Artificial Intelligence for Economics (AI60003)

Quiz 1 (Set 1) **Full Marks: 20** **12th September, 2023**

Name: [redacted] **Roll:** [redacted]

Q1. Suppose I take up the following positions in the market:

- Sell 2 - P_{50} (S_t, T) PUTs.
- Buy 2 - P_{70} (S_t, T) PUTs.
- Buy 3 - C_{90} (S_t, T) CALLs.
- Sell 1 - C_{110} (S_t, T) CALL.
- Sell 2 - C_{120} (S_t, T) CALLs.

i.e I have a portfolio $C^* = -2\,P_{50}(S_t, T) + 2.\,P_{70}(S_t, T) + 3.\,C_{90}(S_t, T) - C_{110}(S_t, T) - 2.\,C_{120}(S_t, T)$

How will my payoff at expiration date look like as a function of the value of the underlying asset on the expiration date? **[10 marks]**

Q2. You are planning a muiti-city tour. Between any two cities you have some transport options. Each option is associated with cost, time and carbon emission (note that the costs etc are symmetric between a pair of locations). You need to consider all these three criteria for planning your travel itinerary. Cast this in the state-space setting and construct the graph. Using multi-criteria A* algorithm, find the set of pareto-optimal paths that cover all 4 cities starting from A. **[10 marks]**

From	To	Mode	Cost	Time	C02 Emission
A	B	Air	5000	2	850
A	B	Rail	200	16	50
A	C	Rail	250	10	30
A	C	Road	50	12	100
B	C	Rail	100	8	35
B	C	Road	80	7	90
B	D	Air	2500	1	400
B	D	Road	600	10	70
C	D	Road	500	8	60

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid-Autumn Semester Examination 2023-24

Date of Examination: 19th September 2023 Session: FN Duration: 2 Hrs Full Marks: 50

Subject No. : AI60003 Subject : Artificial Intelligence for Economics

Department/Center/School: Centre of Excellence in Artificial Intelligence

Specific charts, graph paper, log book etc., required

Special Instructions (if any) :

Q1. Give brief answers to the following problems: **[3 x 5 = 15 marks]**

a) Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by

$$-[(10-a)^2 + (10-b)^2]$$

Suppose prices of both the goods are equal to 1.

i. Solve for the optimal consumption of the consumer when his income is 40.

ii. What happens to his optimal consumption when his income goes down to 10.

b) A researcher has 100 hours of work which have to be allocated between two research assistants Aditya & Gaurav. If Aditya is allocated x hours of work his utility is $-(x-20)^2$. If Gaurav is allocated x hours of work his utility is $-(x-30)^2$. The researcher is considering two proposals:

I. Aditya is given 60 hours of work & Gaurav is given 40 hours.

II. Aditya is given 90 hours of work & Gaurav is given 10 hours.

Which of the following statement is correct.

A. Proposal I is pareto optimal but Proposal II is NOT.

B. Proposal II is pareto optimal but Proposal I is NOT.

C. Both proposals are pareto optimal.

D. Neither proposal is pareto optimal.

c) In the class we saw that the greedy algorithm did NOT lead to stable match. Is it necessary that it will fail? If not, can you provide an example (with 5 boys & 5 girls) where the greedy algorithm succeeds. (Hint: Try finding an appropriate list of preferences.)

d) Two days before the expiration date Harshad wants to sell a CALL with strike price Rs. 100 i.e she wants to go short on C_{100} (S , t). The interest rate is r = 10%. And the current value of the stock is Rs. 120. Use the PUT – CALL parity equation to find a lower bound on the value of C_{100}.

e) We are trying to predict whether there will be a crop shortage or not in a given year. Several factors are recorded from past experience in earlier years and locations, based on which two decision trees are proposed. On the given dataset, which do you think is more suitable?

Government Subsidies (S)	550	437	898	480	754	604	490	725	685	815
Drought (D)	Y	N	N	Y	Y	N	N	N	Y	N
Farming workforce (W) (million)	31	18	25	22	30	28	32	35	21	33
Crop Shortage (C)	Y	Y	Y	Y	Y	N	N	N	N	N

Tree 1: If S > 500: if D=Y then predict Y, else predict N; If S < 500: predict Y
Tree 2: If D = Y: if S>600 then predict N, else predict Y; If D = N: if W<30 then predict Y, else predict N

Q2. Provide brief and clear answers to the following questions

a) Briefly discuss the idea of graph coloring & chromatic number of a graph and how it is applied in scheduling problems. **[5 marks]**

b) Explain the fundamental theorem of linear programming with respect to one linear constrained optimization problem. **[2 marks]**

c) Explain the difference between interior and exterior point methods for non-linear constrained optimization with an example. (Only formulate the problem and explain the solution approach, don't solve actually). **[3 marks]**

d) Explain how Decision Trees and Regularized Linear Regression attempt to identify important features in prediction problems. **[5 marks]**

Provide detailed answers to ANY TWO of the following problems

Q3. Consider an economy where agents are identical and they live for three periods. Suppose in the first period they invest 'e' in their education and become skilled in the second period where the level of skill $h = e^{\delta}$, $0 < \delta < 1$. The investment in education is done via borrowing from the market at a fixed rate R, i.e per unit of borrowing costs $R > 1$ per period. In the second period of their life they work using the acquired skill (h). The total wage of an agent with skill level h is given by w.h where w is the exogenously given wage rate. Once they earn wage income w.h in the second period, they repay their total borrowing for education. Assume that the wage income is sufficient to repay this education loan. Further, for simplicity we assume that they do not consume anything in the first period. In the second period they take their consumption and saving decision for the second and the third period. Gross return on saving is the same R per unit per period. Agents do not work in the third period and live on their saving made in the second period. Suppose the life-time utility function is given by $U = U(c_2) + \beta U(c_3)$; where c_t and c_{t+1} are the consumption levels in the second and third periods respectively. The function U(.) is assumed to be strictly positive and strictly concave with $\beta > 0$.

(a) Write down the utility maximization problem of the agents. Clearly derive the first order conditions with respect to saving 's' and investment in education 'e' in this maximization problem. **[5 marks]**

(b) Derive the optimal level of investment in education (e). Describe its relationship with the wage rate (w) and the rate of interest (R). **[5 marks]**

Q4. Covid pandemic is spreading rapidly in a country. For each month, the government can take two decisions: either declare a lockdown (which reduces disease spread but harms the economy), or let the economy function normally (which increases disease spread). Assume that initially the economy size is X0=100 and number of infections is N0=10. At the beginning of each month, let's say economy was X and number of infections was N. If lockdown is imposed for that month, the economy shrinks to 0.9*X, and the number of infections becomes 3N/2 by the end of the month. If no lockdown is imposed, the economy falls to 0.99*X but the number of infections becomes 4N by month-end. This process continues for 3 months.

(a) Draw the state space transition graph. **[1 marks]**

(b) The government's target is to keep the total number of infections below 100. Using A*, find the sequence(s) of decisions it should take for these 3 months, so that it can achieve this goal while sustaining least harm to the economy (use economic loss as edge cost). **[5 marks]**

(c) The government has two objectives: minimize the number of infections, and minimize the economic loss. Using multi-objective A*, find the pareto-optimal solutions. A solution is ruled out if the number of infections is above 300, irrespective of economic loss. **[4 marks]**

(in both cases, use any heuristic function of your choice, as long as it underestimates the actual cost)

Q5. There are two companies C1, C2. For C1, if it receives a budget of x, it generates revenue R1(x) according to Gaussian distribution N(2x, 20) and creates J1(x) jobs according to Uniform (x, 2x). For same budget, C2 generates revenue R2(x) according to N(x, 40) and creates J2(x)=2x jobs (guaranteed).

The government has a budget of 100 which it can distribute among 2 companies. Let these be denoted as X, 100-X. Denote the total revenue generated and total jobs created by R(X), J(X) respectively. Clearly, R(X)=R1(X)+R2(100-X), J(X)=J1(X)+J2(100-X).

(a) Calculate the expected revenue generated and jobs created as a function of X. **[3 marks]**

Define utility function U(X) = aR(X) + bJ(X). If an event E happens, then (a=1, b=1), but else (a=0, b=2). If event F happens, then X is chosen uniformly between (20, 50), else it is chosen uniformly between (50, 80). Probability of E happening is 60%, while that of F is 30%.

(b) Represent the situation using Bayesian Network. Mention the p(X|pa(X)) type distribution at each node X, where pa(X) are the parents of node X. **[3 marks]**

(c) Calculate the expected value of U. **[4 marks]**

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End-Autumn Semester Examination 2023-24

Date of Examination: 17 November 2023 **Session:** FN **Duration:** 3 Hrs **Full Marks:** 80

Subject No. : AI60003 **Subject :** Artificial Intelligence for Economics

Department/Center/School: Centre of Excellence in Artificial Intelligence

Specific charts, graph paper, log book etc., required

Special Instructions (if any) : _ALL PARTS OF SAME QUESTION MUST BE ANSWERED TOGETHER

PART A (ANSWER ANY 2 QUESTIONS)

Q1. A) Give an example from Economics to illustrate why correlation does not imply causation. ***[2 marks]***

B) Explain the concept of "confounders" with respect to causal graphical models with an example. ***[3 marks]***

C) Discuss the shortcomings of Granger Causality, and how they can be overcome. ***[3 marks]***

D) A state government wants to understand if scaling up public transport results in improvement in urban air quality. Accordingly, they carry out a Randomized Control Trial over 10 towns in that state. The Air Quality Index is measured once before the intervention, and again after it, 6 months later. The details of their experiment are given in the table below. Based on this trial, they conclude that there does exist a causal relation (improvement of public transport-> better air quality). Discuss whether their conclusion is justified or not. [Low Air quality index: better air quality] ***[6 marks]***

Town	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
%Area of Green Cover	24	32	15	8	18	25	21	3	30	22
Population	5Lakh	2L	14L	10L	5L	6L	10L	15L	8L	5L
Initial Bus Fleet Size	80	30	60	120	72	84	140	180	90	75
Initial Air Quality Index	58	42	145	75	65	80	115	245	64	96
Scale-up done	Y	Y	Y	Y	Y	N	N	N	N	N
Present Air Quality Index	52	39	104	78	60	74	105	251	54	89

E) A market survey agency has estimated the "satisfaction score" of 150 different regions, by interviewing people living there. I want to understand the main driving factor(s) behind the "satisfaction score" (Y, measured on a scale 0-100) in different regions using Shapley Value Analysis. My understanding is that the

possible factors that cause satisfaction are X1=Human Development Index, X2=Per Capita Income, X3=Net GDP of the region. In the dataset below of 10 regions, I have estimated two sets of Shapley Values (S_A, S_B) of these factors for 10 regions, based on two different predictive models (f_A, f_B). What is the expected value of Y? Which set of values should I consider, and which factors can I conclude as the main drivers of satisfaction? For each of the 10 regions, discuss the possible values of the 3 factors (relative to their means), and the nature of their contributions to the happiness of the region. **[6 marks]**

ID	1	2	3	4	5	6	7	8	9	10
Y	45	57	23	37	65	44	78	31	28	50
$f_A(X)$	44	59	25	36	69	40	73	35	28	52
$f_B(X)$	45	60	15	45	60	45	75	30	30	45
$S_A(X1)$	-10.7	9.3	-15.4	-8.1	8.8	-5.4	15.4	-14.6	1.7	-3.6
$S_A(X2)$	7.1	-0.5	-12.8	-2.1	6.9	1.1	17.2	-2.8	-16.4	7.4
$S_A(X3)$	0.8	0.2	3.2	-0.8	1.3	0.3	-2.6	0.4	-5.3	-1.8
$S_B(X1)$	-9.6	8.7	-15.2	-7.8	5.9	-3.2	10.6	-7.1	-2.6	1.1
$S_B(X2)$	7.4	-1.4	-12.5	-2.6	7.4	-0.5	16.8	-10.5	-17.1	1.7
$S_B(X3)$	2.2	1.7	3.7	-0.6	3.7	-0.3	2.6	0.6	-0.3	-0.8

Q2 A) The number of new jobs J(t) created, the number of existing jobs abolished F(t), and the number of new persons entering job market E(t) in a country in year 't' depends on the annual growth G(t) of the GDP. Based on the data provided below, represent the situation as a Hidden Markov Model and estimate its state transition and emission parameters based on relative frequencies, using any suitable emission distributions and number of latent states. **[6 marks]**

t	1	2	3	4	5	6	7	8	9	10
GDP	305	306	310	321	322	322	327	334	340	341
New Jobs Created	80	25	65	85	15	10	48	65	72	5
Existing jobs abolished	12	50	10	2	40	85	8	10	5	45
New job candidates	15	5	25	40	10	0	20	28	24	6

B) Consider, the country had 100 employed and 100 unemployed people in the job market at t=0. Assuming that no job post remains vacant, and using the HMM parameters calculated above, estimate the expected number of employed and unemployed persons at t=11. **[4 marks]**

C) You want to test if there is a linear causal relation between number of jobs abolished and the total number of unemployed persons, by considering each time-point as an independent observation. Sketch a causal graph indicating the different variables involved, based on the above data and the HMM model. Explain how you can use the Double-ML technique with Frisch-Waugh-Lovell Theorem to answer the question. **[5 marks]**

D) You want to predict the number of employed persons each year based on the annual GDP growth, knowing the employment numbers at t=0 and initial GDP (t=0) of 296. Formulate an RNN with a 3D hidden state. Explain how you will explain the parameters of the RNN. Show how you will estimate any one parameter, assuming others are known. **[5 marks]**

Q3 A) Give an example to explain why the heuristic function of A* must underestimate the true cost to a goal node. What is the problem if we use 0 as the constant heuristic value at all nodes? **[5 marks]**

B) Two countries A and B, both having a GDP of 100 units initially, engage in a competition to increase their GDP. They do this by increasing their investments in industrial development, which unfortunately results in a decrease in their social sector spending (health, education etc), resulting in a decrease of their Human Development Index and may also increase carbon emissions.

At each step, country A can set their industrial investment XA to 1 or 2 units, while for country B, this investment XB can be 0 or 3 units. In case of A, the GDP GA grows according to the rule GA(new) = GA(old) + 2*XA, while for country B this rule is GB(new) = GB(old) + XB. The Human Development Index HA of country A varies with XA as HA(new) = HA(old) – 0.05*XA, while for country B, this varies as HB(new) = HB(old) – 0.08*XB. The carbon emissions CA of country A increases by an amount that follows N(5*XA, 2), while for country B the carbon emissions CB increases by an amount that follows N(3*XB, 6).

i) Construct a game tree between A and B, showing the state values (GDP) and considering the reduction of HDI as the edge cost at each step. Go upto depth 4 (2 steps for each country) starting with country A. Identify the optimal move for both at each step, considering the valuation of each leaf state as gain of GDP minus 10 times the loss of HDI for A w.r.t. B. **[5+5=10 marks]**

ii) What is the expected change in carbon emissions of both A and B, if both move optimally? If A follows a policy that it will never allow its expected carbon emission to exceed 75% of its maximum value, then which path will be followed? **[2+3=5 marks]**

PART B (ANSWER ALL QUESTIONS)

Q4 Consider an auction with ~~one~~ 5 buyer and ~~1~~ sellers with valuations 10, 12, 15, 5, 9 respectively for the item being sold. However, the valuation of player i is known to that player only. Each player believes that the valuation of every other player is distributed uniformly randomly from the interval [0,20]. Suppose you are the second player and standard game theoretic assumptions apply. Justify your answer in all these cases.

i) Suppose the first price auction is used. Then what will you bid? **[6 marks]**

ii) Suppose the second price auction is used. Then what will you bid? **[5 marks]**

iii) Suppose the third price auction is used, i.e. the lowest bidder wins buy receives the third minimum bid among all the bids as his/her payment. Then what will you bid? **[3 marks]**

iv) Suppose the fourth price auction is used, i.e. the lowest bidder wins buy receives the fourth minimum bid among all the bids as his/her payment. Then what will you bid? **[3 marks]**

v) Suppose the fifth price auction is used, i.e. the lowest bidder wins buy receives the fifth minimum bid among all the bids as his/her payment. Then what will you bid? **[3 marks]**

Q5. A) Two players simultaneously announce an integer from [0, 1, 2, ……., 100]. The player, whose announcement is an integer closer to 2/3 of the average announcement, gets 1 and the other player gets 0. In case of tie, each gets 1/2.

i) Construct the payoff matrix

ii) Find the pure strategy Nash Equilibrium (Equilibria if more than one) **[3+4=7 marks]**

B) Three men – Adway, Palash & Dripto reside on three floors of an apartment building somewhere in Kharagpur. They are deciding whether to employ a security guard at the entrance to the building. Assume that the guard costs 1 per resident. Each resident has a value (not including the cost of hiring) of the guard of either 2 or −2 (it can be negative because some individual resent the loss of privacy). If the guard is NOT hired, all residents receive a net utility of zero (Status Quo!). The three residents adopt the following procedure: they report their value (i.e either 2 or −2) and the guard is hired if the number of residents reporting the positive value, is at least α where α is either one, two or three. Note that different values of α represent different procedures - if α is one, the guard is hired as if at least one resident wants it, if α is two, a majority is required and α is three would mean that hiring takes places only if the residents are unanimous. Show that reporting their value truthfully is a weakly dominant strategy for each resident, for all values of α. **[8 marks]**

[A strategy is weakly dominant if, regardless of what any other players do, the strategy earns a player a payoff at least as high as any other strategy, and, the strategy earns a strictly higher payoff for some profile of other players' strategies. Hence, a strategy is weakly dominant if it is always at least as good as any other strategy, for any profile of other players' actions, and is strictly better for some profile of others' strategies. If a player has a weakly dominant strategy, than all others are weakly dominated. If a strategy is always strictly better than all others for all profiles of other players' strategies, than it is strictly dominant.]

C) In a cricket match between AUS & IND, Virat bets on IND & offers a bet of 30:1 i.e If I bet Re. 1 with Virat that "AUS will win" & AUS indeed wins Virat will pay me Rs. 30 & if IND wins, I will pay Virat Re.1. Steve on the other hand bets on AUS & offers a bet of 7:5 i.e If I bet Re. 1with Steve that "IND will win" & IND indeed wins Steve will pay me Rs. 7/5 & if AUS wins I will pay Steve Re. 1. I have Rs. 100 in my wallet. How should I bet to eliminate risk? What is the maximum guaranteed payoff I can earn? **[5 marks]**