**PHYS265 Lab 02 Report**

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*Introduction*

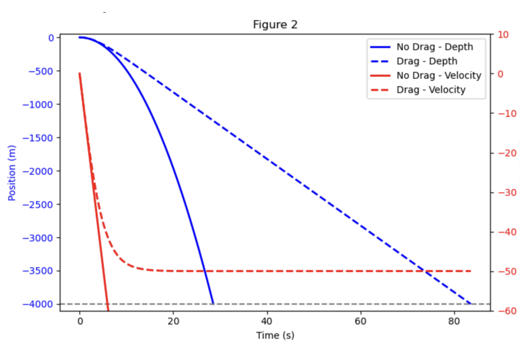
This report explores the feasibility of measuring a deep vertical mine's depth by dropping a test mass and timing its fall. Due to the shaft's extreme depth (4 km), simple free-fall assumptions break down: the effects of air drag, height-dependent gravity, and even Earth's rotation (Coriolis force) become significant. We systematically extend the equations of motion to address these factors, then compare numerical results to analytic estimates. Finally, we apply the same framework to hypothetical shafts that traverse Earth entirely and to a lunar mine, highlighting how variations in planetary density influence fall times.

*Calculation of Fall Time*

To measure the shaft depth, a 1 kg test mass is dropped from rest at the Earth's surface (y = 0 km). Three scenarios were considered to calculate fall times:

1. Constant Gravity, No Drag: Using a constant gravitational acceleration $g=9.81\ m/s^2$ and no air resistance ($\alpha=0$), the test mass takes 28.6 seconds to reach the bottom of the 4 km shaft. This matches precisely the analytical free-fall calculation.

2. Variable Gravity, No Drag: When gravitational acceleration varies linearly with depth ($g(r)=g0*r/R$), the numerical solution yielded a fall time of 28.6 seconds, nearly identical to the constant gravity case. The discrepancy (approximately 0.0015 seconds) is negligible given the shallow shaft depth compared to Earth's radius.

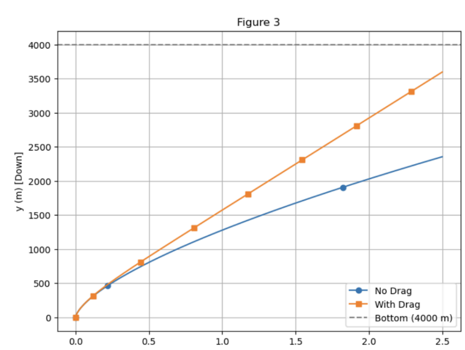



3. Variable Gravity with Drag: Introducing drag (quadratic speed dependence, $\gamma=2$), with a calibrated drag coefficient ($\alpha=0.004$), drastically increased fall time to 84.3 seconds, a 194.8% increase relative to the no-drag scenario. This substantial difference is due to the test mass reaching a terminal velocity of approximately 50 m/s, significantly slowing descent.

The effect of drag is clearly demonstrated in Figure 2, showing how velocity plateaus at terminal velocity, greatly extending the fall duration.

*Depth Measurement Feasibility*

The Earth's rotation introduces a significant Coriolis force on a mass dropped within a vertical shaft, notably affecting horizontal displacement. At the equator, Earth's angular velocity ($\Omega=7.272\times10^{-5}$ rad/s) generates horizontal accelerations proportional to the vertical velocity, resulting in substantial sideward drift.




This horizontal motion is modeled by the Coriolis acceleration:

$a_x=2\Omega v_y$, $a_y=g_0-2\Omega v_x$

With no drag (α=0), the mass collides with the shaft wall after 21.9 s, reaching a depth of 2353.9 m (59% of total shaft depth). Including drag (α=0.004), the collision occurs at 29.7 s, significantly shallower (1296.6 m, 32% depth). This difference arises as drag slows vertical descent, increasing exposure to horizontal acceleration, exacerbating lateral displacement.

Given a shaft width of only 5 m (±2.5 m radius), these results illustrate that the test mass will inevitably strike the shaft walls well before reaching the intended 4 km depth. Consequently, accurately measuring shaft depth by timing vertical descent is impractical without greatly increasing shaft dimensions or implementing additional stabilization methods. This is shown above in Figure 3.

### *IV. Calculation of Crossing Times for Homogeneous and Non-homogeneous Earth*

For an infinitely deep, frictionless tunnel through Earth's diameter, crossing times were calculated considering variations in Earth's density distribution, modeled by:

$$\rho(r) = \rho_n \left(1 - \frac{r^2}{R_\oplus^2}\right)^n$$

with n=0 (homogeneous Earth) and n=9 (highly non-uniform density).

Homogeneous Earth (n=0):

- Crossing time: 1267.34 s

Highly Non-homogeneous Earth (n=9):

- Crossing time: 943.77 s

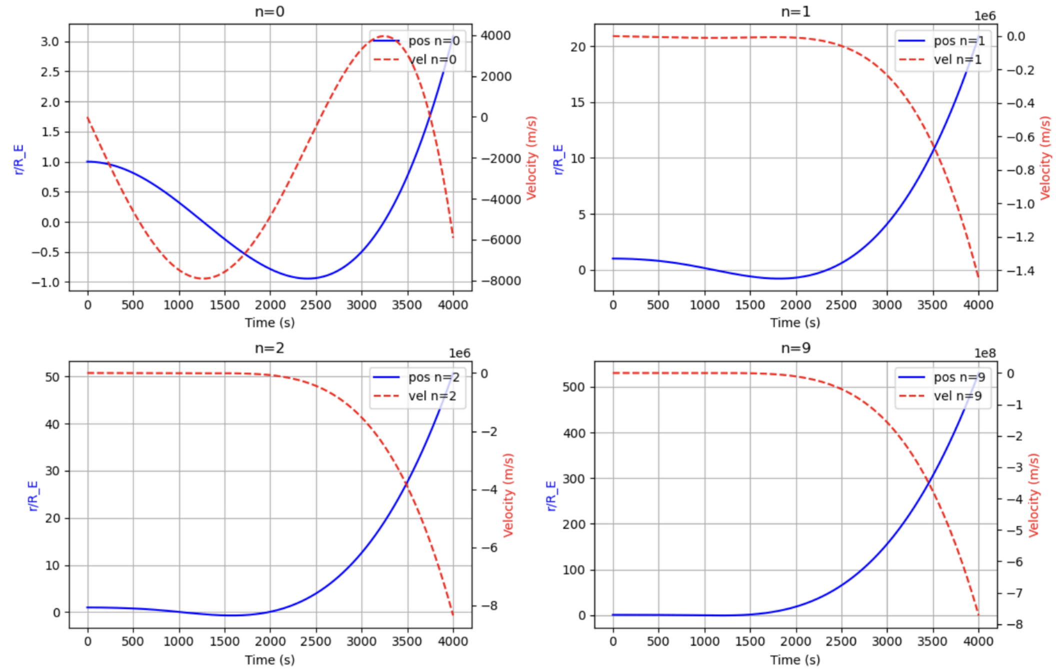
The reduced crossing time with increased central density (n=9) is due to the stronger gravitational acceleration near Earth's core, causing higher peak velocities and shorter transit duration. This emphasizes that planetary density distribution critically impacts traversal times.

Repeating this analysis for the Moon (assuming uniform density) yields a significantly different crossing time due to its lower mass and smaller radius.

The substantial difference arises primarily due to density disparities. Earth's calculated uniform density (approx. 5494.9 kg/m³) results in shorter crossing times relative to the Moon’s lower density (approx. 3341.8 kg/m³). Specifically, the crossing time scales inversely with the square root of planetary density: $T_{earth}/T_{moon}$ = sqrt(pmoon/pearth)

Numerical validation confirms this density dependence precisely, meaning there is an importance of density considerations when evaluating transit times through planetary bodies.

Figure 7




Position and velocity of a mass traversing Earth's diameter for density profiles n=0,1,2,9. Higher central density (larger n) significantly reduces crossing time by increasing gravitational acceleration and thus peak velocity near Earth's center. There is a drastically shorter crossing duration for n=9 compared to homogeneous density (n=0), emphasizing the critical role of internal density structure on traversal dynamics.

## *V. Discussion and Future Work*

The analysis shows that measuring shaft depth by timing a test mass's free-fall is not feasible given current shaft dimensions. Coriolis forces due to Earth's rotation introduce significant horizontal displacement, causing the mass to collide with the shaft walls well before reaching the intended depth. Including drag forces amplifies this effect, resulting in earlier and shallower wall collisions. Thus, without substantially widening the shaft or providing additional stabilization methods, this measurement technique cannot be recommended.

Several simplifying assumptions impact the accuracy of the presented calculations. The Earth was modeled as a perfect sphere with uniform rotation and simplified density distribution, neglecting its actual oblate shape and geological complexities. The drag coefficient (\(\alpha=0.004\)) was assumed constant, overlooking altitude-dependent variations in air pressure and the object's geometry. Additionally, calculations assumed precise equatorial or polar positioning, an idealization rarely achievable in practice.

Future work should incorporate more accurate gravitational models reflecting Earth's true shape and internal density variations, alongside altitude- and geometry-dependent drag models. Given the identified limitations, alternative depth-measurement techniques should be explored to develop practical solutions unaffected by these significant constraints.