IT575 - Computational Shape Modeling
Assignment 1: Procrustes Analysis

1. Show that the full Procrustes distance d_F is a metric on the shape space. Assume that $d([p],[q]) = \cos^{-1}(|\langle p_c, q_c \rangle|)$, where p_c, q_c denotes the representation of any landmark configuration in the equivalence class $[p],[q]$ respectively as a vector in $\mathbb{C}^n$, is a valid metric on the shape space.

2. Write a MATLAB function `myProcrustesAlign.m`, to be used as `pa = myProcrustesAlign(p,q)` that will align the set of landmarks p to landmarks in q. The inputs p, q and output pa should be $k \times 2$ matrices, where k is the number of landmarks.

3. Write a MATLAB function `myProcrustesMean.m`, to be used as `mnshp = myProcrustesMean(p,thr)` that outputs the mean shape as a $k \times 2$ matrix, with the input `p` being a $k \times 2n$ matrix for n objects represented as k landmarks. The x and y coordinates will be assumed to be put in alternating columns of matrix p. The mean shape should be computed using the iterative algorithm mentioned in class, wherein the iterations should stop once the norm of difference of successive mean estimates is below the threshold given as an input in the variable `thr`.

4. The shape of coke bottles has gone through a few changes over the years. You will find some representative images of these bottles in the folder `bottles`. For marking the landmark points, use the MATLAB function `getpointsASM.m`. Type `help getpointsASM` to see its usage and related information. Use `imread.m` (available in MATLAB) for reading image files.

   (a) Justify the number of landmarks you will use to compute shape related information for these bottles. Compute and plot the mean shape of the coke bottle using `myProcrustesMean.m`.

   (b) Compute the shape covariance matrix and plot the eigenvalue profile. Find out the number of eigenvectors required to summarize 95% variance of the entire dataset.

   (c) Let v_1 and v_2 be the first two *principal components* (eigenvectors). Plot variations in the mean shape along v_1 and v_2, i.e, $\mu + k\Delta v_i$, where μ is the mean shape, $k \in [-2,-1,0,1,2]$, $i = 1,2$, and Δ is an appropriate scalar.

5. Repeat the above exercise for car shape using images provided in the folder `cars`. If you think that only the input changes from bottle landmarks to car landmarks, you are missing something.

Submission Instructions

1. Please do not paste your codes in the report.
2. Submissions will not be accepted if the report is not written in LaTeX. Use the template provided on the course web page.
3. Put the files `YourID_report.pdf`, `myProcrustesAlign.m`, `myProcrustesMean.m` in a folder `YourID_asg1`, and submit the folder as a zip file on the course web page.
4. You are allowed to work out the assignment in a group of at most two. The group should remain the same throughout the course. Make one submission per group. Please mention the ID nos. and names of students in the group.
5. The submission deadline is Thursday 19^{th} January, 18:00 hrs.