Discrete Mathematics (BCSC 1010)

Module 2: Assignment-1

Q1. Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

Q2. In [Sm78] Smullyan posed many puzzles about an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people A and B. What are A and B if A says "B is a knight" and B says "The two of us are opposite types?"

Q3. A father tells his two children, a boy and a girl, to play in their backyard without getting dirty. However, while playing, both children get mud on their foreheads. When the children stop playing, the father says "At least one of you has a muddy forehead," and then asks the children to answer "Yes" or "No" to the question: "Do you know whether you have a muddy forehead?" The father asks this question twice. What will the children answer each time this question is asked, assuming that a child can see whether his or her sibling has a muddy forehead, but cannot see his or her own forehead? Assume that both children are honest and that the children answer each question simultaneously.

Q4. Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science."

Q5. Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letters. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter?

Q6. Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.). Justify your answer in detail.

Q7. Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

Q8. Let P(x) be the statement "x can speak Russian" and let Q(x) be the statement "x knows the computer language C++." Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

b) There is a student at your school who can speak Russian but who doesn't know C++.

c) Every student at your school either can speak Russian or knows C++.

d) No student at your school can speak Russian or knows C++.

Q9. What rule of inference is used in each of these arguments?

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn.

Q 10. 6. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

Q11. Let $S = N \times N$, the set of ordered pairs of positive integers, with the operation $*$ defined by

$$(a, b) * (c, d) = (a + c, b + d)$$

(a) Find $(2, 5) * (6, 8)$ and $(4, 9) * (2, 8)$.

(b) Show that $*$ is associative. (Hence that S is a semigroup.)

Q12. The set $Zm = \{0, 1, 2, \ldots, m - 1\}$ under the operation of addition and multiplication modulo m is a ring; it is called the ring of integers modulo m. If m is a prime, then Zm is a field. On the other hand, if m is not a prime then Zm has zero divisors. Explain with example.

Q13. Suppose that there are m men and n women on an island. Each person has a list of members of the opposite gender acceptable as a spouse. We construct a bipartite graph $G = (V1, V2)$ where V1 is the set of men and V2 is the set of women so that there is an edge between a man and a woman if they find each other acceptable as a spouse. A matching in this graph consists of a set of edges, where each pair of endpoints of an edge is a husband-wife pair. A maximum matching is a largest possible set of married couples, and a complete matching of V1 is a set of married couples where every man is married, but possibly not all women. Explain.

Q14. Suppose that there are five young women and five young men on an island. Each man is willing to marry some of the women on the island and each woman is willing to marry any man who is willing to marry her. Suppose that Sandeep is willing to marry Tina and Vandana; Barry is willing to marry Tina, Xia, and Uma; Teja is willing to marry Tina and Zelda; Anil is willing to marry Vandana and Zelda; and Emilio is willing to marry Tina and Zelda. Use Hall's theorem to show there is no matching of the young men and young women on the island such that each young man is matched with a young woman he is willing to marry.

Q15. Form a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry (using alphabetical order).

Q16. Consider the positive integers N. Then $(N, +)$ and $(N, \times)$ are semigroups since addition and multiplication on N are associative. In particular, $(N, \times)$ is a monoid since it has the identity element 1. However, $(N, +)$ is not a monoid. Justify.

Q17. Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

Q18. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material. What rule of inference is used in each of this argument?

Q19. Freedonia has fifty senators. Each senator is either honest or corrupt. Suppose you knowthat at least one of the Freedonian senators is honest and that, given any two Freedonian senators, at least one is corrupt. Based on these facts, can you determine how many Freedonian senators are honest and how many are corrupt? If so, what is the answer?

Q20. A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.