1 Category of Vector Spaces

- **Objects**

  Each vector space is an object

- **Morphisms**

  Transformations (represented as matrices) between vector spaces

- **Identities**

  The identity matrix for each vector space

- **Composition**

  Matrix multiplication

2 Monoid of Integers on Addition

This is a singleton category - it contains only one object

- **Objects**

  Integer

  Note the abstraction here. We are not specifying which integer. So, for example both 20 and 25 are things which happen to be represented by the same object *Integer* in this category.

- **Morphisms**

  Set of integers, Z

  This is interesting. Here, the morphisms are just simple integers. So, for example to go from an object, say 20 to an object, say 25 (both are same objects because of our abstraction), we apply the morphism 5. In case of a monoid category, we always start and end at the same object since there is only one object.

- **Identities**

  The integer 0 for our only object *Integer*

- **Composition**

  Integer addition

  That is, to compose say two morphisms 4 and 6, we add them to get a third morphism 10

3 Rubik's cube: Another Monoid

- **Objects**

  Rubik's cube

  The abstraction here allows for any configuration of Rubik's cube to be represented by the singleton object

- **Morphisms**

  An action on Rubik's cube is a morphism. We can see that its possible to combine two actions to produce a more complex action, and so on. In that way, from any configuration of Rubik's cube, there always exists a single (sufficiently complex) action, or morphism that solves the cube. Again a solved cube is no different than an unsolved cube as far as its significance in category is concerned

- **Identities**

  A no-op action, which actually does not change the cube's configuration at all. We can think of it as maybe an action of touching the cube and putting it back. This is something I have performed many times :)

- **Composition**

  Composition here is equivalent to sequencing the morphisms. I guess now its getting clearer that not always can we have a composition that forgets its parts

4 Factors

- **Objects**

  A set of natural numbers, N

- **Morphisms**

  There is a single morphism between any two natural numbers a and b if b is a multiple of a

- **Identities**

  Since every natural number is a multiple of itself (we get the same number when we multiply it by 1), there is an identity for each object in this category

- **Composition**

  Composition here will simply represent a transitive relationship. If a is a factor of b and b is a factor of c then it implies that a is a factor of c, which means there exists a morphism between a and c