Indian Institute of Technology, Kharagpur

Date..................... Time: 60 mins Full Marks: 10

Class Test (Autumn) Semester 2022-23 No. of Students: 176 Sub. No. MA 61019/ MA 30227

Subject Name: Optimization Techniques

Instruction: Answer as many as you can, but the maximum score is 10. Notations used are as explained in the class.

1. **[2 mark]** A city hospital has the following daily requirements of nurses in the COVID period at the minimal level:

Period	Clock Time (24 hours a day)	Minimal number of nurses required
1	6a.m. - 10a.m.	3
2	10a.m. - 2p.m.	8
3	2p.m. - 6p.m.	16
4	6p.m. - 10p.m.	9
5	10p.m. - 2a.m.	21
6	2a.m. - 6a.m.	7

Nurses report to the hospital at the beginning of each period and work for 8(eight) consecutive hours. The hospital wants to determine minimal number of nurses to be employed, so that sufficient number of nurses are available for each period. Formulate this as Linear Programming model by setting up appropriate constraints and objective function.

2. **[2 mark]** Show graphically that the L.P.P

$$\text{Maximize } z = 5x + 3y$$

$$\begin{aligned} \text{Subject to } x + y &\leq 2, \\ 5x + 2y &\leq 10 \\ 3x + 8y &\leq 12 \\ \text{and } x, y &\geq 0. \end{aligned}$$

admits of a degenerate optimal basic feasible solution.
(In case of degeneracy, more than one constraints pass through the same solution point.)

$\lambda_1 = -8$ $\lambda_2 = 0$, $\lambda_2 = 2$ λ_3

3. **[2 marks]** Given the linear system

$\lambda_2 = 2$ $\lambda_3 = 9$ $2\lambda_1 - \lambda_2 + 2\lambda_3 = 0$

$2\lambda_1 - \lambda_2 =$

$\lambda_2 - 2\lambda_1 = 2\lambda_3$

$$\begin{aligned} 2x_1 - x_2 + 2x_3 &= 10, \\ x_1 + 4x_2 &= 18 \end{aligned}$$

$\lambda_1 + 4\lambda_2 = 0$

λ_3

and the non negativity restriction $x_1, x_2, x_3 \geq 0$, obtain a basic feasible solution starting from the feasible solution $x_1 = 2$, $x_2 = 4$, $x_3 = 5$.

$2\lambda_1 + \lambda_3 = 9$

$2\lambda_1 + 4\lambda_2 + 5\lambda_3$

$\lambda_1' = -8$

$-3x_1$

$2\lambda_1 + 4\lambda_2 + \frac{5}{2}\lambda_2 - 5\lambda$

4. [2 mark] The following incomplete tableau represents the second stage in the solution of an L.P.P. by the simplex method. All variables corresponding to zero coefficients of the objective function are slack variables. Complete the tableau. Also state the original problem assuming it to be a maximization problem. (The notations have their usual meanings.)

$4x - y = 15$

$c_j \rightarrow$			1	4	-2	3	-1	0	0	0
$\mathbf{c}_B$	Vectors in the basis	b	$\mathbf{a}_1$	$\mathbf{a}_2$	$\mathbf{a}_3$	$\mathbf{a}_4$	$\mathbf{a}_5$	$\mathbf{a}_6$	$\mathbf{a}_7$	$\mathbf{a}_8$
0	$\mathbf{a}_6$	9	13	0	1	-1	9	1	0	3
0	$\mathbf{a}_7$	4	-2	0	0	4	1	0	1	1
4	$\mathbf{a}_2$	2	4	1	0	-1	1	0	0	1
	$z_j - c_j$	8	15	0	2	-7	5	0	0	4

$4x - y$ $\quad -4 \; -3 \quad 4 \; -1 \quad 16 + 1$

5. [2 mark] Apply simplex method (without using simplex table) to show that the following linear programming problem will admit of an alternative optimal solution:

$$\text{Maximize } z = 2x_1 - x_2 + 3x_3$$

$$\begin{aligned} \text{Subject to } x_1 - x_2 + 5x_3 &\leq 10, \\ 2x_1 - x_2 + 3x_3 &\leq 40 \\ \text{and } x_1, x_2, x_3 &\geq 0. \end{aligned}$$

6. [2 mark] Use Charnes Big M-method to

$$\text{Maximize } z = 5x_1 - 2x_2 + 3x_3$$

$$\begin{aligned} \text{Subject to } 2x_1 + 2x_2 - x_3 &\geq 2, \\ 3x_1 - 4x_2 \qquad &\leq 3, \\ x_2 + 3x_3 &\leq 5, \\ \text{and } x_1, x_2, x_3 &\geq 0. \end{aligned}$$

7. [2 mark] Using simplex method, obtain the inverse of the matrix

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$$

8. [2 mark] Find the dual of the following L.P.P.
Maximize $z = x_1 - x_2 + 3x_3 + 2x_4$
Subject to $x_1 + x_2 \geq -1,$
$x_1 - 3x_2 - x_3 \leq 7$
$x_1 + x_3 - 3x_4 = -2$
$x_1, x_4 \geq 0$ and x_2, x_3 are unrestricted in sign.

——The End——

Indian Institute of Technology, Kharagpur

Date.................... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 7

Mid (Autumn) Semester 2023-24, Deptt: MA Sub. No. MA 61019/MA 30227

Subject Name: Optimization Techniques

Instruction: Answer **all** questions.

Question 1 [$5 \times 2 = 10$ marks]

a) A firm wants to expand Rs. 10 lakhs for advertising one of its products on radio, TV and a newspaper. It can purchase radio time for Rs. 2000 per spot, TV time for Rs. 10,000 per spot and newspaper advertisement for Rs. 4000 per insertion. The payoff from the advertisement is a measure of the audience reached. Based on past experience, a radio spot is given 40 audience points, a TV spot 160 points and a newspaper insertion 300 points. The firm wishes to determine the money to allocate to each medium so that the number of total audience points is maximised. Based on subjective consideration, the firm decides not to spend more than Rs. 2.5 lakhs on radio and at least Rs. 4 lakhs on TV. Also, the firm wants to keep newspaper allocation to not exceed 50% of allocation to TV. Formulate the problem as a L.P.P.

b) Solve the following L.P.P. graphically:

$$\text{Minimize } z = 4x_1 + 2x_2$$

$$\begin{aligned} \text{subject to } 3x_1 + x_2 &\leq 27, \\ -x_1 - x_2 &\leq -213, \\ x_1 + 2x_2 &\geq 303 \\ \text{and } x_1, x_2 &\geq 0. \end{aligned}$$

c) Convert the following L.P.P. to standard form:

$x_1 = |x_1' - x_1''|$

$x_i' > 0$

$$\text{Minimize } z = |x_1| - 2|x_2| + |x_3|$$

$$\begin{aligned} \text{subject to } x_1 + x_2 - x_3 &\leq 10, \\ x_1 - 3x_2 + 2x_3 &\geq 5 \end{aligned}$$

d) Write down the dual of the following L.P.P. :

$$\text{Maximize } z = 2x_1 + 3x_2 - 4x_3$$

$$\begin{aligned} \text{subject to } 3x_1 + x_2 + x_3 &\leq 2, \\ -4x_1 \qquad + 3x_3 &\geq 4, \\ x_1 - 5x_2 + x_3 &= 5, \end{aligned}$$

$x_1 \geq 0, x_2 \geq 0$ and x_3 is unrestricted in sign.

——P.T.O.——

e) Find the extreme points of the convex set of the feasible solutions of the L.P.P.

$$\text{Minimize } z = 2x_1 + 3x_2 + 4x_3 + 5x_4$$
$$\text{subject to } 2x_1 + 3x_2 + 5x_3 + 6x_4 = 16,$$
$$x_1 + 2x_2 + 2x_3 + 3x_4 = 9$$
$$\text{and } x_1, x_2, x_3, x_4 \geq 0.$$

Question 2 [$3 \times 4 = 12$ marks]

a) The following incomplete table represents the second stage in the solution of an L.P.P. by the simplex method. All variables corresponding to zero coefficients of the objective function are slack variables. Complete the table. Also, state the original problem assuming it to be a maximization problem. (The notations have their usual meanings.)

		c_j					0	0	0
B	c_B	b	$\mathbf{a}_1$	$\mathbf{a}_2$	$\mathbf{a}_3$	$\mathbf{a}_4$	$\mathbf{a}_5$	$\mathbf{a}_6$	$\mathbf{a}_7$
$\mathbf{a}_5$	0	8	-1	-13	20				-6
$\mathbf{a}_6$	0	26	7	13	-11				5
$\mathbf{a}_4$		2	1	2	-3				1
$z_j - c_j$		12	4	15	-16				6

b) $x_1 = 1, x_2 = 2, x_3 = 1, x_4 = 0$ is a feasible solution of the set of equations

$$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6,$$
$$15x_1 + 3x_2 - 12x_3 + 5x_4 = 9$$

Reduce the feasible solution to two different basic feasible solutions. Clearly specify if there is any degenerate basic feasible solution.

c) Find a basic feasible solution, if there is any, of the following set of linearly independent equations and if such solution exists, taking that basis as an admissible basis, calculate all y_j, $z_j - c_j$ $[j = 1, 2, 3]$ and the value of the objective function corresponding to that B.F.S. (without using simplex method)

$$\text{Maximize } z = 2x_2 + x_3$$
$$\text{subject to } x_1 + x_2 - 2x_3 \leq 7,$$
$$-3x_1 + x_2 + 2x_3 \leq 3$$
$$\text{and } x_1, x_2, x_3 \geq 0.$$

d) Solve the following equations by simplex method:

$$3x_1 + x_2 = 7,$$
$$x_1 + x_2 = 3$$

——P.T.O.——

Question 3 [3 + 3 + 2 = 8 marks]

a) Mention two situations of degeneracy that may occur in solving a L.P.P. Prove that a situation of degeneracy occurs in solving the L.P.P. given by

$$\text{Maximize } z = 22x_1 + 30x_2 + 25x_3$$

$$\begin{aligned} \text{subject to } 2x_1 + 2x_2 &\leq 100, \\ 2x_1 + x_2 + x_3 &\leq 100, \\ x_1 + 2x_2 + 2x_3 &\leq 100 \\ \text{and } x_1, x_2, x_3 &\geq 0. \end{aligned}$$

Mention clearly the method to resolve the degeneracy you are applying in this problem.

b) Prove the conditions for existance of alternative non-basic optimal solutions for a maximization L.P.P.

c) What are the conditions for existance of alternative basic optimal solutions for a maximization L.P.P.

——-The End——

Indian Institute of Technology, Kharagpur

Date.................... FN/AN Time: 3 Hrs Full Marks: 50 No. of Students: 176
End (Autumn) Semester 2023-24, Deptt: MA Sub. No. MA 61019/ MA 30227
Subject Name: Optimization Techniques

Instruction: Answer all questions.

Question I [$3+3+4=10$ marks]

1. State and prove the Complementary Slackness Theorem in the duality theory.
2. What is the maximum number of basic variables for a transportation problem involving m origins and n destinations? Give justification for your answer.
3. Discuss whether the following sets are convex or not where x_1, x_2 are real.

   (a) $X = \{(x_1, x_2) |\ x_2^2 \geq 4x_1\}$

   (b) $X = \{(x_1, x_2) |\ x_2 - 3 \geq -x_1^2, x_1, x_2 \geq 0\}$

Question II [$5 \times 8 = 40$ marks]

1. Solve the dual of the following L.P.P by dual simplex method and find the optimal solution of this L.P.P :

$$\begin{aligned} \text{Maximize, } z &= 3x_1 + 2x_2 \\ \text{subject to } 2x_1 + x_2 &\leq 5, \\ x_1 + x_2 &\leq 3 \\ \text{and } x_1, x_2 &\geq 0 \end{aligned}$$

2. Solve the following problem by revised simplex method :

$$\begin{aligned} \text{Maximize, } z &= x_1 + x_2 \\ \text{subject to } 2x_1 + 5x_2 &\geq 6, \\ x_1 + x_2 &\geq 2 \\ \text{and } x_1, x_2 &\geq 0 \end{aligned}$$

3. Use Gomory's cutting plane method to find the optimal solution of the following L.P.P:

$$\begin{aligned} \text{Maximize, } z &= 3x_1 + 4x_2 \\ \text{subject to } 3x_1 + 2x_2 &\leq 8, \\ x_1 + 4x_2 &\leq 10, \\ x_1, x_2 \geq 0 &\text{ and are integers.} \end{aligned}$$

——P.T.O.——

4. Consider the problem of assigning five operators to five machines. The assignment costs in rupees are given in the following table. Operator B cannot be assigned to machine 2 and operator E cannot be assigned to machine 4. Find the optimal cost of assignment.

	1	2	3	4	5
A	8	4	2	6	1
B	0	-	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	-	5

5. A machine operator processes five types of items on his machine each week and must choose a sequence for them. The setup cost per change depends on the item presently on the machine and the setup to be made according to the following table:

		TO ITEM				
		A	B	C	D	E
FROM ITEM	A	∞	4	7	3	4
	B	4	∞	6	3	4
	C	7	6	∞	7	5
	D	3	3	7	∞	7
	E	4	4	5	7	∞

If he processes each type of item once and only once each week, how should he sequence the items on his machine in order to minimize the total setup cost?

6. Four products are produced in three machines and their profit margins (machine to product) on sale are given in the following table:

		PRODUCTS				
		P_1	P_2	P_3	P_4	CAPACITIES
MACHINES	M_1	6	4	1	5	14
	M_2	8	9	2	7	18
	M_3	4	3	6	2	7
DEMANDS		6	10	15	8	39

4 7+ 39

Demands of the products and capacities of the machines per week are shown in the table. Find a suitable production plan of the products in the machines to maximize the profit. Find any other optimal solution to the problem, if any.

——P.T.O.——

7. Solve the following transportation problem:

	D_1	D_2	D_3	a_i
O_1	0	2	1	5
O_2	2	1	5	10
O_3	2	4	3	5
b_j	5	5	10	

8. Following is the optimal table for an L.P.P:

					c_3	c_4	c_5
		c_j	2	1	1	2	0
c_B	B	x_B	y_1	y_2	y_3	y_4	y_5
2	a_1	3	1	0	-1	3	2
1	a_2	4	0	1	4	-1	-2
			0	0	1	3	2

(a) Find the limitations of the values of c_3, c_4, c_5 for which the current solution will remain optimal.

(b) Find the optimal solution to the problem if c_3 increases to 3.

——The End——