IT575 Computational Shape Modeling **Assignment - 2**

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1. Let $\sigma : W \subset R^2 \to U \subset R^3$ be a surface patch for any given regular embedded surface. Let $p = \sigma(u,v) \in S$ be a point in S. The patch σ can be reparameterized using the map $\phi(u,v) = (\widetilde{u},\widetilde{v}) = (f(u,v), g(u,v))$, where $\phi : W \to \widetilde{W}$ is a diffeomorphism, such that $\widetilde{\sigma}(\widetilde{u},\widetilde{v}) = \sigma(u,v)$. Figure out the relation between the first fundamental form at p in terms of σ and $\widetilde{\sigma}$.

   Answer : We did not get the whole answer but we got something that has to do with Jacobian of partial derivative of $\widetilde{u}$ and u. The matrix that we got was decomposible but we did not get any simple form for that.

2. Let $\sigma(u,v) = (u,v,f(u,v))$ be a parameterization of a surface for $(u,v) \in R^2$, where f is a degree n polynomial in the two variables u and v:

   $$f(u,v) = a^0 + a^1u + a^2v + a^3u^2 + a^4uv + a^5v^2 + ... + a^ku^n + a^{k+1}u^{n1}v + ... + a^mv^n \quad (1)$$

   The function f can be represented by the coefficient vector $a = (a^0, a^1, ..., a^m)$. Write a MATLAB function *mypolysurface.m* that takes an input vector a (of any length), a vector (u^0, v^0) specifying parameter values, and

   (a) plots the surface $(u,v,f(u,v))$ for $u \in [u_0 - 1, u_0 + 1]$, $v \in [v_0 - 1, v_0 + 1]$ as an appropriately sampled mesh (use *mesh.m* or *surf.m*)

   (b) plots the basis of the tangent space σ_u, σ_vat the point $\sigma(u_0, v_0)$, and the unit normal, (use *quiver3*),

   (c) plots the principal curvature directions scaled by the principal curvatures at $\sigma(u_0, v_0)$,

   (d) and outputs the Gaussian and Mean curvature at $\sigma(u_0, v_0)$ on the screen.

   Answer : *mypolysurface.m* is with the submission.

3. Given a surface patch $\sigma(u,v) = (u,v,f(u,v))$ where $f : R^2 \to R$ is an arbitrary smooth function, compute the

   (a) First fundamental form and unit normal to the surface at a point $\sigma(u,v)$,
   Answer : First fundamental form:

   $$F_I = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$$
   $$= \begin{bmatrix} \langle \sigma_u, \sigma_u \rangle & \langle \sigma_u, \sigma_v \rangle \\ \langle \sigma_v, \sigma_u \rangle & \langle \sigma_v, \sigma_v \rangle \end{bmatrix}$$
   $$\sigma_u = \left(1, 0, \frac{\partial f(u,v)}{\partial u}\right)$$

$\sigma_v = \left(0, 1, \frac{\partial f(u,v)}{\partial v}\right)$

$\langle \sigma_u, \sigma_u \rangle = 1 + \left(\frac{\partial f(u,v)}{\partial u}\right)_{(u,v)}$

$\langle \sigma_v, \sigma_v \rangle = 1 + \left(\frac{\partial f(u,v)}{\partial v}\right)_{(u,v)}$

$\langle \sigma_u, \sigma_v \rangle = \left(\frac{\partial f(u,v)}{\partial u}\right)_{(u,v)} \times \left(\frac{\partial f(u,v)}{\partial v}\right)_{(u,v)}$

Now, $N = \left(-\frac{\partial f(u,v)}{\partial u}, \frac{\partial f(u,v)}{\partial v}, 1\right)$

$|N| = \left(\left(\frac{\partial f(u,v)}{\partial u}\right)^2 + \left(\frac{\partial f(u,v)}{\partial v}\right)^2 + 1\right)^{0.5}$

(b) The Second fundamental form and principal curvatures κ_1 ,κ_2 at a point $\sigma(u,v)$.

Answer :

$\sigma_{uv} = \left(0, 0, \frac{\partial f(u,v)}{\partial u \partial v}\right)$

$\sigma_{vv} = \left(0, 0, \frac{\partial f(u,v)}{\partial v^2}\right)$

$\sigma_{uu} = \left(0, 0, \frac{\partial f(u,v)}{\partial u^2}\right)$

$$F_{II} = \begin{bmatrix} L & M \\ M & N \end{bmatrix}$$
$$= \begin{bmatrix} \langle \sigma_{uu}, N \rangle & \langle \sigma_{uv}, N \rangle \\ \langle \sigma_{vu}, N \rangle & \langle \sigma_{vv}, N \rangle \end{bmatrix}$$
$$= \begin{bmatrix} \frac{\partial f(u,v)}{\partial u^2 . |N|} & \frac{\partial f(u,v)}{\partial uv . |N|} \\ \frac{\partial f(u,v)}{\partial uv . |N|} & \frac{\partial f(u,v)}{\partial v^2 . |N|} \end{bmatrix}$$

$S = F_I^{-1} F_{II}$

(c) Plot examples of surfaces (using Question 2) with points where:

$\sigma(u,v) = (u, v, f(u,v))$. **surface will be determined by** $f(u,v)$. **So changing the polynomial in following ways.**

Points to match the following conditions are the points where tangents and principal curves are drawn in the plot.

i. $\kappa_1 > 0, \kappa_2 > 0$.
Answer : This case happens if for both principal curvatures are positive which means both directions(u,v) z coordinate of the surface(for the given $(u, v, f(u, v))$ case) increases with increase in u and v. The following curve has $k1$ and $k2 > 0$, which is of a simple curve when $f(x, y) = u + u^2 + v + v^2$.

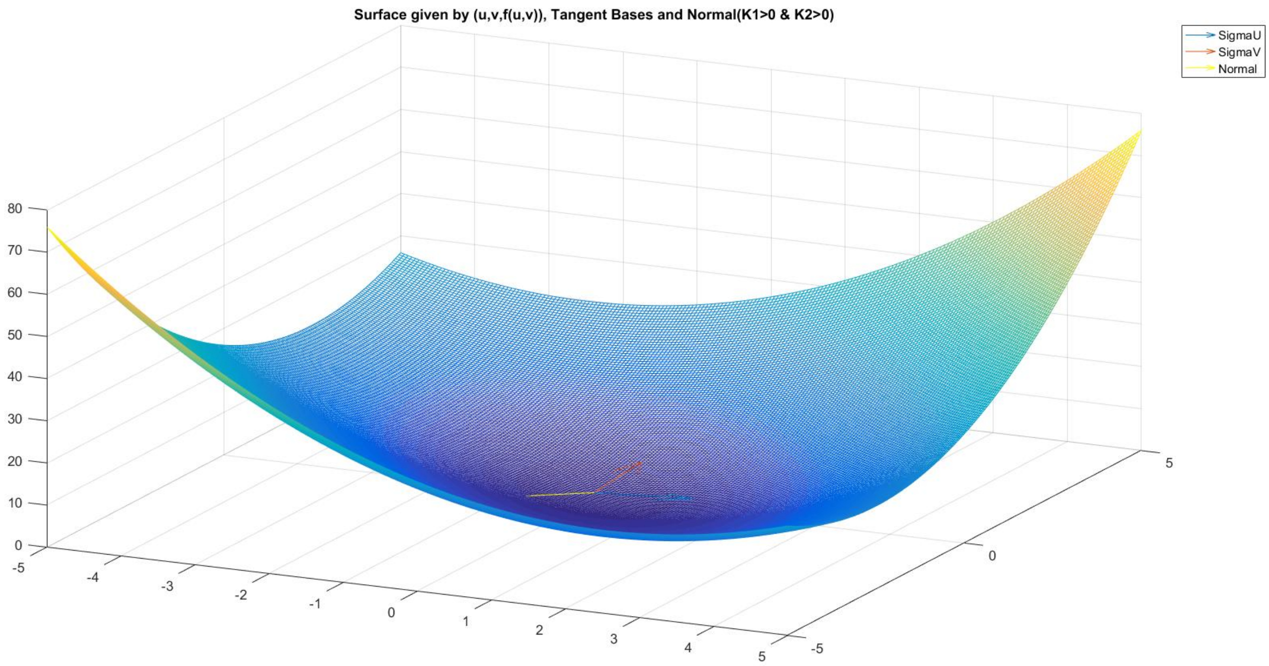

Figure 1: **K1> 0 and K2>0**

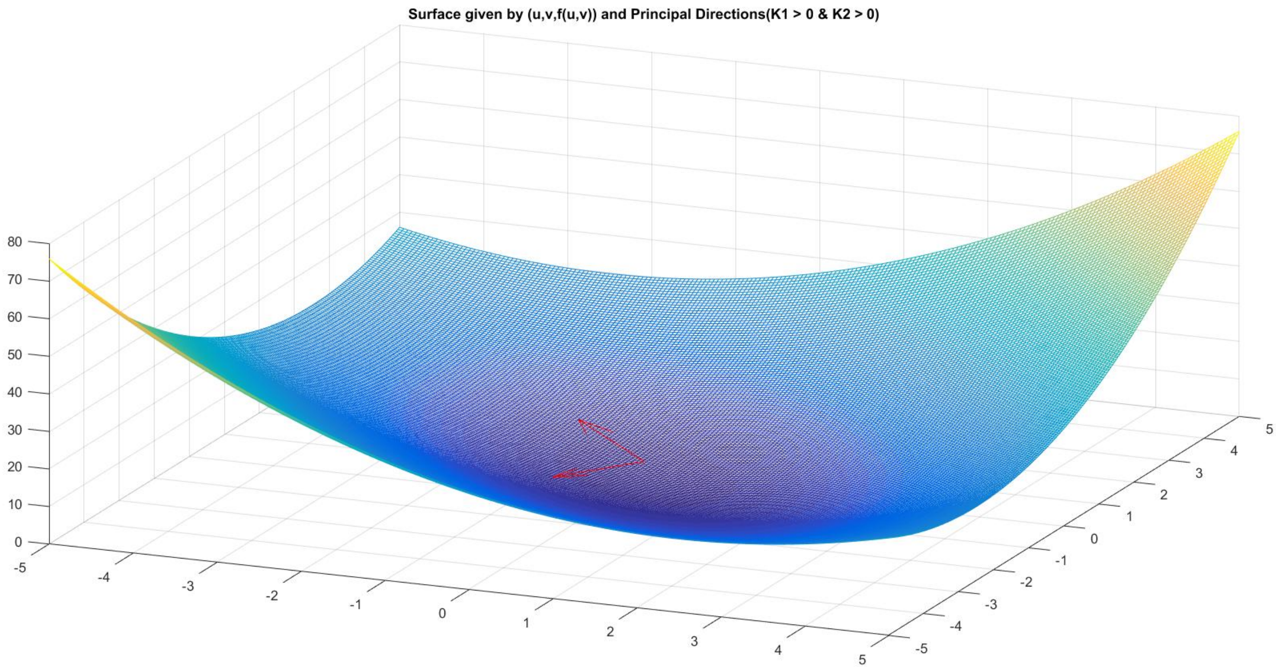

Figure 2: **K1> 0 and K2>0**

ii. $\kappa_1 < 0$, $\kappa_2 > 0$.

Answer : $k1 > 0$, $k2 < 0$, this happens when both principal directions are opposite, which means value of z coordinate shows different behaviour for increase u,v. A simple curve u$^4 v^3 typecurve(termhavingodd, evenpowersdominate) whentakenapointkindof$(-a,-b),a,b>0.$Followingcurveisgeneratedbydominanceofterm$v$^5 u^4$.

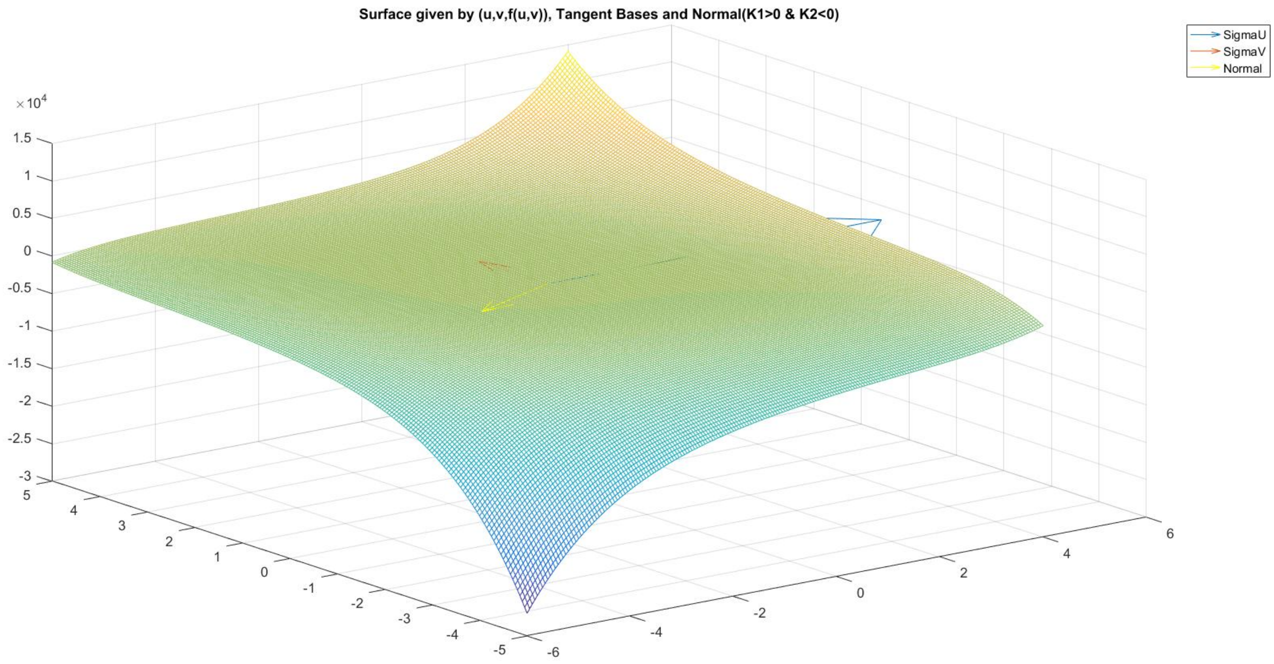

Figure 3: **K1> 0 and K2<0**

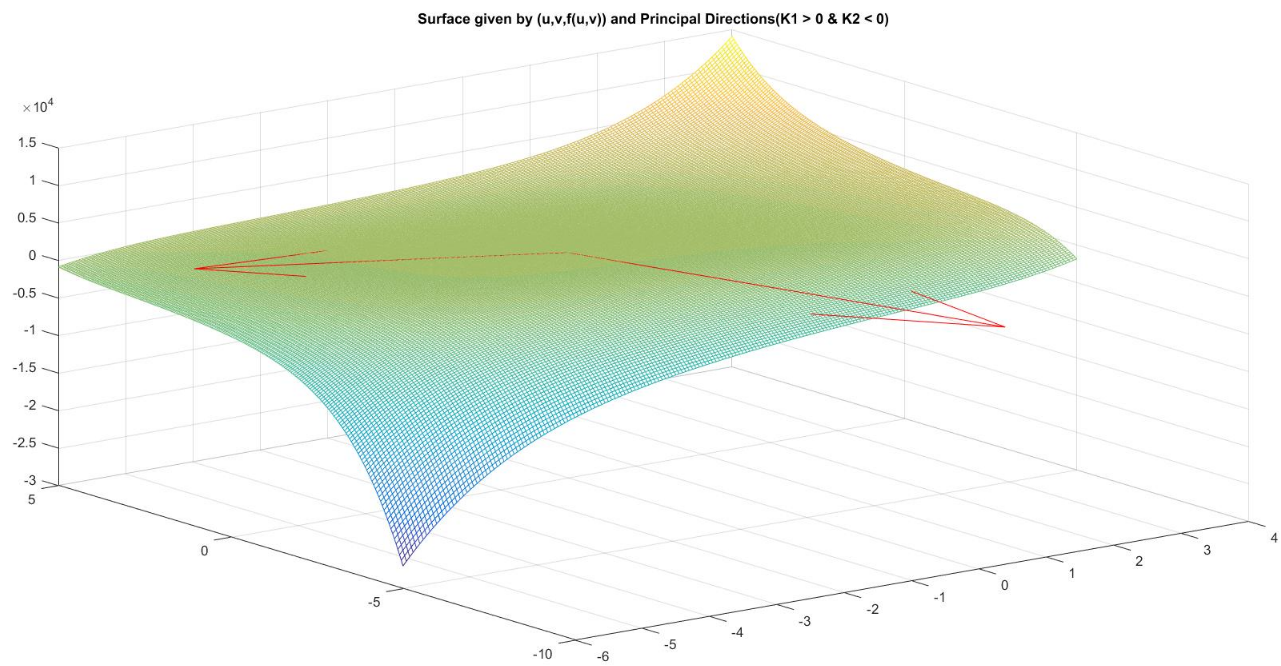

Figure 4: **K1> 0 and K2<0**

iii. $\kappa_1 > 0$, $\kappa_2 = 0$.

Answer : This happens when one principal direction is zero which simply means $f(u,v)$ doesn't depend on one variable. Curve v^2 is shown here, which exhibits asked behaviour.

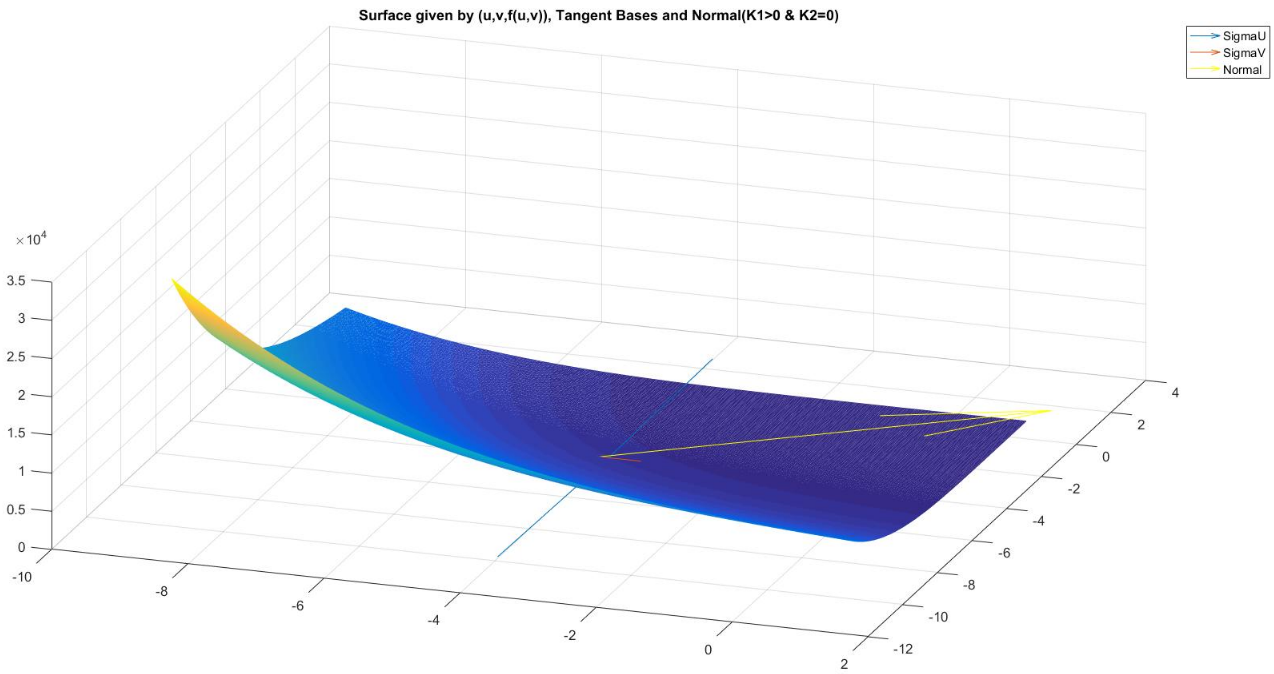

Figure 5: **K1> 0 and K2=0**

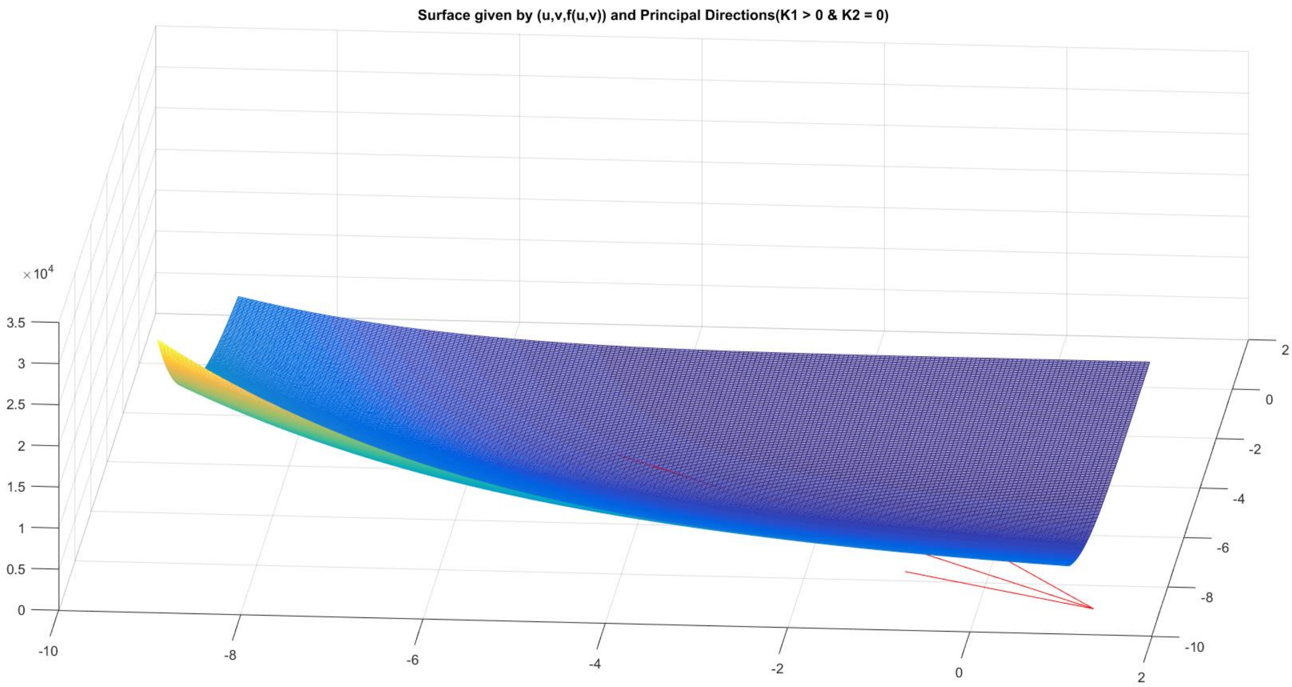

Figure 6: **K1> 0 and K2=0**

iv. $\kappa_1 = \kappa_2 = 0$, but the surface is not a flat plane.

Answer : This happens when a surface similar to planer surface at the given point and changes curvature values for other points(Not planer). Curves like f(u,v) = $v - v^2$ exhibit these kind of property but momentarily with values not exactly zero, but below 0.5.