FE / Sem I (ALL BRANCHES)
Sub - Applied mathematics-I 30/11/10

Con. 5543-10.

GT-7770

(3 Hours)

[Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** from remaining **six** questions.
(3) Figures to the **right** indicate **full** marks.
(4) Assume the **suitable** data if needed with **justification**.

1. (a) Express $(1+7i)(2-i)^{-2}$ in the form of $r(\cos\theta + i\sin\theta)$ and prove that its fourth power is a real negative number. 5

(b) If $u = z\tan^{-1}\frac{y}{x}$ find the value of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}$ 5

(c) Find $\nabla\cdot\overline{F}$ & $\nabla\times\overline{F}$ where $\overline{F} = \frac{1}{(x^2+y^2)}(x\hat{i} - y\hat{j})$ 5

(d) Test the convergence of the series $\sum\frac{\sqrt{n}}{\sqrt{n^2+1}}x^n$ 5

2.(a) Prove that $\tan^{-1}x = x - \frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+ - - - - -$

Hence expand $\log(1+x^2)$ *in powers of* x 8

(b) $If\ z = f(x,y), x = u\cosh v, y = u\sinh v, prove\ that$ $\left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2}\left(\frac{\partial z}{\partial v}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2$ 6

(c) Show that for real values of a and b, $e^{2ai\cot^{-1}b}\left[\frac{bi-1}{bi+1}\right]^{-a} = 1$ 6

3.(a) State Euler's theorem for a function of 3 variables and hence verify the same for $u = \log\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$ 8

(b) Determine $y_n(0)$ $if\ y = \frac{x^3}{x^2-1}$ 6

(c) Find the constants a, b, c if the normal to the surface $ax^2 + yz + bxz^3 = c$ at P(1,2,1) is parallel to the normal to the surface $y^2 + xz = 61$ at Q(10,1,6). 6

4.(a) Find the cube roots of unity. If w is a complex cube root of unity, prove that

(i) $1+w+w^2=0$ (ii) $\frac{1}{1+2w}+\frac{1}{2+w}-\frac{1}{1+w}=0$ 8

(b) Use Rolle's Theorem to prove that the equation $ax^2+bx=\frac{a}{3}+\frac{b}{3}$ has a root between 0 & 1. 6

(c) Find $xu_x+yu_y+zu_z$ for $u=\cos\frac{xy+yz}{x^2+y^2+z^2}+\sin(\sqrt{x}+\sqrt{y}+\sqrt{z})$ 6

5.(a) $If\ \frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$,

prove that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$ 8

(b) Find a, b, c given that $\lim_{x\to 0}\frac{ae^x-be^{-x}-c}{x-\sin x}=4$ 6

(c) Prove that $\sin h^{-1}(\tan\theta)=\log\left(\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right)$. 6

6.(a) if $y^{\frac{1}{m}}+y^{\frac{-1}{m}}=2x$,
Prove that $(x^2-1)y_{n+2}+x(2n+1)y_{n+1}+(n^2-m^2)y_n=0$ 8

(b) The diameter and altitude of a can in shape of right circular cylinder are measured as 4cm & 6cm respectively. The possible error in each measurement is 0.1cm. Find approximately the maximum possible error in the value computed for the volume & lateral surface. 6

(c) If $x+iy=c\cot(u+iv)$, show that $\frac{x}{\sin 2u}=\frac{-y}{\sinh 2v}=\frac{c}{\cosh 2v-\cos 2u}$ 6

7.(a) Find the stationary values of $x^3+3xy^2-15x^2-15y^2+72x$ 8

(b) If $a=\cos 3\alpha+i\sin 3\alpha, b=\cos 3\beta+i\sin 3\beta, c=\cos 3\gamma+i\sin 3\gamma$ then prove that

$$\sqrt[3]{\frac{ab}{c}}+\sqrt[3]{\frac{c}{ab}}=2\cos(\alpha+\beta-\gamma)$$ 6

(c) if $\bar{r}=a\cos t\,\hat{i}+a\sin t\,\hat{j}+at\tan\alpha\,\hat{k}$, then prove that

(i) $[\dot{\bar{r}}\ \ddot{\bar{r}}\ \dddot{\bar{r}}]=a^3\tan\alpha$ (ii) $|\dot{\bar{r}}\times\ddot{\bar{r}}|=a^2\sec\alpha$ 6