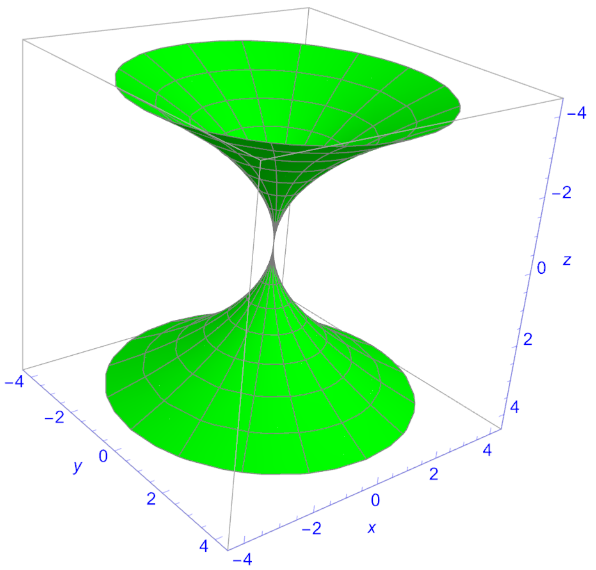
Obtaining Surface of Revolution of Curves

Obtain the surface of revolution of parabola $y^2 = 4ax$.

Solution :

In[∘]:=
```
RevolutionPlot3D[{t^2, 2 t}, {t, -2, 2}, AxesLabel → {x, y, z},
 RevolutionAxis → {0, 0, 1}, PlotStyle → Green, AxesStyle → Blue, Background → White]
```

In[∘]:=

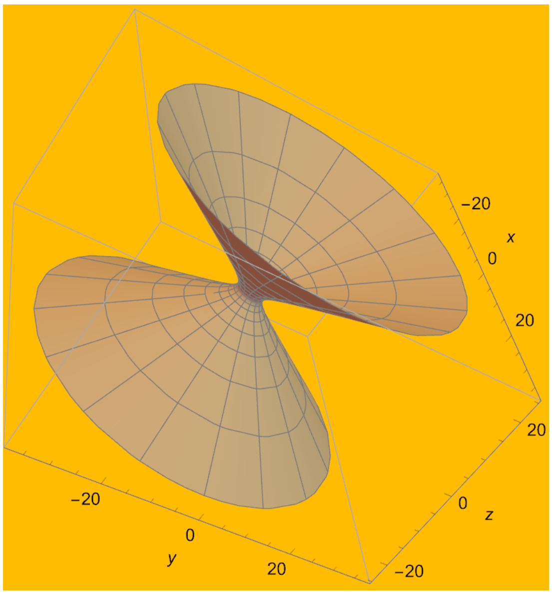


Obtain the surface of revolution of hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$.

Solution :

In[∘]:=
```
RevolutionPlot3D[{3 Cosh[t], 2 Sinh[t]}, {t, -Pi, Pi}, AxesLabel → {x, y, z},
 RevolutionAxis → {0, 0, 1}, PlotStyle → Brown, Background → Orange]
```

Out[∘]=

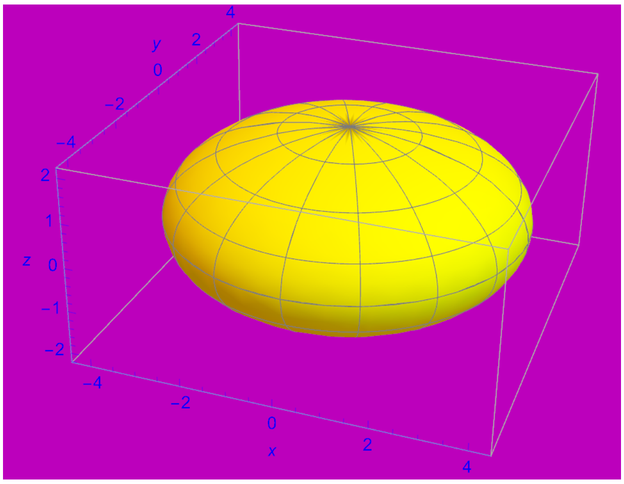


Obtain surface of revolution of ellipse.

Solution :

In[∘]:=
```
RevolutionPlot3D[{4 Cos[t], 2 Sin[t]}, {t, -Pi, Pi}, AxesLabel → {x, y, z},
 RevolutionAxis → {0, 0, 1}, PlotStyle → Yellow, AxesStyle → Blue, Background → Purple]
```

Out[∘]=

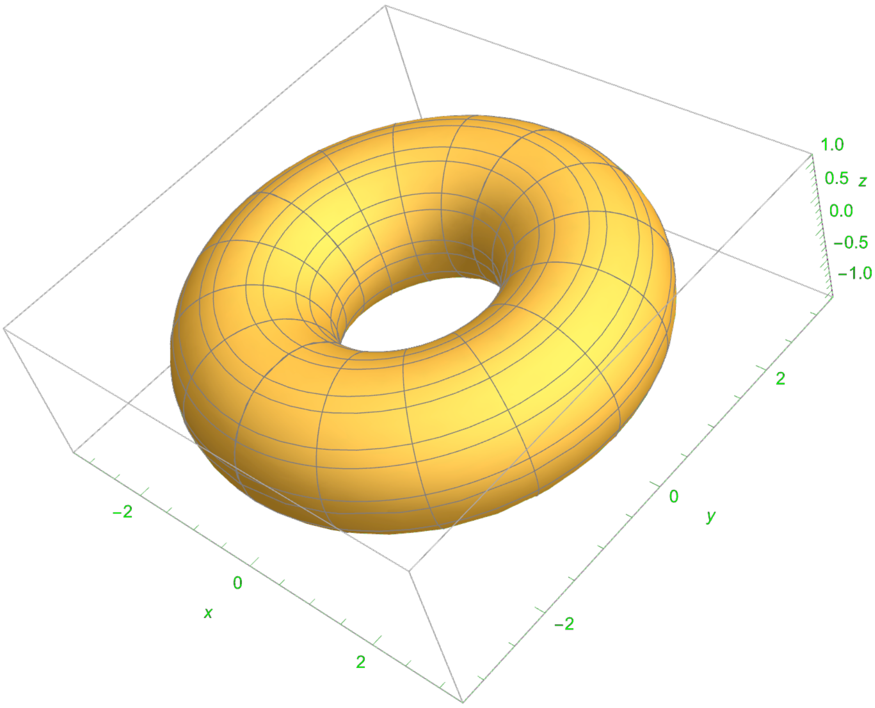


Torus (or Donut)

Solution :

```
RevolutionPlot3D[{2 + Cos[t], Sin[t]},
 {t, -2 Pi, Pi}, AxesLabel → {x, y, z}, AxesStyle → Green]
```

Out[∘]=

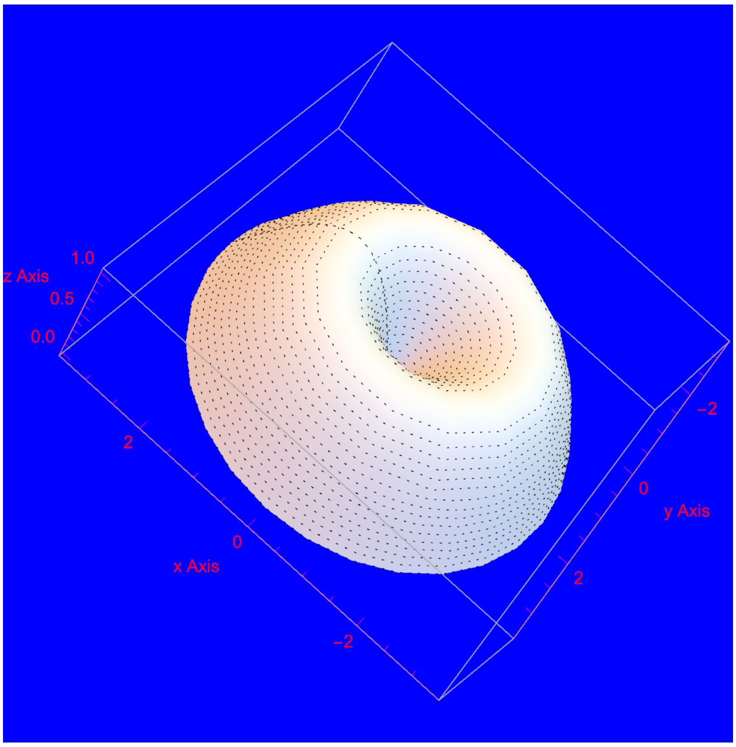


Obtain surface of revolution of curves Sinx, Cosx, Tanx.
Solution :

Sinx

In[ ]:=
```
RevolutionPlot3D[Sin[x], {x, 0, Pi}, AxesLabel → {"x Axis", "y Axis", "z Axis"},
 RevolutionAxis → {0, 0, 1}, PlotStyle → White, MeshFunctions → {#3 &},
 MeshStyle → Dotted, AxesStyle → Red, Background → Blue]
```

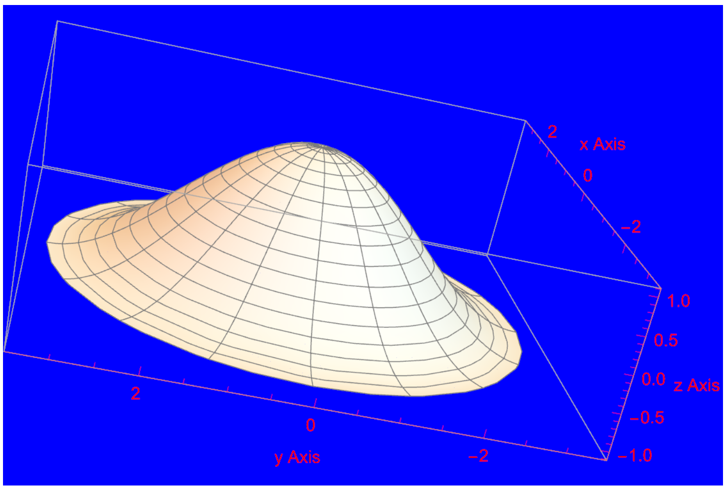
Out[ ]=



Cosx

In[ ]:=
```
RevolutionPlot3D[Cos[x], {x, 0, Pi}, AxesLabel → {"x Axis", "y Axis", "z Axis"},
 RevolutionAxis → {0, 0, 1}, PlotStyle → White, AxesStyle → Red, Background → Blue]
```

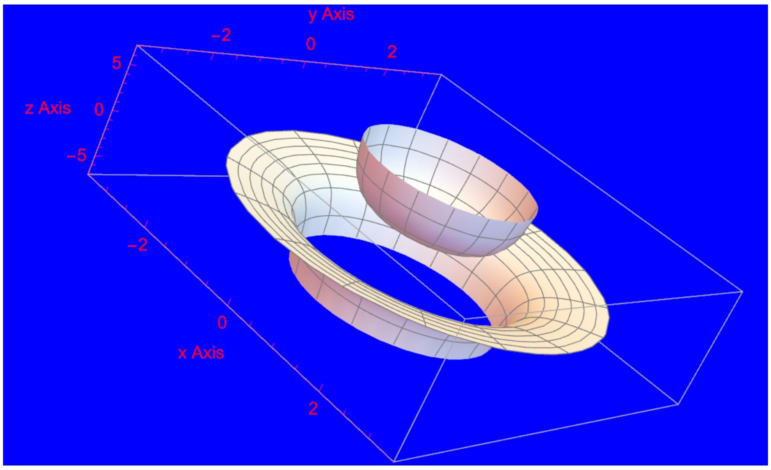
Out[ ]=



Tanx

In[∘]:=
```
RevolutionPlot3D[Tan[x], {x, 0, Pi}, AxesLabel → {"x Axis", "y Axis", "z Axis"},
 RevolutionAxis → {0, 0, 1}, PlotStyle → White, AxesStyle → Red, Background → Blue]
```

Out[∘]=

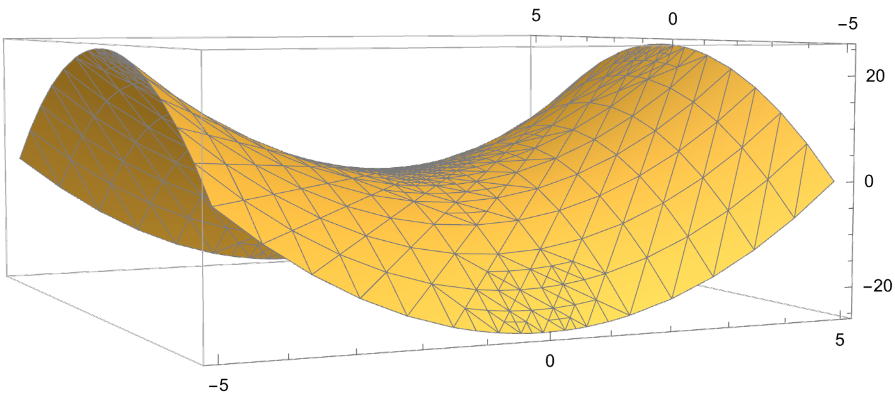


Drawing the surface and finding its level curves from the given height

Equation : $f(x) = x^2 - y^2$

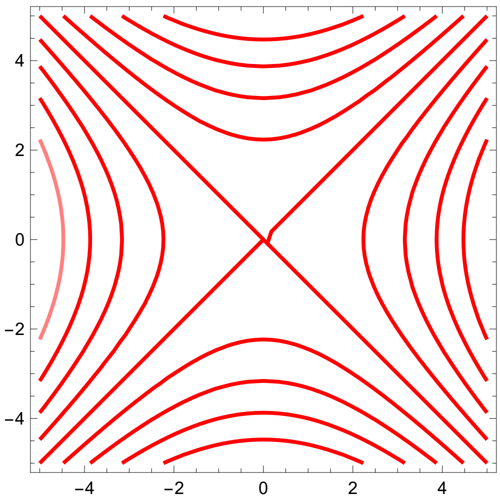
In[∘]:=
```
Plot3D[x^2 - y^2, {x, -5, 5}, {y, -5, 5}, Mesh → All, MeshFunctions → (#3 &)]
```

Out[∘]=



In[∘]:=
```
ContourPlot[x^2 - y^2, {x, -5, 5}, {y, -5, 5},
  ContourShading → False, ContourStyle → {{Thick, Red}}]
```

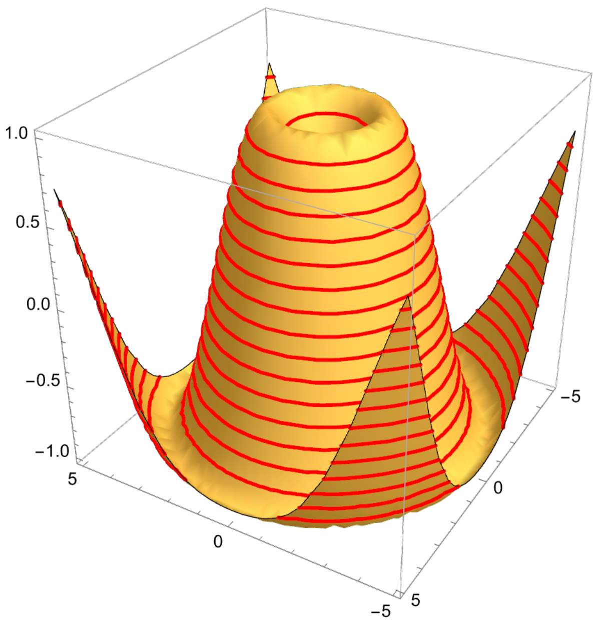
Out[∘]=



Equation : $f(x) = \mathrm{Sin}\sqrt{x^2 + y^2}$

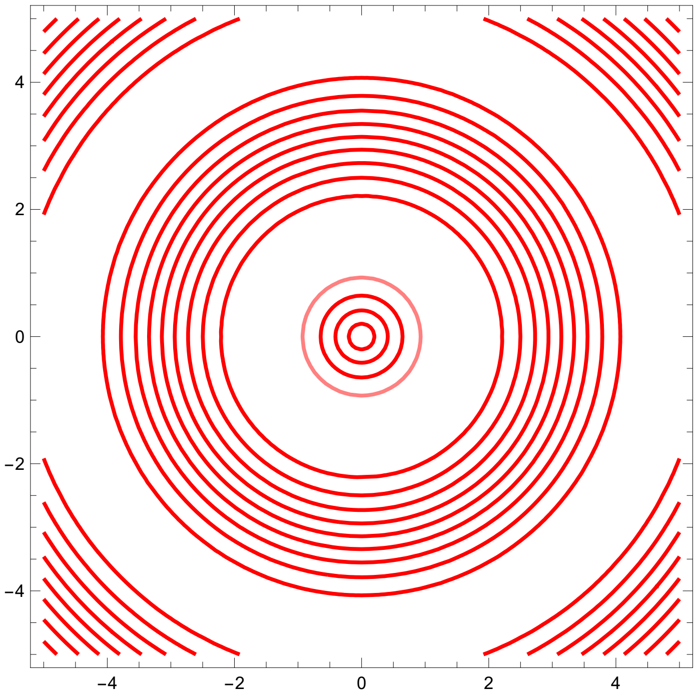
In[∘]:=
```
Plot3D[Sin[√(x^2 + y^2)], {x, -5, 5}, {y, -5, 5},
  MeshFunctions → (#3 &), BoxRatios → {2, 2, 2}, MeshStyle → {Thick, Red}]
```

Out[∘]=



In[ ]:= ContourPlot[Sin[$\sqrt{x^2 + y^2}$], {x, -5, 5}, {y, -5, 5},
ContourShading → False, ContourStyle → {{Thick, Red}}]

Out[ ]=



Heat Equation

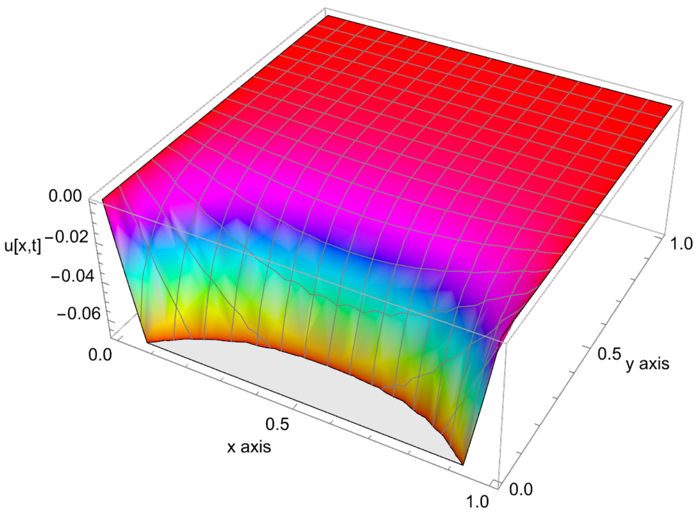
Equation : $\frac{\partial U[X, T]}{\partial T} - \frac{\partial^2 U[X, T]}{\partial X^2} = 0$, U[X, 0] = x^2 − x ,
U[0, t] = 0, u[l, t] = 0, 0 < x < l l = 1

In[ ]:=
```
heqn1 = {D[u[x, t], t] - D[u[x, t], x, x] == 0, u[x, 0] == x^2 - x, u[0, t] == 0, u[1, t] == 0};
sol1 = u[x, t] /. NDSolve[heqn1, u[x, t], {x, 0, 1}, {t, 0, 1}]
Plot3D[sol1, {x, 0, 1}, {t, 0, 1},
 ColorFunction → Hue, AxesLabel → {"x axis", "y axis", "u[x,t]"}]
```

Out[ ]=

{InterpolatingFunction[Domain: {{0., 1.}, {0., 1.}} Output: scalar][x, t]}

Out[ ]=



Wave Equation

Equation : $\dfrac{\partial^2 U[X, T]}{\partial T^2} - 4\dfrac{\partial^2 U[X, T]}{\partial X^2} = 0,$

$U[X, 0] = 0,\ U_t[X, 0] = X[1 - X],\ U[0, T] = 0,\ U[1, T] = 0$

In[∘]:=
```
weqn2 = {D[u[x, t], t, t] - 4 D[u[x, t], x, x] == 0, u[x, 0] == 0,
   u[0, t] == 0, u[1, t] == 0, Derivative[0, 1][u][x, 0] == x (1 - x)}
sol2 = u[x, t] /. NDSolve[weqn2, u[x, t], {x, 0, 1}, {t, 0, 1}]
Plot3D[sol2, {x, 0, 1}, {t, 0, 1},
 AxesLabel → {"x axes", "y axes", "u[x,t]"}, AxesOrigin → {0, 0}, ColorFunction → Hue]
```

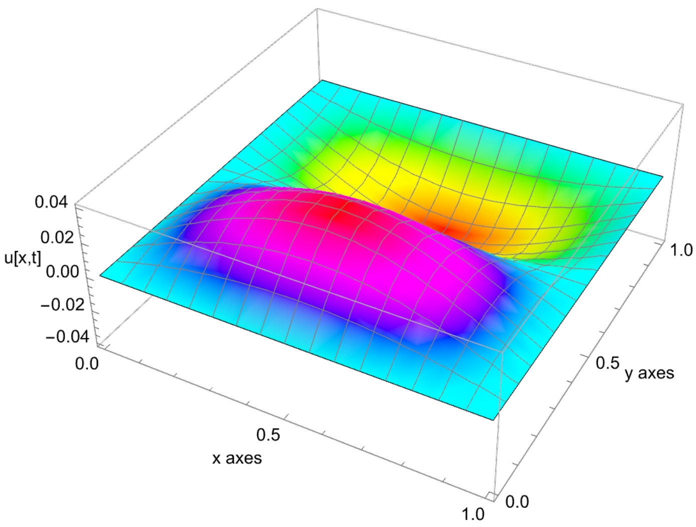
Out[∘]=

```
{u^(0,2)[x, t] - 4 u^(2,0)[x, t] == 0, u[x, 0] == 0,
 u[0, t] == 0, u[1, t] == 0, u^(0,1)[x, 0] == (1 - x) x}
```

Out[∘]=

```
{InterpolatingFunction[ Domain: {{0., 1.}, {0., 1.}} Output: scalar ][x, t]}
```

Out[∘]=



Growth Model (Exponential Case)

Exponential Growth : -

If a function x(t) grows continuously at a rate k>0, then x(t) has the form

$$x(t) = x_0\, e^{kt}$$

where x_0 is the initial amount and t is the time. In this case , the quantity x(t) is said to exhibit exponential growth, and k is the growth rate. It’s differential model is given by 2%

$$\frac{dx}{dt} = kx\,,\ k > 0$$

Suppose that the population of a certain country grows at an annual rate of 2 × %.If the current population is 3 million, what will the population be in 10 years ? Also plot the graph of the solution.

Solution : Here x_0 = 3, k = 2 × % = 0.02, t = 10 years, x[t] = ?

In[ ]:=
```
Sol = DSolve[x'[t] == k x[t], x[t], t]
Sol1 = Evaluate[x[t] /. Sol[[1]] /. {k → 0.02, C[1] → 3}]
Plot[Sol1, {t, -50, 50}, PlotStyle → {Pink, Thickness[0.03]},
 AxesLabel → {t, x[t]}, AxesStyle → Arrowheads[{-.03, .03}]]
x[10] = Evaluate[Sol1 /. {t → 10}]
```

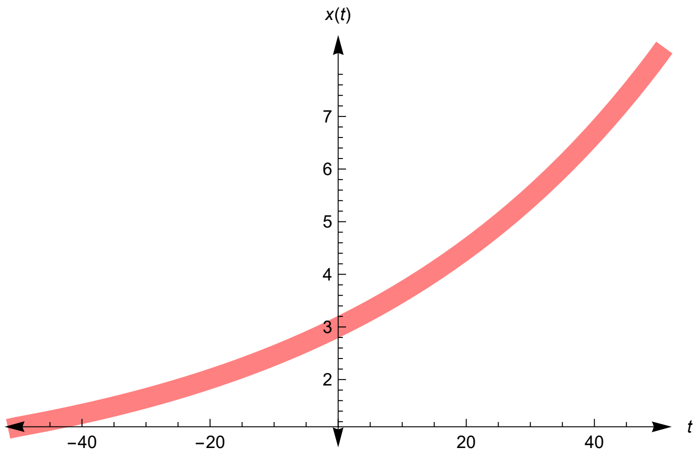
Out[ ]=

$\{\{x[t] \to e^{k\,t}\, c_1\}\}$

Out[ ]=

$3\, e^{0.02\,t}$

Out[ ]=



Out[ ]=

3.66421

Conclusion : Hence population after 10 years will be 3.66421.

Decay Model (Exponential Case)

Exponential Decay :
If a function x (t) decreases continuously at a rate k > 0, then x (t) has the form

$$x(t) = x_0\, e^{-kt}$$

where x_0 is the initial amount x_0. In this case, the quantity x (t) is said to exhibit exponential decay, and k is the decay rate. It ' s differential model is given by $\frac{dx}{dt} = -kx$, k > 0.

Suppose that a certain radioactive element has an annual decay rate of 10 × %. Starting with a 200 gram sample of the element , how many grams will be left in 3 years ?

Solution : Here k = 10 × % = 0.1, x_0 = 200, t = 3, x (t) = ?

In[∘]:=
```
Sol = DSolve[x'[t] == -k x[t], x[t], t]
Sol1 = Evaluate[x[t] /. Sol[[1]] /. {k → 0.1, C[1] → 200}]
Plot[Sol1, {t, -50, 50}, PlotStyle → {Green, Thickness[0.02]}, AxesLabel → {t, x[t]},
 AxesStyle → Arrowheads[{-.03, .03}], PlotLabel → "Decay Model"]
x[3] = Evaluate[Sol1 /. {t → 3}]
```

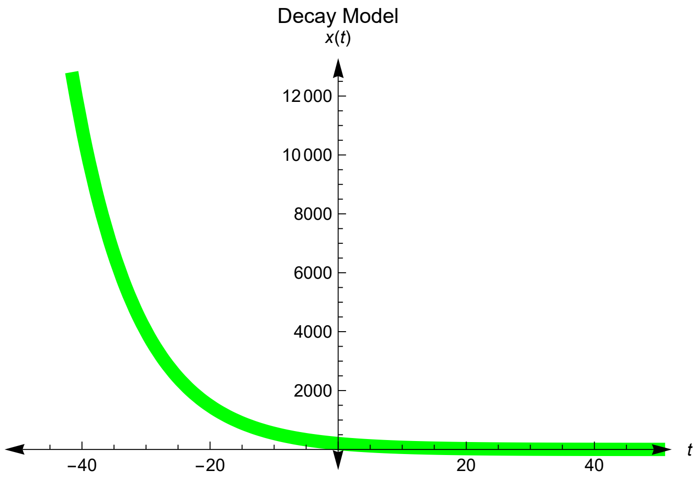
Out[∘]=

$\{\{x[t] \to e^{-k\,t}\, c_1\}\}$

Out[∘]=

$200\, e^{-0.1\,t}$

Out[∘]=



Out[∘]=

148.164

Conclusion : Hence 148.164 gms of radioactive element will be left after 3 years.