');
```

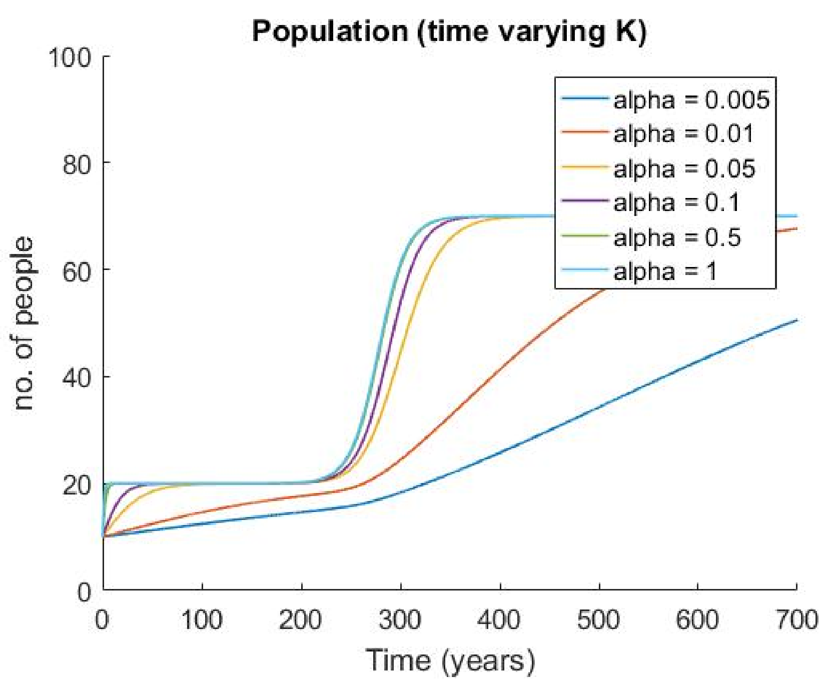

Figure 26

Part D:

Compare and contrast the current case with the situation in which the carrying capacity is a constant. Code:

```
close all;

clear all;

dt = 0.1;

total = 700;set(gca,'fontsize',13)

hold on

iter = total / dt + 1;

t = zeros(iter, 1);

x = 0.07;

alpha = [0.005, 0.01, 0.05, 0.1, 0.5, 1] ;

p = zeros(iter, 1);

for i = 2 : iter

    t(i) = t(i - 1) + dt;

end;

k = 60;

for j = 1 : 6

    p(1) = 10;

    for i = 2 : iter

        p(i) = p(i - 1) + dt * alpha(j) * p(i - 1) * (1 - p(i - 1) / k);

    end;

    plot(t, p,'lineWidth',1.2)

    hold on;

end;

ylim([0, 100]);

xlim([0, 700]);

legend('alpha = 0.005', 'alpha = 0.01', 'alpha = 0.05', 'alpha = 0.1', 'alpha = 0.5', 'a

title('Population (constant K)');

xlabel('Time (years)');
```

```
ylabel('no. of people');
```

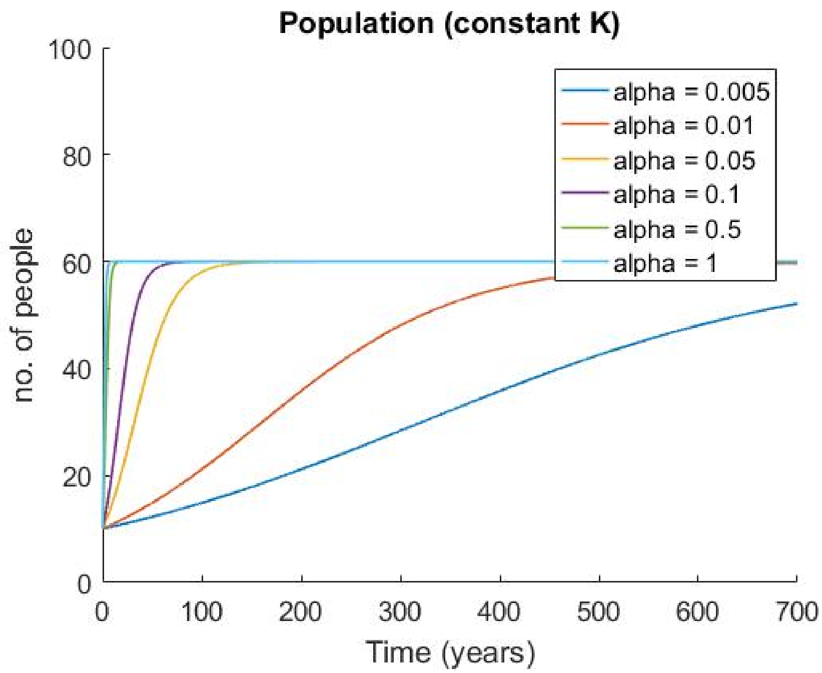

Figure 27

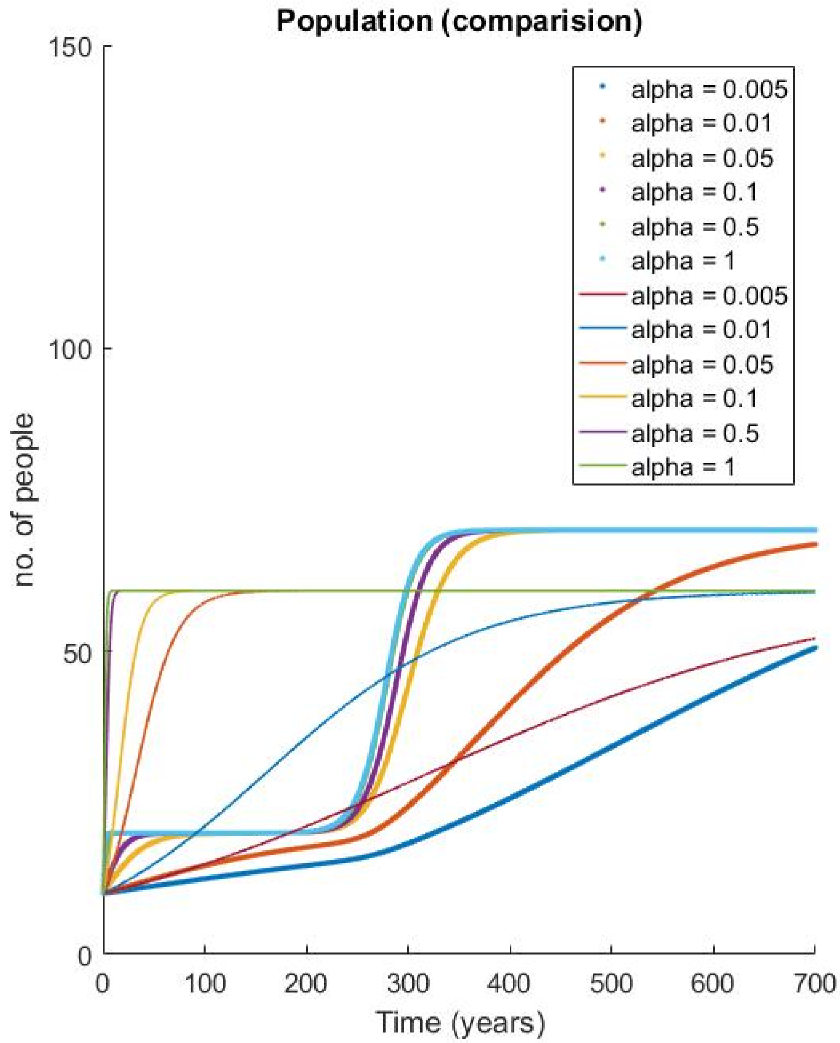

Figure 28

Here we can see that, when carrying capacity is constant the increase in population is steep. But when the carrying capacity is variable, we can see curve which is more realistic. Because it takes the technological advancement in account.

End of the Document