Con. 5530-10.   GT-6234

(3 Hours)   [Total Marks : 100

N.B. (1) Question No. **1** is **compulsory**.
     (2) Attempt any **four** questions out of remaining **six** questions.
     (3) **Figures** to the **right** indicate **full** marks.

1. (a) Find Z-transform of $\{ \cos (ax + b) \}$, $k \geq 0$ 5

   (b) Evaluate $\int_{0}^{\infty} \dfrac{\sin 2t}{t}\, dt$ using Laplace transform.   5

   (c) Express the matrix $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ as the sum of symmetric and skew-   5

   symmetric matrix.

   (d) If $f(x) = c_1\,\phi_1(x) + c_2\phi_2(x) + c_3\phi_3(x)$ where c_1, c_2, c_3 are constants and ϕ_1, ϕ_2, ϕ_3 5

   are orthonormal sets on (a, b). Show that $\int_{a}^{b} [\, f(x)\,]^2\, dx = c_1^2 + c_2^2 + c_3^2$

2. (a) Find $L[\sin h^5 t]$ 6
   (b) Find Fourier sine transform of $f(x)$ 6

   if $f(x) = \begin{cases} 0 & 0 < x < a \\ x & a \leq x \leq b \\ 0 & x > b \end{cases}$

   (c) Find a Fourier series of $f(x) = x^2$ in $(0, 2\pi)$ and hence deduce that—   8

   $$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$

3. (a) Express the function $f(x) = \begin{cases} -e^{kx} & \text{for } x < 0 \\ e^{-kx} & \text{for } x > 0 \end{cases}$ as a Fourier integral and   6

   hence prove that $\int_{0}^{\infty} \dfrac{w \sin wx}{w^2 + k^2}\, dw = \dfrac{\pi}{2}\, e^{-kx}$ if $x > 0$, $k > 0$

   (b) Find $L\left[\dfrac{\cos 2t \sin t}{e^t} \right]$ 6

   (c) Find the inverse of B and then the matrix BAB^{-1} where—   8

   $$B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } A = \frac{1}{2}\begin{bmatrix} 5 & 2 & 1 \\ 1 & 4 & -1 \\ -1 & -2 & 3 \end{bmatrix}.$$

[TURN OVER

4. (a) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$. **6**

(b) Find inverse Laplace transform of $\dfrac{s}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}$ by using convolution **6**
theorem.

(c) Find the rank of matrix A by reducing it to normal form where— **8**

$$A = \begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

5. (a) Find Z transform of $f(k) = 5^k$ for $k < 0$ **6**
$= 3^k$ for $k \geq 0$

(b) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(0, a)$. **6**

(c) Find the inverse Laplace transform of— **8**

(i) $\dfrac{3s+1}{(s+1)\left(s^2 + 2\right)}$ (ii) $\dfrac{s^2}{(s+a)^3}$

6. (a) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ find two matrices P and Q such that PAQ is in normal form. **6**

(b) Find $L\left[t \cdot \sqrt{1+ \sin t}\right]$ **6**

(c) Find Fourier expansion for $f(x) = x - x^2$, $-1 < x < 1$. **8**

7. (a) Obtain half range cosine series for— **6**
$f(x) = x$ in $0 < x < 2$

(b) Solve the following system of equations— **6**
$x + y + z = 3$
$2x + 5y + 7z = 14$
$2x + y - z = 2$

(c) Use Laplace transform to solve : **8**

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1 \quad \text{where } y(0) = 0, \ y'(0) = 1.$$