Ans 1:

Ans 1> 1a)

Let the polynomial be expressed as

$$a_0 + a_1x + a_2x^2 + a_3x^3$$

Now simply substituting the points & finding coefficients

$$a_0 + a_1(-2) + 4a_2 - 8a_3 = 15$$

$$a_0 = -1$$

$$a_0 + a_1 + a_2 + a_3 = 0$$

$$a_0 + 3a_1 + 9a_2 + 27a_3 = -2$$

Now solving these eqn's to get the coefficients

$$-2a_1 + 4a_2 - 8a_3 = 16$$

$\boxed{-a_1 + 2a_2 - 4a_3 = 8}$ $\boxed{a_1 + a_2 + a_3 = 1}$

$\boxed{3a_1 + 9a_2 + 27a_3 = -1}$

$3a_2 - 3a_3 = 9$ $\boxed{a_2 - a_3 = 3}$

$$-3a_1 + 6a_2 - 12a_3 = 24$$
$$3a_1 + 9a_2 + 27a_3 = -1$$
$$15a_2 + 25a_3 = 23$$

$$15(3 + a_3) + 25a_3 = 23$$
$$45 + 15a_3 + 25a_3 = 23$$
$$40a_3 = -22$$

$\boxed{a_0 = -1}$ $\boxed{a_1 = -\frac{8}{15}}$ $\boxed{a_2 = \frac{34}{15}}$ $\boxed{a_3 = -\frac{11}{15}}$

$$\boxed{p(x) = -1 - \frac{8}{16}x + \frac{34}{15}x^2 - \frac{11}{15}x^3} = \frac{-15 - 8x + 34x^2 - 11x^3}{15}$$

$x \to$	-2	0	1	3
$y \to$	15	-1	0	-2

$$\cancel{p(x) = 15\left[\frac{(x-0)(x-1)(x-3)}{(15-0)(15-1)(15-3)}\right] + (-1)\left[\frac{(x+2)(x-1)(x-3)}{(-1+2)(-1-1)(-1-3)}\right]}$$

$$\cancel{+\,0 + (-2)\left[\frac{(x+2)(x-0)(x-1)}{\;}\right]}$$

$$p(x) = 15\left[\frac{(x-0)(x-1)(x-3)}{(-2-0)(-2-1)(-2-3)}\right] + (-1)\left[\frac{(x+2)(x-1)(x-3)}{(0+2)(0-1)(0-3)}\right]$$

$$+\,0 + (-2)\left[\frac{(x+2)(x-0)(x-1)}{(3+2)(3-0)(3-1)}\right]$$

$-2 \times -3 \times -5$

$2 \times -1 \times -3$

$5 \times 3 \times 2$

$$p(x) = 15 \cdot \frac{x(x-1)(x-3)}{-30} - 1 \cdot \frac{(x+2)(x-1)(x-3)}{6}$$

$$-2 \cdot \frac{(x+2)(x)(x-1)}{30}$$

$$\Rightarrow -\frac{x(x-1)(x-3)}{2} - \frac{(x+2)(x-1)(x-3)}{6} - \frac{x(x-1)(x+2)}{15}$$

(c)

x	$f(x)$	1st	2nd	3rd
-2	$15 \to b_1$			
		$\frac{-1-15}{0+2} \Rightarrow -8 \to b_2$		
0	-1		$\frac{1+8}{1+2} = 3 \to b_3$	
		$\frac{0+1}{1-0} \Rightarrow 1$		$\frac{-\frac{2}{3}-3}{3+2} \Rightarrow \frac{-11}{15} \to b_4$
1	0		$\frac{-1-1}{3-0} \Rightarrow -\frac{2}{3}$	
		$\frac{-2-0}{3-1} \Rightarrow -1$		
3	-2			

$$p(x) = b_1 + b_2(x-x_1) + b_3(x-x_1)(x-x_2) + b_4(x-x_1)(x-x_2)(x-x_3)$$

$$p(x) = 15 - 8(x+2) + 3(x+2)(x-0) - \frac{11}{15}(x+2)(x-0)(x-1)$$

$$p(x) = 15 - 8(x+2) + 3(x+2)(x) - \frac{11}{15}(x+2)(x)(x-1)$$

(d) Simplifying polynomials in each case turn to

$$\boxed{p(x) = \frac{-15 - 8x + 34x^2 - 11x^3}{15}}$$

Ans 2:

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(a) $\int_0^1 \frac{1}{1+x^2}\,dx = \frac{\pi}{4}$

Midpoint Quadrature:

$$\Rightarrow \int_a^b f(t)\,dt \approx (b-a)\, f\left(\frac{a+b}{2}\right) \qquad f(t) = \frac{1}{1+t^2} \qquad \frac{a+b}{2} = \frac{0+1}{2}$$

$$\Rightarrow (1-0)\left[\frac{1}{1+(1/2)^2}\right] \Rightarrow 1\cdot\left[\frac{1}{1+\frac{1}{4}}\right] \Rightarrow \frac{4}{5} = 0.8$$

Trapezoid Quadrature:

$$\Rightarrow \int_a^b f(t)\,dt \approx \left(\frac{b-a}{2}\right)\left[f(a) + f(b)\right]$$

$$f(t) = \frac{1}{1+t^2}$$

$$\Rightarrow \left(\frac{1-0}{2}\right)\left[\frac{1}{1+(0)^2} + \frac{1}{1+(1)^2}\right] \Rightarrow \frac{1}{2}\left[\frac{1\times 2}{1\times 2} + \frac{1}{2}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{3}{2}\right] \Rightarrow \frac{3}{4} = 0.75$$

$1 + \frac{1}{2}$

Simpson's Quadrature:

$$\Rightarrow \int_a^b f(t)\cdot dt \approx \left(\frac{b-a}{6}\right)\left[f(a) + 4 f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$\frac{3}{2} + \frac{16}{5}$, $\frac{15+32}{10}$, $\frac{47}{10}$

$$\Rightarrow \left(\frac{1-0}{6}\right)\left[\frac{1}{1+(0)^2} + 4\times\frac{1}{1+\left(\frac{1}{2}\right)^2} + \frac{1}{1+(1)^2}\right]$$

$\frac{1}{1+\frac{1}{4}} = \frac{4}{5}$

$$\Rightarrow \frac{1}{6}\left[1 + 4\times\frac{4}{5} + \frac{1}{2}\right] \Rightarrow \frac{1}{6}\left[1 + \frac{16}{5} + \frac{1}{2}\right] = \frac{1}{6}\left[\frac{47}{10}\right] \Rightarrow \frac{47}{60} = 0.78$$

Ans 2

2-Point Gaussian Quadrature:

$$\int_a^b f(t)\,dt \approx C_1 f(x_1) + C_2 f(x_2)$$

$$\Rightarrow C_1 = \frac{b-a}{2}, \quad C_2 = \frac{b-a}{2}$$

$$x_1 = \left(\frac{b-a}{2}\right)\left(\frac{-1}{\sqrt{3}}\right) + \frac{b+a}{2}$$

$$x_2 = \left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}$$

$$C_1 = \frac{1-0}{2} = C_2$$

$$x_1 = \left(\frac{1-0}{2}\right)\left(\frac{-1}{\sqrt{3}}\right) + \frac{1+0}{2} \Rightarrow \frac{1}{2} \times \frac{-1}{\sqrt{3}} + \frac{1}{2} \Rightarrow \frac{1}{2}\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \frac{\sqrt{3}-1}{2\sqrt{3}}$$

$$x_2 = \left(\frac{1-0}{2}\right)\left(\frac{1}{\sqrt{3}}\right) + \frac{1+0}{2} \Rightarrow \frac{1}{2\sqrt{3}} + \frac{1}{2} \Rightarrow \frac{1}{2}\left(\frac{1}{\sqrt{3}} + 1\right) \Rightarrow \frac{\sqrt{3}+1}{2\sqrt{3}}$$

$$\Rightarrow \frac{1}{2} f\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right) + \frac{1}{2} f\left(\frac{\sqrt{3}+1}{2\sqrt{3}}\right)$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{1+\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right)^2}\right] + \frac{1}{2}\left[\frac{1}{1+\left(\frac{\sqrt{3}+1}{2\sqrt{3}}\right)^2}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{12}{12+3+1-2\sqrt{3}} + \frac{12}{12+3+1+2\sqrt{3}}\right] \Rightarrow \frac{1}{2}\left[\frac{12}{16-2\sqrt{3}} + \frac{12}{16+2\sqrt{3}}\right]$$

$$\Rightarrow \frac{6}{16-2\sqrt{3}} + \frac{6}{16+2\sqrt{3}} \Rightarrow \frac{3}{8-\sqrt{3}} + \frac{3}{8+\sqrt{3}}$$

$$\Rightarrow \frac{3}{6.2679} + \frac{3}{9.7320}$$

$$\Rightarrow 0.4786 + 0.3082$$

$$\Rightarrow 0.7868$$

(b) $$\int_0^1 \sqrt{x}\,\log x \cdot dx = -\frac{4}{9}$$ [Considering log to be natural log [ln], log with base (e)]

Midpoint Quadrature:

$$\Rightarrow \int_a^b f(t)\,dt \simeq (b-a)\, f\left(\frac{a+b}{2}\right)$$

$$\Rightarrow (1-0)\left[\sqrt{\frac{0+1}{2}} \cdot \log\left(\frac{0+1}{2}\right)\right]$$

$$\Rightarrow \frac{1}{\sqrt{2}} \log \frac{1}{2} = -0.4901$$

Trapezoidal Quadrature:

$$\Rightarrow \int_a^b f(t)\,dt \simeq \left(\frac{b-a}{2}\right)\left(f(a) + f(b)\right)$$

$$\Rightarrow \left(\frac{1-0}{2}\right)\left[0+0\right] \Rightarrow 0.5$$

Simpson's Quadrature

$$\int_a^b f(t)dt \simeq \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$\Rightarrow \left(\frac{1-0}{6}\right)\left[0 + 4\left(\frac{1}{\sqrt{2}}\log\frac{1}{2}\right) + 0\right]$$

$$\Rightarrow \frac{1}{6}\left[\frac{4}{\sqrt{2}}\log\frac{1}{2}\right] = \frac{2}{3}[-0.4901] = -0.3267$$

2-Point Gaussian Quadrature

$$\int_a^b f(t)dt \simeq C_1 f(x_1) + C_2 f(x_2)$$

$$C_1 = \frac{b-a}{2}, \quad C_2 = \frac{b-a}{2}$$

$$x_1 = \frac{b-a}{2}\left(-\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}$$

$$x_2 = \frac{b-a}{2}\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}$$

$$C_1 = \frac{1-0}{2} = \frac{1}{2} = C_2$$

$$x_1 = \frac{1}{2}\left(-\frac{1}{\sqrt{3}}\right) + \frac{1}{2} \Rightarrow \frac{1}{2}\left(1 - \frac{1}{\sqrt{3}}\right) \Rightarrow \frac{\sqrt{3}-1}{2\sqrt{3}}$$

$$x_2 = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right) + \frac{1}{2} \Rightarrow \frac{1}{2}\left(\frac{1}{\sqrt{3}} + 1\right) \Rightarrow \frac{\sqrt{3}+1}{2\sqrt{3}}$$

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$$\Rightarrow \frac{1}{2} f\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right)+\frac{1}{2} f\left(\frac{\sqrt{3}+1}{2\sqrt{3}}\right)$$

$$\Rightarrow \frac{1}{2}\left[\sqrt{\frac{\sqrt{3}-1}{2\sqrt{3}}} \log\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right)+\sqrt{\frac{\sqrt{3}+1}{2\sqrt{3}}} \log\left(\frac{\sqrt{3}+1}{2\sqrt{3}}\right)\right]$$

$$\Rightarrow \frac{1}{2}\left(\sqrt{0.211} \log(0.211)+\sqrt{0.788} \log(0.788)\right]$$

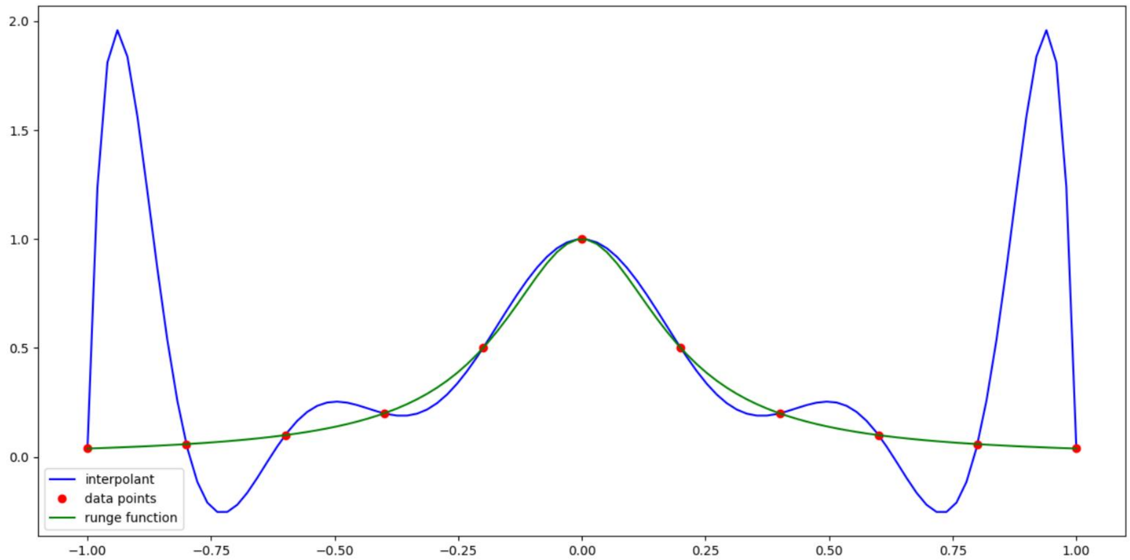
$$\Rightarrow \frac{1}{2}\left(0.459 \times (-1.55)+0.887 \times (-0.238)\right]$$

$$\Rightarrow \frac{1}{2}\left[-0.71145-0.211106\right]=-0.4612$$

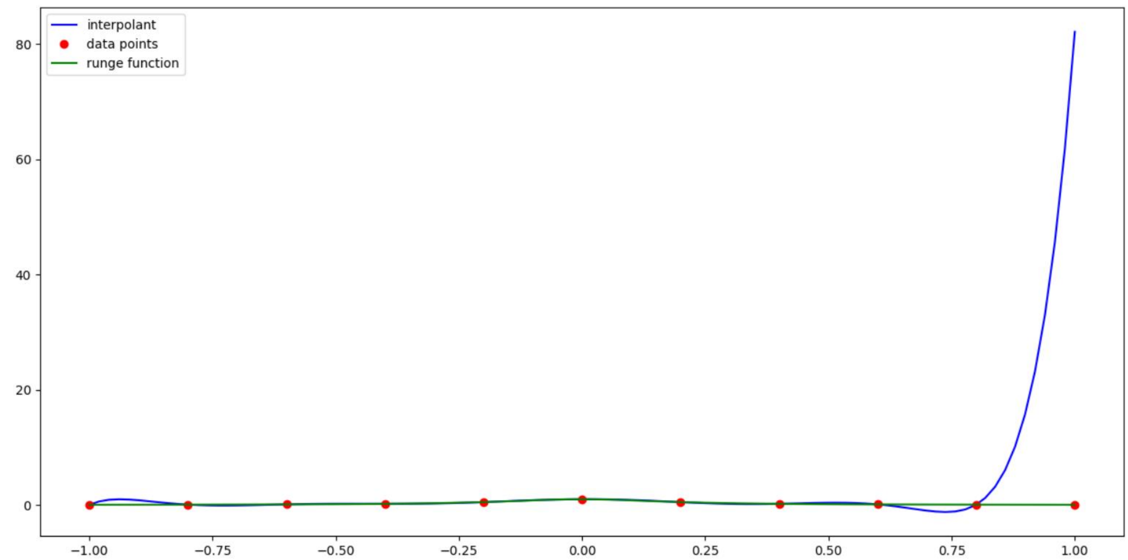
Ans 3:

N=11:

A. Polynomial Interpolant:

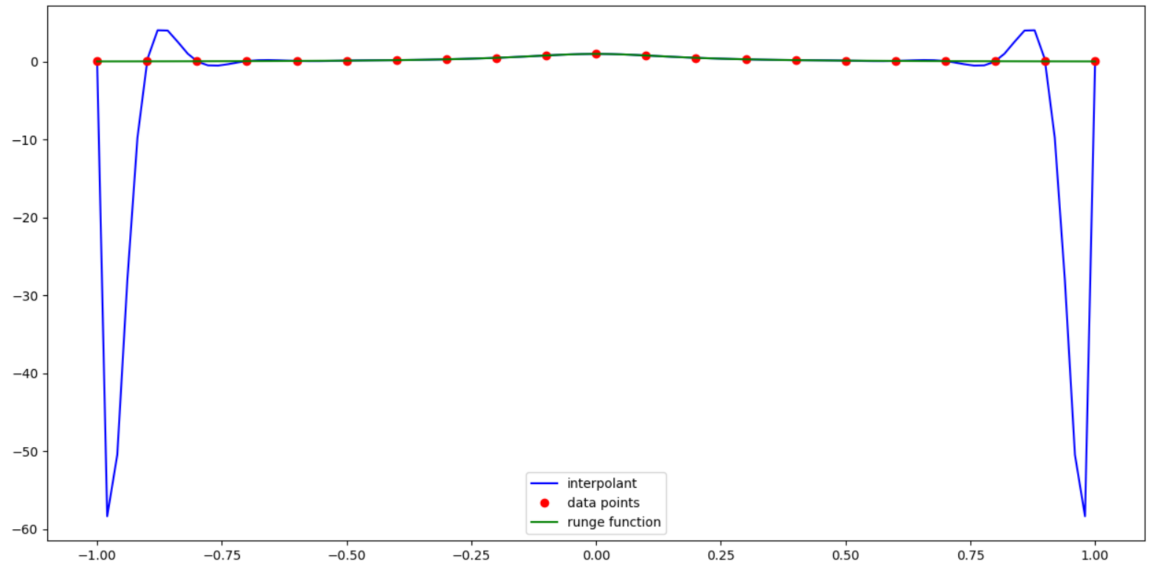


B. Cubic Spline Interpolant:

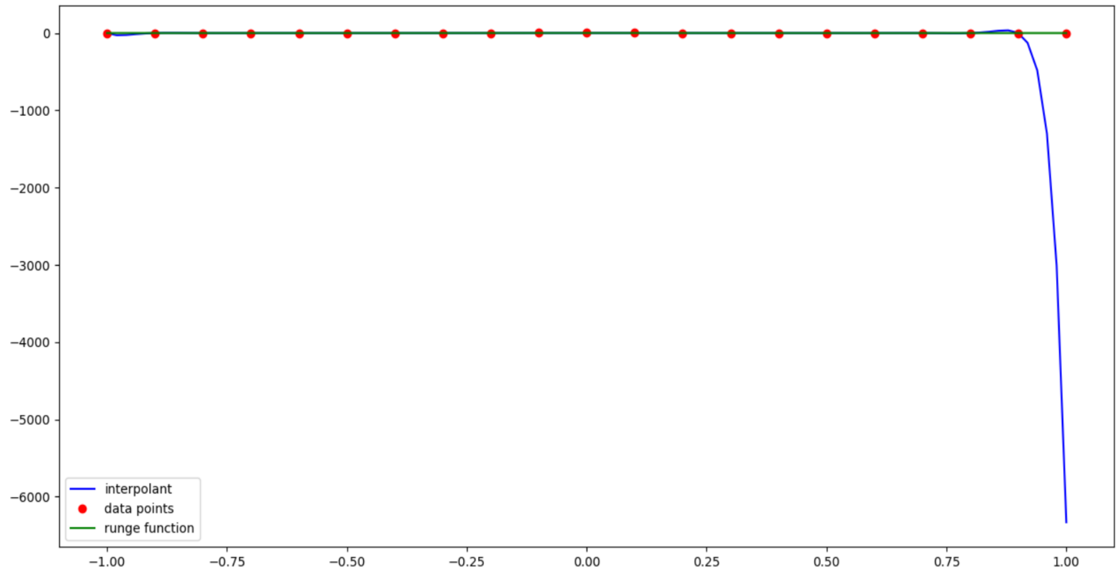


N=21:

A. Polynomial Interpolant:



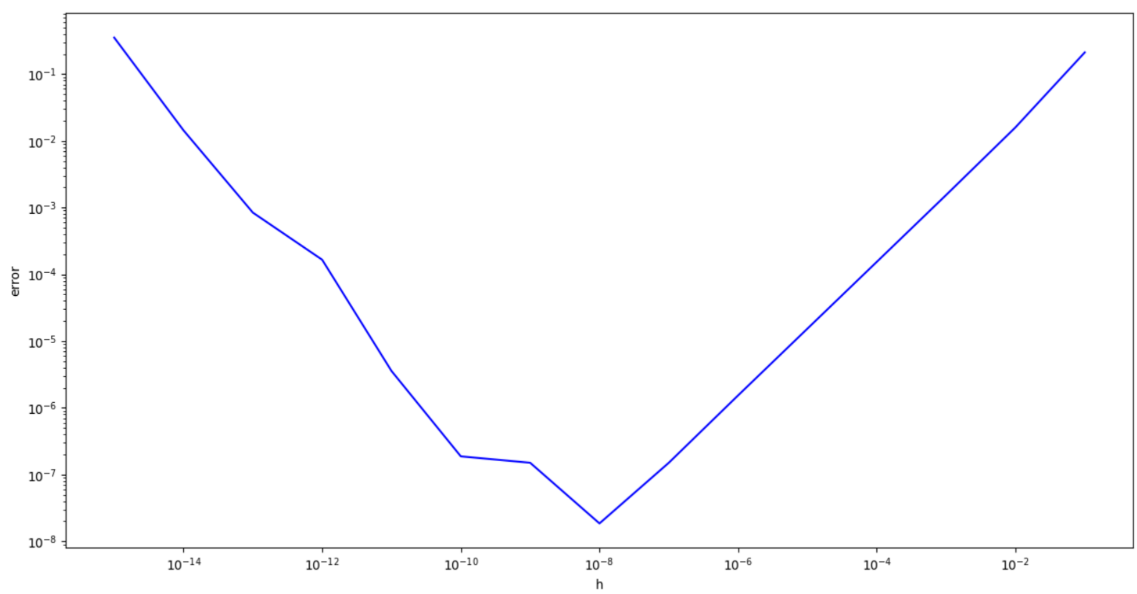
B. Cubic Spline Interpolant:



Ans 4:

```
PS C:\Users\91991> python -u "c:\Users\91991\Desktop\SC\Q4.py"
n = 2
I = -78.32340199221684
Iexact = 0.7853981633974483
Absolute error = 100.72445269467931
Relative Error = 10072.445269467931 %
n = 4
I = -41.826043216559654
Iexact = 0.7853981633974483
Absolute error = 54.25457222312565
Relative Error = 5425.457222312565 %
n = 8
I = -23.852918513553384
Iexact = 0.7853981633974483
Absolute error = 31.370479108801646
Relative Error = 3137.0479108801646 %
n = 16
I = -19.95145600107054
Iexact = 0.7853981633974483
Absolute error = 26.402982755606686
Relative Error = 2640.2982755606686 %
n = 32
I = -19.07910928186856
Iexact = 0.7853981633974483
Absolute error = 25.292276415998746
Relative Error = 2529.2276415998745 %
n = 64
I = -18.86803235807917
Iexact = 0.7853981633974483
Absolute error = 25.023524929649042
Relative Error = 2502.3524929649043 %
```

Ans 5:



```
h = 0.1
Absolute error = 0.21137797735668845
h = 0.01
Absolute error = 0.01591901492553915
h = 0.001
Absolute error = 0.0015439997955671379
h = 0.0001
Absolute error = 0.00015392519915594116
h = 1e-05
Absolute error = 1.538777641720602e-05
h = 1e-06
Absolute error = 1.5387641349049007e-06
h = 1e-07
Absolute error = 1.5043434748227398e-07
h = 1e-08
Absolute error = 1.8566686231707894e-08
h = 1e-09
Absolute error = 1.5049441793568456e-07
h = 1e-10
Absolute error = 1.8762779039910032e-07
h = 1e-11
Absolute error = 3.5688498737469492e-06
h = 1e-12
Absolute error = 0.00016549225429364 55
h = 1e-13
Absolute error = 0.0008417366709632153
h = 1e-14
Absolute error = 0.01436662500435461
h = 1e-15
Absolute error = 0.35248883333913933
```

The total error increases for larger value of ‘h’ because of the turncation error occurring in them and it increases for large value of ‘h’ because of increasing rounding error in them.

We are getting minimum error when ‘h’ equals 10^-8,