1 Definition

Given two functors, $F, G : \zeta_1 \to \zeta_2$ which map objects and morphisms from ζ_1 to ζ_2, a natural transformation is a bunch of morphisms in ζ_2 that maps objects and morphism produced by F into objects and morphisms produced by G

Notes

- It is denoted by greek letters like α and really represents multiple morphisms under the hood, which live in ζ_2
- Since there might not always be morphisms in ζ_2 that can be utilized for natural transformation, sometimes there is no natural transformation available/possible.

2 Constraints

- **Diagram must commute**

  For each morphism in ζ_1, there will be two morphisms in ζ_2 (one due to F and another due to G). These two morphism and the components (specific to this morphism) of a candidate natural transformation from F to G create a diagram. This diagram must commute for the candidate natural transformation to be a natural transformation

  Notes

  - One mistake I made initially while trying to understand natural transformations was that I assumed unconditional existence of natural transformations. Now I realize that the existence of natural transformation is subject to availability of morphisms in the target category (ζ_2) which can be used as components to build a natural transformation. If no such morphisms exist, a natural transformation cannot exist.
  - For programming it means if there is information loss due to abstraction, or the diagram does not commute, then a natural transformation might not exist between two functors.