ANALYSIS FOR APOLLO 11 MISSION

I. Introduction

This report provides an introduction to the fundamental physics behind the Apollo program, offering insights into the gravitational forces, energy fields, and propulsion principles that made a Moon landing possible. To successfully navigate space, engineers needed to understand how the Earth and Moon exert gravitational influence on spacecraft and how rockets generate the necessary thrust to escape Earth's gravity.

To support NASA Director Gene Kranz in securing funding, this report presents an analysis of gravitational potential fields, force interactions between celestial bodies, and the predicted performance of the Saturn V rocket. By breaking down these complex topics, this report aims to make the mission's technical feasibility clear to both technical experts and policymakers in Congress.

II. The Gravitational Potential of the Earth-Moon System

The gravitational potential describes the potential energy per unit mass at any point in a gravitational field and is given by:

$$\Phi(r) = -\frac{GM}{r}$$

where G is the gravitational constant, M is the mass of the body creating the gravitational field, and R is the distance from the center of the mass.

For this analysis, a Python function was implemented to calculate the gravitational potential at any evaluation point given the mass and position of the Earth (approximated as a point mass) to avoid singularities at the Earth's center.

A 1D plot was generated showing how the gravitational potential magnitude decreases logarithmically as the distance from the Earth's surface increases, extending out to 1.5 times the Earth-Moon distance.

Additionally, a 2D color-mesh plot visualizing the Earth's gravitational potential was created. This plot shows the distribution of gravitational potential in two-dimensional space around Earth, clearly demonstrating how potential decreases rapidly with increasing distance and illustrating Earth's gravitational influence.

III. The Gravitational Force of the Earth-Moon System

The gravitational force field represents the direction and magnitude of the acceleration a spacecraft will experience at various points in the Earth-Moon system. The analysis produced a force vector field visualization, which clearly demonstrates gravitational influences from both Earth and Moon. Near Earth, gravitational force vectors point radially inward toward Earth's center, whereas in the region between Earth and Moon, the Moon's gravitational influence becomes significant. The force magnitude, depicted through a color gradient, ranges approximately from 16 to 19 Newtons per kilogram near Earth and decreases with distance, consistent with the inverse-square law.

Although not explicitly said, the visualization implies the existence of critical equilibrium points known as Lagrange points. These points mark regions where gravitational forces from the two bodies balance out. It is pretty important to understand this as this will help plan precise navigation paths for Apollo 11, ensuring accurate launch, lunar orbit insertion, and safe return trajectories.

IV. Projected Performance of the Saturn V Stage 1

To evaluate the performance of the Saturn V rocket's first stage, we used the Tsiolkovsky rocket equation. Numerical integration was applied to determine the altitude reached at the burnout time.

Our calculation yielded a burn time of 126.92 seconds, which is shorter than the observed test data of approximately 160 seconds. Additionally, the calculated altitude at burnout was found to be 64.55 km, compared to the test data altitude of 70 km. These differences suggest that our simplified model, which did not include atmospheric drag or variations in gravity and thrust during ascent, underestimates the rocket's actual performance.

V. Discussion and Future Work

Our current model approximates performance reasonably well but displays noticeable discrepancies compared to NASA's recent prototype testing data. Specifically, our calculated burn time ($T \approx 127$ s) underestimates the actual burn duration (160 s) by roughly 21%, and our predicted altitude ($h \approx 64.5$ km) is approximately 8% below the observed altitude of 70 km. These deviations primarily arise from neglecting atmospheric drag, assuming constant gravitational acceleration, and modeling thrust as constant.

Neglecting atmospheric drag likely caused an altitude overestimation, as ignoring resistance artificially boosts predicted ascent rates. Conversely, the constant thrust assumption, not accounting for real-world thrust variations (due to fuel depletion and engine efficiency loss), likely contributed to underestimating the actual burn duration and altitude. Future work should address these simplifications by incorporating detailed altitude-dependent aerodynamic drag modeling, variable gravitational acceleration models, and realistic engine performance data for dynamic thrust analysis. Integrating these enhancements will yield significantly more accurate predictions, ensuring reliable mission planning and robust justification for continued NASA funding.