Indian Institute of Technology Kharagpur

Mid-Autumn Semester Examination 2023-24

Advanced Macroeconomics (HS60243)

Full Marks: 30

Answer All Questions

1. **Consider, an Overlapping Generation (OLG) model discussed in the class, where population grows at the rate, $0 < n < 1$; and capital fully depreciates in each period; $\delta = 1$. Production function of the economy is, $Y_t = K_t^{\alpha} N_t^{1-\alpha}$; $0 < \alpha < 1$. K_t is the amount of capital at time, t; and N_t is the amount of labour at time, t. An individual is endowed with 1 unit of labour when young, and dies next period as old. Old does not have any endowment of labour. As a result, N_t is also the number of population at time, t. Suppose, the consumption of an individual when young at time t is, c_t^1; and when old at time $t+1$ is, c_{t+1}^2. Lifetime utility function of an individual is, $u(c_t^1, c_{t+1}^2) = \log(c_t^1) + \beta \log(c_{t+1}^2)$; $0 < \beta < 1$. β is the discount factor. The wage rate of the economy at time, t is w_t; and the rental rate at time t is, r_t. (25)**

a) Write down the budget constraint of an individual when young, and also when old. Also, write down the budget constraint in the present discounted value format. (1+1+2)

b) Using the profit maximization of the firm show that, $w_t = (1-\alpha)k_t^{\alpha}$, and $r_t = \alpha k_t^{\alpha-1}$ (1+1)

c) Set, the relevant Lagrangian and derive the Euler Equation. Derive the demand function for, c_t^1, c_{t+1}^2. Also derive the optimal savings scheme of the young at time, t, s_t. (2+1+1+1)

d) Derive the difference equation of the percapita capital stock at time; k_{t+1}, and calculate the steady state percpita capital stock, k_{ss} when $\alpha = \frac{1}{3}$; $n = 0.05$; $\beta = 0.99$. (3+2)

e) Derive the resource constraint of the economy at the steady state, and calculate the golden rule level of percapita capital stock, k_g when $\alpha = \frac{1}{3}$; $n = 0.05$; $\beta = 0.99$. (3+2)

f) Comment on the dynamic efficiency of the competitive equilibrium when, $\alpha = \frac{1}{3}$; $n = 0.05$; $\beta = 0.99$. (4)

2. **Consider the infinite horizon Neo-classical growth model discussed in the class. Suppose, the utility function is, $u(c_t) = log(c_t)$; where c_t is the percapita consumption. Suppose, the production function in the percapita form is, $y_t =$**

$f(k_t) = k_t^{\alpha}$; $\alpha = \frac{1}{3}$. **Suppose, the depreciation of capital stock, $\delta = 1$, and the discount factor, $\beta = 0.99$. Also assume that, the rate of growth of population is, $n = 0.05$. (5)**

1. Calculate the steady state real interest rate (1)
2. Calculate, the steady state percapita consumption and investment. Calculate the Golden rule level of percapita capital stock, k_g (1+1+2)

Indian Institute of Technology Kharagpur

Department of Humanities and Social Sciences

End-Autumn Semester Examination 2023-24

Advanced Macroeconomics (HS60243)

Duration: 3 Hours; Full Marks: 50

Answer All Questions

1. **Consider the system of first order difference equation given below. Suppose, the eigenvalues of the coefficient matrix are λ_1, and λ_2.** Derive the stable solution of $\hat{c}_t$, and $\hat{k}_t$ when (i) $0 < \lambda_1 < 1$; $0 < \lambda_2 < 1$, (ii) $\lambda_1 > 1$; $\lambda_2 > 1$, and (iii) $\lambda_1 > 1$; $0 < \lambda_2 < 1$. Identify the globally stable, saddle path stable, and globally unstable system. **(4+2+1+3=10)**

$$\begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \end{bmatrix}$$

2. **Consider the Neo-Classical growth model discussed in the class. Suppose, the utility function of the individual is, $u(c_t) = \frac{c_t^{1-\theta}-1}{1-\theta}$; $0 < \theta \leq 1$, and the Production function is, $y_t = k_t^{\alpha}$. There is 100% depreciation of the capital stock. Moreover, there is no population growth, and the initial population is normalized to unity. The life time utility function is, $\sum_{t=0}^{\infty} \beta^t \; u(c_t)$; $0 < \beta < 1$. (40)**

   a) Write down the resource constraint. (1)
   b) Write down the initial condition, and the terminal condition of the problem. (1+1)
   c) State the Optimization problem. (3)
   d) Set up the relevant Lagrangian, and derive the First Order Conditions. Suppose, the Euler Equation is, Intuitively Interpret the Euler Equation, $\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = \beta\left[\frac{\alpha y_{t+1}}{k_{t+1}}\right]$. (2+1+1+1+2)
   e) Calculate, $\frac{y_{ss}}{k_{ss}}$, and $\frac{c_{ss}}{k_{ss}}$ as a function of the parameters of the model (economic fundamentals). (1+2)

f) Derive the log-linearized Euler Equation, and the Log-linearized resource constraint around the steady state, and write them in the format of a system of first order difference equation. What is the interpretation of θ? Intuitively discuss the impact of the rise of θ on the current and future consumption from the Log-linearized Euler Equation. (1+1+1+2)

g) Suppose, the coefficient matrix of the system of difference equation is, A. Show that, $|A| > 1$. (2)

h) Show that, one of the characteristic roots of the coefficient matrix, A is more than unity, and the other is less than unity. (5)

i) Derive the solution of $\hat{c}_t$ and $\hat{k}_t$. Analyze the Stability of the system using the phase diagram. Draw the Phase Diagram by intuitively identifying the direction of the arrows in the Phase Diagram from the log-linearized system. (3+1+4)

j) What is the slope of the saddle path? Is it negatively sloped? (1+3)