$$x_1^2 + 2x_2^2 + 3x_3^2 + (x_1 - x_2 + x_3 - 1)^2 + (-x_1 - 4x_2 + 2)^2$$

Ans/- (a)

$$\Rightarrow (x_1)^2 + (\sqrt{2}x_2)^2 + (\sqrt{3}x_3)^2 + (x_1 - x_2 + x_3 - 1)^2 + (-x_1 - 4x_2 + 2)^2$$

$$\Rightarrow (x_1 + 0x_2 + 0x_3)^2 + (0x_1 + \sqrt{2}x_2 + 0x_3)^2 + (0x_1 + 0x_2 + \sqrt{3}x_3)^2 + (x_1 - x_2 + x_3 - 1)^2 + (-x_1 - 4x_2 + 2)^2$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \\ 1 & -1 & 1 \\ -1 & -4 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -2 \end{bmatrix}$$

(b) $(-6x_2 + 4)^2 + (-4x_1 + 3x_2 - 1)^2 + (x_1 + 8x_2 - 3)^2$

$$A = \begin{bmatrix} 0 & -6 & 0 \\ -4 & 3 & 0 \\ 1 & 8 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -4 \\ 1 \\ 3 \end{bmatrix}$$

(c) $2(-6x_2 + 4)^2 + 3(-4x_1 + 3x_2 - 1)^2 + 4(x_1 + 8x_2 - 3)^2$

$$\Rightarrow (-6\sqrt{2}x_2 + 4\sqrt{2})^2 + (-4\sqrt{3}x_1 + 3\sqrt{3}x_2 - \sqrt{3})^2 + (2x_1 + 16x_2 - 6)^2$$

$$A = \begin{bmatrix} 0 & -6\sqrt{2} & 0 \\ -4\sqrt{3} & 3\sqrt{3} & 0 \\ 2 & 16 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -4\sqrt{2} \\ \sqrt{3} \\ 6 \end{bmatrix}$$

(d) $x^T x + \|Bx - d\|_2^2$

$\Rightarrow \|x\|_2^2 + \|Bx - d\|_2^2$ $\qquad \|x\|_2^2 = x^T \cdot x$

$\Rightarrow \|(B+I)x - d\|_2^2$ $\qquad [\|A+B\| \leq \|A\| + \|B\|]$

$\boxed{A = B+I \qquad b = d}$

(e) $x^T D x + \|Bx - d\|_2^2$

$\Rightarrow \|Dx\|_2 + \|Bx - d\|_2^2$

$\Rightarrow \|(B+D)x - d\|_2$

$A = B+D \qquad b = d$

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Q2.

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \simeq \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

Given the matrix we can see that it would have residual $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ which implies that the euclidean norm is 1

$$\hookrightarrow \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1+1 \\ 0+1 \\ 0+0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

And in order to compute the Sol^n we would consider the matrix

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\boxed{\text{Which gives } x_1 = x_2 = 1}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Ans 3 (a)

$$Z = \begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix}_{(m+n)\times(m+n)}$$

In order to show that the matrix Z is non-Singular, we can show that the inverse of matrix Z exists

Let the inverse of Z be Y

$$Y = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$$ Where each of P, Q, R & S are a matrix

Also, we know than $ZY = I$

$$\begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix}\begin{bmatrix} P & Q \\ R & S \end{bmatrix} = I = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

Simplifying by multiplying the matrix we get

$I \cdot P + A \cdot R = I$ $\quad\quad$ $I \cdot Q + A \cdot S = 0$

$A^T \cdot P = 0$ $\quad\quad$ $A^T Q = I$

Simplifying the above expression, we get

$\boxed{Q = (A^T)^{-1}}$

$(A^T)^{-1} + A \cdot S = 0$

$A\, S = -(A^T)^{-1}$

$S = -(A^{-1})(A^T)^{-1}$

$\boxed{\Rightarrow -(A^T A)^{-1} = S}$

$P + AR = I$

Multiply both sides by A^T $\qquad A^T P = 0$

$A^T P + A^T A R = A^T$

$A^T A R = A^T$

$$\boxed{R = (A^T A)^{-1} A^T}$$

$$Y = \begin{bmatrix} I - A(A^T A)^{-1} A^T & (A^T)^{-1} \\ (A^T A)^{-1} A^T & -(A^T A)^{-1} \end{bmatrix}$$

$P + A\left[(A^T A)^{-1} A^T\right] = I$

$$\boxed{P = I - A(A^T A)^{-1} A^T}$$

Now since A has linearly independent columns $(A^T A)^{-1}$ will exist.

Thus, we can say that inverse of matrix Z would exists which means Z is not singular.

(b) $\begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix}\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$

$\hat{x} + A\hat{y} = b \qquad A^T\hat{x} = 0$

Multiplying both sides by A^T

$A^T\hat{x} + A^TA\hat{y} = A^Tb$

$A^TA\hat{y} = A^Tb \Rightarrow \boxed{\hat{y} = (A^TA)^{-1}A^Tb}$

$A^T(b - A\hat{y}) = 0 \Rightarrow (b - A\hat{y})$ is null

$\because$ A has linearly independent columns

Which shows that it is a linear least square problem & thus we need to minimize $\|b - A\hat{y}\|_2^2$.

Which would be minimized when $A\hat{y} = b$. Now comparing it with $Ax = b$

We can conclude that $\hat{y} = x$ is sol^n of the Linear Least Square Prob^m.

Thus, minimizing $\|Ax - b\|^2$ we get

$\hat{x} = b - A\hat{y}$

$\boxed{\hat{x} = b - Ax}$

4.) Simplifying the QR factorisation we get

$$[A\ b] = [q_1\ q_2\ \text{———}\ q_n\ q_{n+1}]\begin{bmatrix} R_{11} & R_{12} & R_{1n} & R_{1,n+1} \\ 0 & & & \\ 0 & 0 & 0 & R_{n+1,n+1} \end{bmatrix}$$

$$[A\ b] = [R_{11}q_1\quad R_{12}q_1 + R_{22}q_2\ \text{———}\ R_{nn}q_{n} \ldots\ R_{n+1,n+1}q_{n+1}]$$

Comparing both the Sides we get

$$A = [R_{11}q_1\quad R_{12}q_1 + R_{22}q_2\ \text{———}]$$

$$b = [R_{1,n+1}q_1 + R_{2,n+1}q_2 + \text{———} R_{n+1,n+1}q_{n+1}]$$

(a) From above it can be clearly shown that $\|b\|$ can be computed only using last column of R.

(b) Simplifying A further, we know that q_1, q_2 ——— are orthogonal to one another which means they are linearly Independent.
Also R_{11}, R_{12}, ——— are cofficents

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Thus if linearly independent vectors are expressed so the only possible Solⁿ would be $R_{11} = R_{12} = 0$.

Which means $A = [R_{n} q_{1}]$

Which means $\|A\hat{x}\|_2$ Can also be Computed only using last column of R

(ii) Combining the reasoning from above two parts, we can conclude that it Can also be expressed/solved using last column of R

Ans 5) (a) $y_i = \frac{1}{1+e^{-(\alpha t_i+\beta)}}$

$1+e^{-(\alpha t_i+\beta)} = \frac{1}{y_i}$

$e^{-(\alpha t_i+\beta)} = \frac{1-y_i}{y_i}$

$e^{(\alpha t_j+\beta)} = \frac{y_i}{1-y_j}$

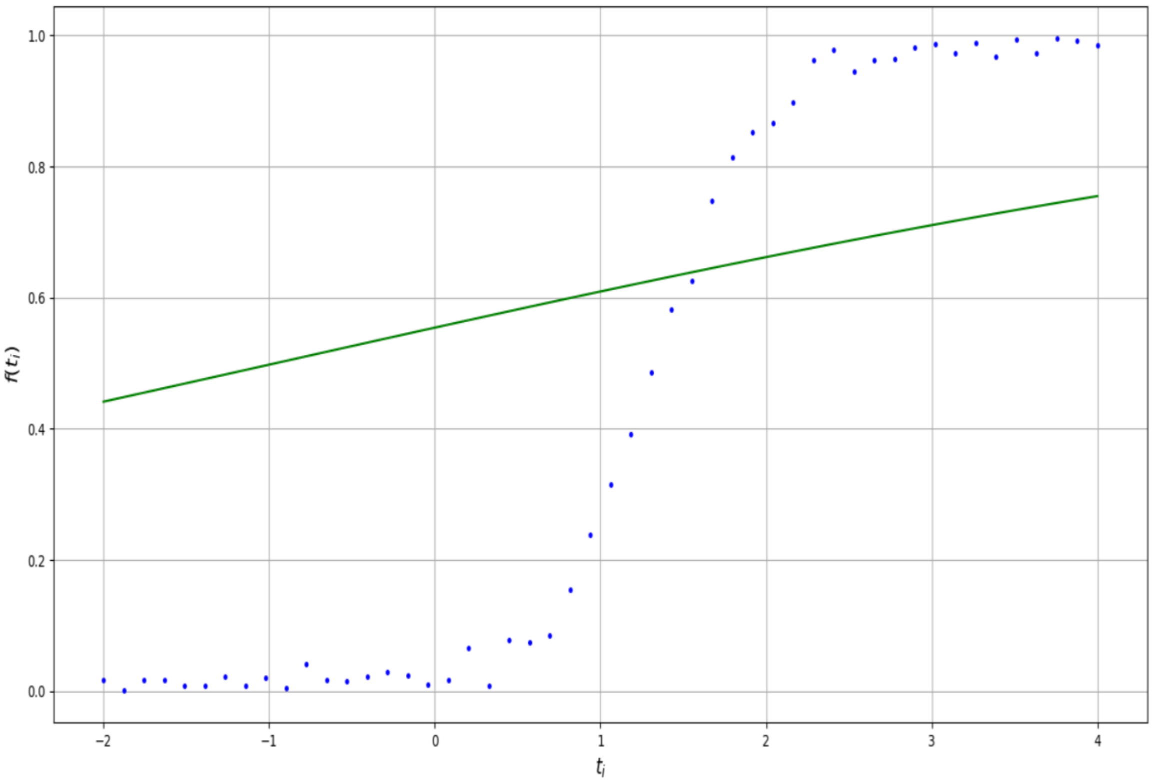
$\alpha t_j+\beta = \ln\left(\frac{y_i}{1-y_i}\right)$

Let $\ln\left(\frac{y_i}{1-y_i}\right) = Y'_i$

$\boxed{\Rightarrow \alpha t_i + \beta = Y'_i} \rightarrow$ Which is clearly a linear least Square Problem.

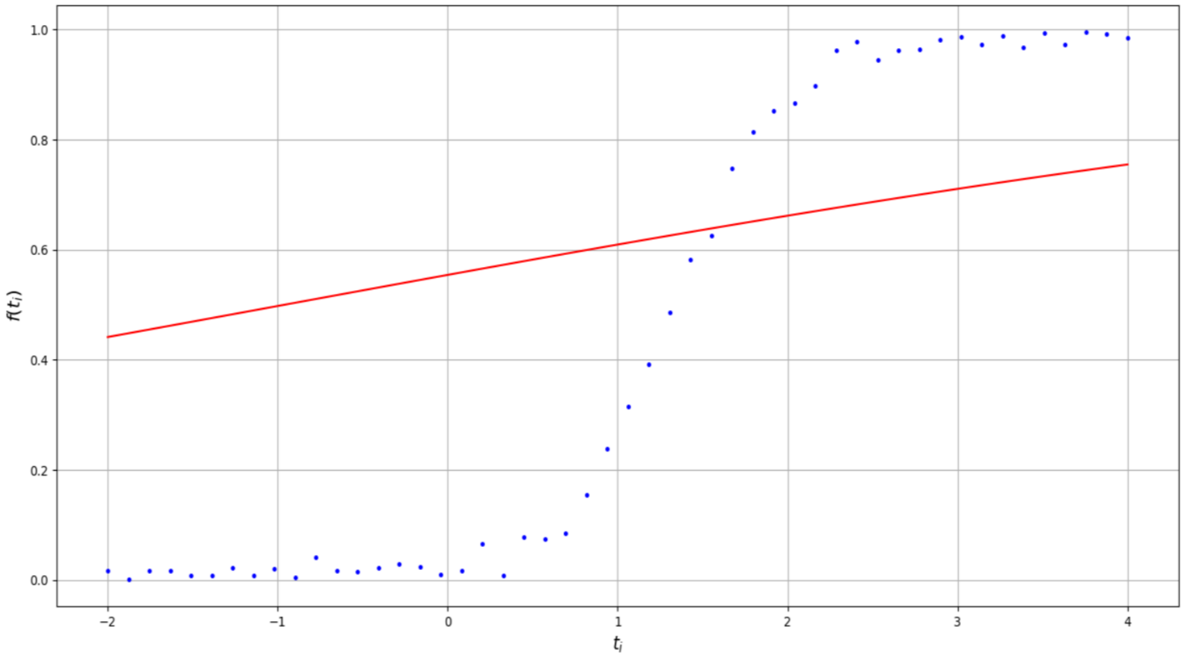
ANS 5-b:

i.



Error: 1.1041146302323548

ii.



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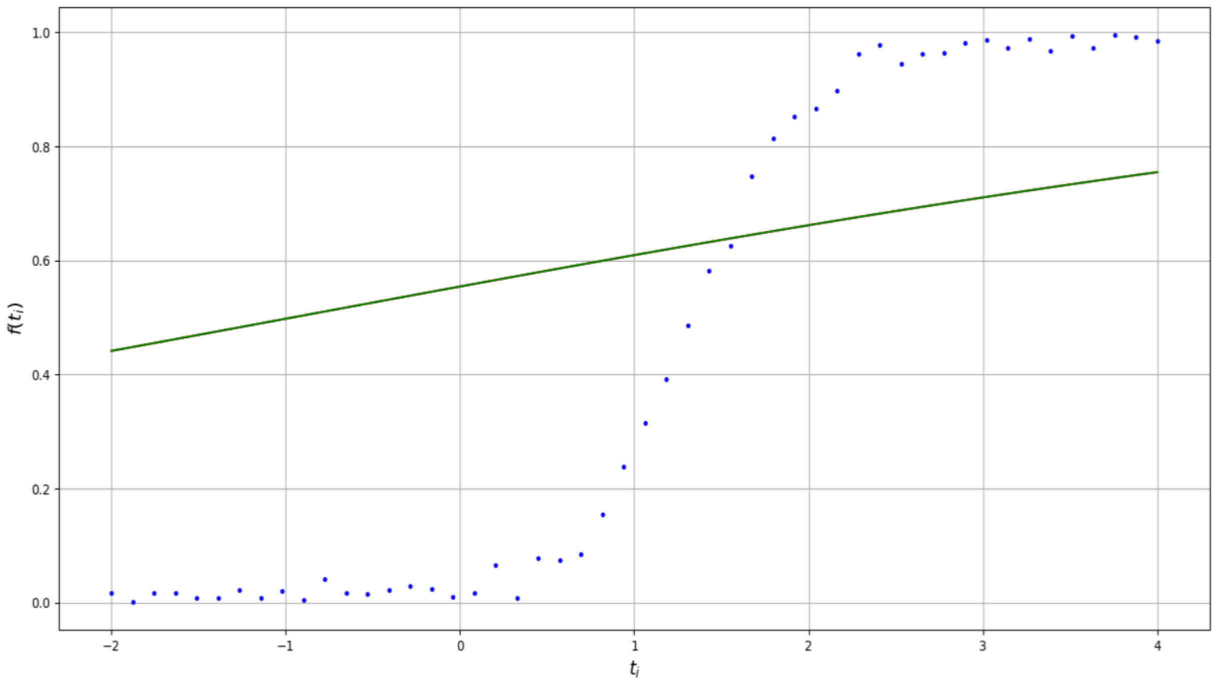
Plotting both of them on a single plot:



Ans6 → (a) $A = \begin{bmatrix} 1 & 1 \\ 10^{-k} & 0 \\ 0 & 10^{-k} \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 10^{-k} & 0 \\ 1 & 0 & 10^{-k} \end{bmatrix}$

<u>Normal Eq^n</u>

$A^T A x = A^T b$

$A^T A = \begin{bmatrix} 1 & 10^{-k} & 0 \\ 1 & 0 & 10^{-k} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 10^{-k} & 0 \\ 0 & 10^{-k} \end{bmatrix}$

$= \begin{bmatrix} 1+10^{-2k} & 1 \\ 1 & 1+10^{-2k} \end{bmatrix}$

$\underbrace{\begin{bmatrix} 1+10^{-2k} & 1 \\ 1 & 1+10^{-2k} \end{bmatrix}}_{C} x = \begin{bmatrix} -10^{-k} \\ 1+10^{-k} \\ 1-10^{-k} \end{bmatrix}$

$x = C^{-1} A^T B$

$\Rightarrow (A^T A)^{-1} A^T B$

$\Rightarrow A^{-1} A^{T^{-1}}$

$\Rightarrow A^{-1} B$

$C^{-1} = \frac{1}{|C|} \, adj(C)$

$\Rightarrow |C| = (1+10^{-2k})^2 - 1$

$\Rightarrow 1 + 10^{-4k} + 2\times10^{-2k} - 1$

$\Rightarrow 10^{-4k} + 2\times10^{-2k}$

$$adj(C) = \begin{bmatrix} 1+10^{-2k} & -1 \\ -1 & 1+10^{-2k} \end{bmatrix}$$

$$C^{-1} = \frac{1}{10^{-4k}+2\times10^{-2k}} \begin{bmatrix} 1+10^{-2k} & -1 \\ -1 & 1+10^{-2k} \end{bmatrix}_{2\times2}$$

$$A^T b = \begin{bmatrix} 1 & 10^{-k} & 0 \\ 1 & 0 & 10^{-k} \end{bmatrix}_{2\times3} \begin{bmatrix} -10^{-k} \\ 1+10^{-k} \\ 1-10^{-k} \end{bmatrix}_{3\times1}$$

$-10^{-k}+10^{-k}$

$-10^{-k}+10^{-k}-1$

$$\Rightarrow \begin{bmatrix} 10^{-2k} \\ -10^{-2k} \end{bmatrix}$$

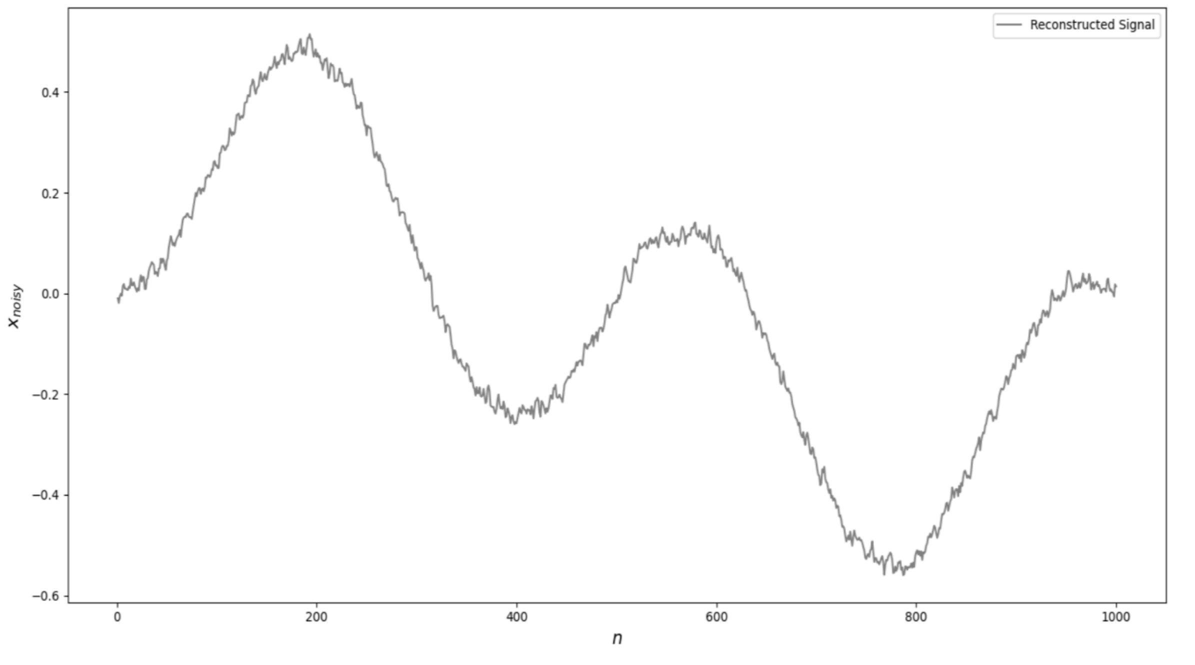
$$x = \frac{1}{10^{-4k}+2\times10^{-2k}} \begin{bmatrix} 1+10^{-2k} & -1 \\ -1 & 1+10^{-2k} \end{bmatrix}_{2\times2} \begin{bmatrix} 10^{-2k} \\ 10^{-2k} \end{bmatrix}_{2\times1}$$

$$x = \frac{1}{10^{-4k}+2\times10^{-2k}} \begin{bmatrix} \cancel{10^{-2k}}+10^{-4k}-\cancel{10^{-2k}} \\ -\cancel{10^{-2k}}+\cancel{10^{-2k}}+10^{-4k} \end{bmatrix}$$

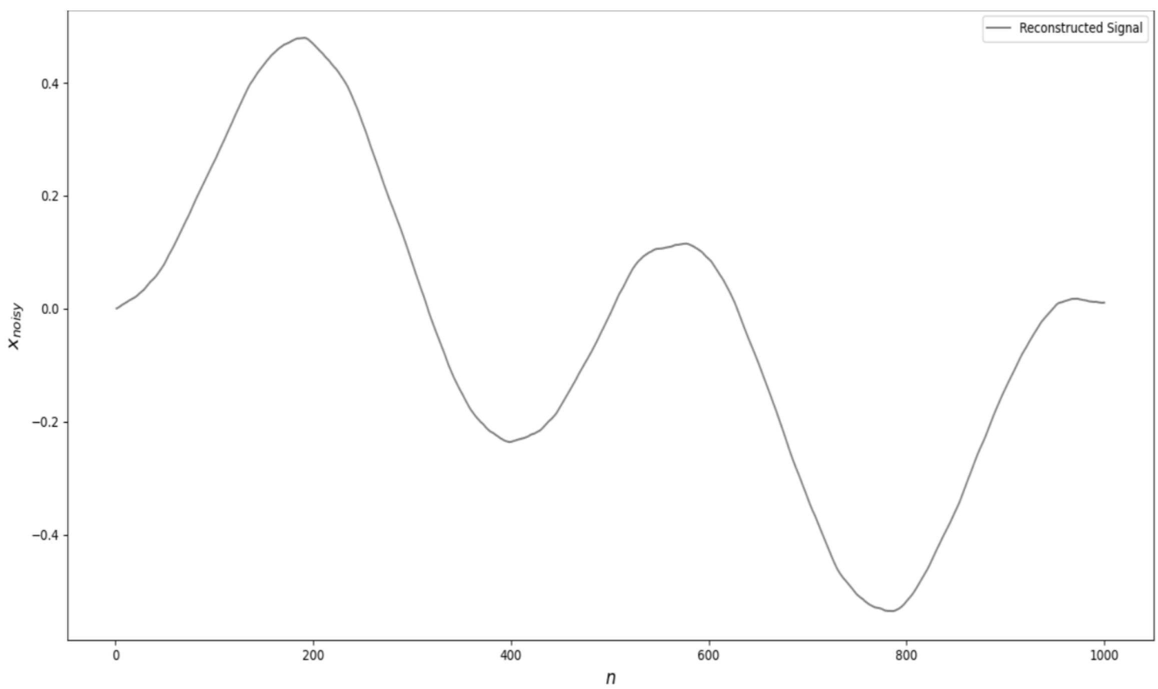
$$x = \frac{10^{2k}}{2+10^{-2k}} \begin{bmatrix} 10^{-4k} \\ 10^{-4k} \end{bmatrix} \Rightarrow \frac{1}{2+10^{-2k}} \begin{bmatrix} 10^{-2k} \\ 10^{-2k} \end{bmatrix}$$

Ans 7:

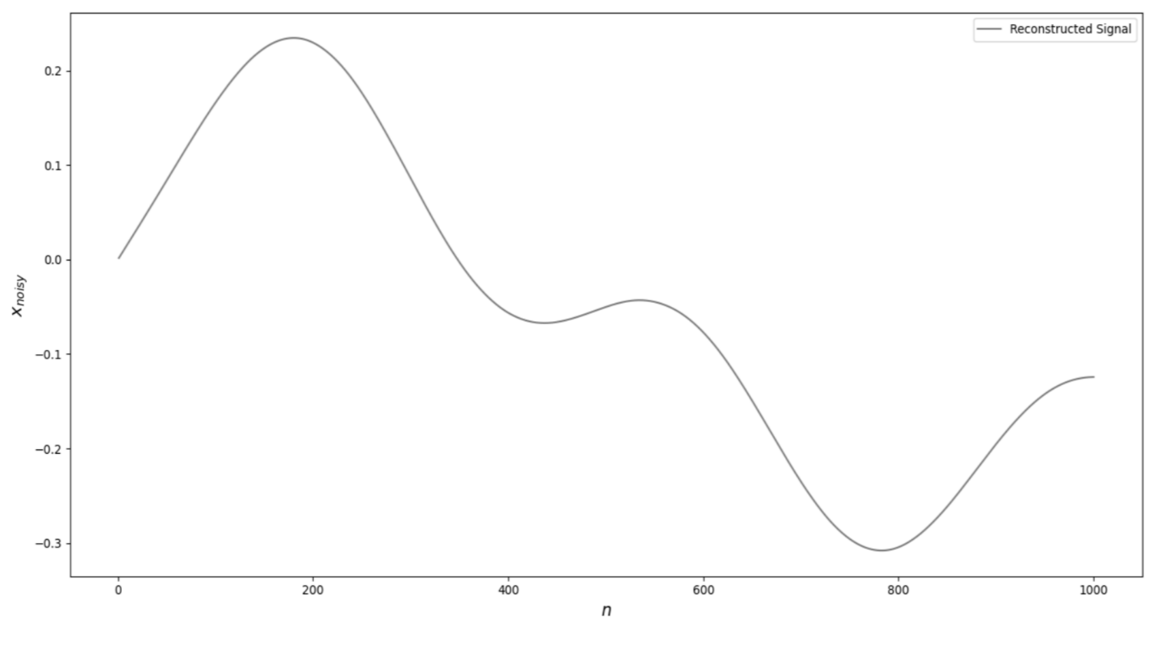
Lambda=1:



Lambda=100:



Lambda=10000:



From the values of lambda we can clearly see that as the value of lambda is increasing the signal is becoming more clear from the noise.