S.E./Com/Sem IV

Applied Maths IV

Con. 5532-10. **GT-6465**

(3 Hours) [**Total Marks : 100**

N.B. (1) Question No. **1** is **compuylsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Diagonalize the Hermitian matrix **5**

$$\begin{bmatrix} -3 & 2+2i \\ 2-2i & 4 \end{bmatrix}$$

(b) Find the analytic function f(z) whose real part is $r^2 \cos 2\theta - r \sin \theta$ **5**

(c) Show that $\int_C \log z \, dz = 2\pi i$, where C is the unit circle in plane. **5**

(d) Find all basic feasible solutions of the following system of equations **5**

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

2. (a) Verify cayley-Hamilton Theorem for matrix A and Hence find A^{-1}, where **6**

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(b) Prove that $f(z) = x^3 - 3xy^2 + 2xy + i(3x^2y - x^2 + y^2 - y^3)$ is analytic and find f(z) in terms of Z. **6**

(c) Construct dual of the following LPP and solve its dual **8**

Minimize $Z = 0{\cdot}7x_1 + 0{\cdot}5x_2$

Subject to $x_1 \geq 4$,
$x_2 \geq 6$,
$x_1 + 2x_2 \geq 20$,
$2x_1 + x_2 \geq 18$,
$x_1, x_2 \geq 0$.

3. (a) If $A = \begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix}$, find cos A. **6**

(b) Solve the following LPP by Simplex method **6**

Maximize $Z = x_1 + 4x_2$

Subject to $2x_1 + x_1 \leq 3$
$3x_1 + 5x_2 \leq 9$
$x_1 + 3x_2 \leq 5$
$x_1, x_2 \geq 0$.

(c) Show that $\displaystyle\int_0^{\pi} \frac{d\theta}{3 + 2\cos\theta} = \frac{\pi}{\sqrt{5}}$ **8**

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4. (a) Find a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i\,(cx^2 + 2dxy + y^2)$ is analytic. **6**
   (b) Find the bilinear transformation which maps the points $Z = \infty, i, 0$ onto the points $0, i, \infty$. **6**
   (c) Find eigen values and eigen vectors of the matrix A where **8**

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

5. (a) Show that $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory. **6**
   (b) Find the image of the region bounded by **6**
   $x = 0$, $x = 2$, $y = 0$, $y = 2$ in the Z-plane under transformation :
   $W = (1 + i)\,Z$.
   (c) Using the method of Lagranges Multipliers, solve the following NLPP **8**

$$\begin{aligned} &\text{Optimize} && Z = x_1^2 + 5x_2^2 \\ &\text{Subject to} && x_1 + 5x_2 = 7 \\ & && x_1, x_2 \geq 0. \end{aligned}$$

6. (a) Find the orthogonal trajectory of the family of the curves $x^3y - xy^3 = c$. **6**
   (b) Use the Kuhn-Tucker condition to solve the following NLPP. **6**

$$\begin{aligned} &\text{Maximize} && Z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2 \\ &\text{Subject to} && 2x_1 + x_2 \leq 5 \\ & && x_1, x_2 \geq 0. \end{aligned}$$

   (c) Evaluate $\int_C \dfrac{z+6}{z^2-4}\,dz$ where C is the circle. **8**

   (i) $|z| = 1$ (ii) $|z - 2| = 1$ (iii) $|z + 2| = 1$

7. (a) Find Laurents series for $f(z) = \dfrac{2}{(z-1)(z-2)}$ when $1 < |z| < 2$. **6**
   (b) By using residue theorem evaluate $\int_C \dfrac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3}\,dz$ where C is $|z| = 1$. **6**
   (c) Use the dual simplex method to solve the following LPP **8**

$$\begin{aligned} &\text{Minimize} && Z = x_1 + x_2 \\ &\text{Subject to} && 2x_1 + x_2 \geq 2, \\ & && -x_1 - x_2 \geq 1, \\ & && x_1, x_2 \geq 0. \end{aligned}$$