SVKM'S

Mithibai College of Arts, Chauhan Institute of Science & Amrutben Jivanlal College of Commerce and Economics (Autonomous)

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Programme: B.Sc. Computer Science **Max. Marks:** 75

Course Name: Linear Algebra with Python **Course Code:** USMACS405

Date: **Time:** **Duration:** 2 hours 30 minutes

MODEL ANSWER PAPER

Q1 ATTEMPT ANY THREE 21

A Given u = (2, -1, 2, 1, 4), v = (-1, -3, 2, 2, -3) find 7

i. distance between the vectors **u** and **v**: **2*(-1)+(-1)(-3)+2*2+1*2+4(-3)=-5 (1 mark)**

ii. angle between the vectors **u** and **v**:

$$\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{-5}{(\sqrt{26})\cdot(3\sqrt{3})} = -\frac{5\sqrt{78}}{234}$$ **(2 marks)**

iii. projection between the vectors **u** and **v**:

$$\frac{-5}{(3\sqrt{3})^2}\cdot\langle -1,-3,2,2,-3\rangle$$ **(2 marks)**

iv. norm of the vector **v**: $|-1|^2+|-3|^2+|2|^2+|2|^2+|-3|^2=27$. **(2 marks)**

B Given **z** and **w** are complex numbers where **z = 3 -2i** and **w = -1 – 4i** then find 7

i. z + w: 2−6i **(1 mark)**

ii. zw: (3(-1)-(-2)(-4))+(3(-4)-2(-1))i =-11-10i **(2 mark)**

iii. conjugate of z: 3+2i **(1 mark)**

iv. w / z : {(-1-4i)(3+2i)}/{(3-2i)(3+2i)} ={5}/{13}-{14}/{13}i **(2 mark)**

v. |z|: √{9+(-2)²} = √13 **(1 mark)**

C Given that 7

$$A=\begin{bmatrix} -2 & 1 & 4 \\ 2 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad B=\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & -3 & 2 \end{bmatrix}$$

i. Find the difference A – B **(2 mark)**

$$\begin{bmatrix} -2 & 1 & 4 \\ 2 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & -3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} (-2)-(2) & (1)-(1) & (4)-(1) \\ (2)-(1) & (1)-(-2) & (-2)-(3) \\ (0)-(1) & (1)-(-3) & (1)-(2) \end{bmatrix} = \begin{bmatrix} -4 & 0 & 3 \\ 1 & 3 & -5 \\ -1 & 4 & -1 \end{bmatrix}$$

ii. Find the transpose of matrix A **(1 mark)**

$$\begin{bmatrix} -2 & 2 & 0 \\ 1 & 1 & 1 \\ 4 & -2 & 1 \end{bmatrix}$$

iii. Find the product of matrices **(4 mark)**

$$\begin{bmatrix} -2 & 1 & 4 \\ 2 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & -3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} (-2)\cdot(2)+(1)\cdot(1)+(4)\cdot(1) & (-2)\cdot(1)+(1)\cdot(-2)+(4)\cdot(-3) & (-2)\cdot(1)+(1) \\ (2)\cdot(2)+(1)\cdot(1)+(-2)\cdot(1) & (2)\cdot(1)+(1)\cdot(-2)+(-2)\cdot(-3) & (2)\cdot(1)+(1)\cdot \\ (0)\cdot(2)+(1)\cdot(1)+(1)\cdot(1) & (0)\cdot(1)+(1)\cdot(-2)+(1)\cdot(-3) & (0)\cdot(1)+(1)\cdot \end{bmatrix}$$

$$\begin{bmatrix} 1 & -16 & 9 \\ 3 & 6 & 1 \\ 2 & -5 & 5 \end{bmatrix}$$

D Given that **7**

$$A=\begin{bmatrix} -2 & -3 & 2 \\ -4 & 1 & 4 \\ -1 & -1 & -1 \end{bmatrix} \quad B=\begin{bmatrix} -1 & 2 & -1 \\ -1 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

i. Find the sum of the matrices **(2 marks)**

$$\begin{bmatrix} -2 & -3 & 2 \\ -4 & 1 & 4 \\ -1 & -1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & -1 \\ -1 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} (-2)+(-1) & (-3)+(2) & (2)+(-1) \\ (-4)+(-1) & (1)+(1) & (4)+(-2) \\ (-1)+(1) & (-1)+(2) & (-1)+(1) \end{bmatrix} = \begin{bmatrix} -3 & -1 & 1 \\ -5 & 2 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

ii. Find the inverse of matrix B from its adjugate matrix only **(5 mark)**

Calculate the determinant of the matrix: it equals -4

Calculate the adjugate of the matrix: it is $\begin{bmatrix} 5 & -4 & -3 \\ -1 & 0 & -1 \\ -3 & 4 & 1 \end{bmatrix}$

The inverse matrix is the adjugate matrix divided by the determinant.

Thus, the inverse matrix is $\begin{bmatrix} -\frac{5}{4} & 1 & \frac{3}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{3}{4} & -1 & -\frac{1}{4} \end{bmatrix}$.

Q2 ATTEMPT ANY THREE **21**

A Solve the following system of linear equations using Gaussian Elimination **7**

$x + 2y - 3z = 1$ — (1)

$2x + 5y - 3z = 4$ — (2)

$3x + 8y + 3z = 7$ — (3)

Subtract (2) from (1)

$x + 2y - 3(z) = 1$

$-2x - 5y + 3z = -4$

$-x - 3y = -3$ (4)

2 marks

Adding (2) & (3)

$2x + 5y + 3z = 4$

$3x + 8y + 3z = 7$

$5x + 13y = 11$ (5)

2 Marks

Mul. (4) by 5 & add with (5)

$-5x - 15y = -15$

$5x + 13y = 11$

$y = 2.$

2 marks

Substituting y in (4).

$-x - 3(2) = -3$

$x = -3$

Substituting x, y in (1)

$-3 + 2(2) - 3z = 1$

$z = 0$

The solⁿ is $(-3, 2, 0)$

1 mark.

B Reduced the following matrix to it echelon form and then to its row-canonical form **7**

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 1 & 2 & -2 \end{bmatrix}$$

(1 mark for each step)

Subtract row 1 multiplied by 2 from row 2: $R_2 = R_2 - 2R_1$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -7 \\ 1 & 2 & -2 \end{bmatrix}$$

Subtract row 1 from row 3: $R_3 = R_3 - R_1$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -7 \\ 0 & 0 & -5 \end{bmatrix}$$

Divide row 2 by -3: $R_2 = -\frac{R_2}{3}$.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{7}{3} \\ 0 & 0 & -5 \end{bmatrix}$$

Subtract row 2 multiplied by 2 from row 1: $R_1 = R_1 - 2R_2$.

$$\begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & \frac{7}{3} \\ 0 & 0 & -5 \end{bmatrix}$$

Divide row 3 by -5: $R_3 = -\frac{R_3}{5}$.

$$\begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & \frac{7}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Add row 3 multiplied by $\frac{5}{3}$ to row 1: $R_1 = R_1 + \frac{5R_3}{3}$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{7}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Subtract row 3 multiplied by $\frac{7}{3}$ from row 2: $R_2 = R_2 - \frac{7R_3}{3}$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

C Find the basis and the rank of following matrix using row space of the matrix: **7**

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 2 & 2 & 2 \\ -2 & -1 & 3 & -2 \\ -1 & 3 & 3 & -2 \end{bmatrix}$$

(1 mark for each step)

The row echelon form of the matrix is $\begin{bmatrix} 1 & 2 & -2 & -1 \\ 0 & -2 & 7 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -17 \end{bmatrix}$

The row space is a space spanned by the nonzero rows of the reduced matrix.

Thus, the row space is $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 7 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -17 \end{bmatrix} \right\}$.

The rank is 4

D i. Consider the basis S = {(1,2), (4,7)} of R^2 and let **v** = (5,8) presented in the standard basis. Find the coordinates of **v** in the basis S, that is find $[\mathbf{v}]_S$. **(3 Marks)** 7

We set (5,8) = $c_1(1,2) + c_2(4,7)$ or

$$c_1 + 4c_2 = 5$$
$$2c_1 + 7c_2 = 8$$

We get the matrix equation

$$\begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

The matrix is just the matrix whose columns are the basis vectors of S. The solution to this is

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}^{-1}\begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$c_1 = -3$ $\quad$ $c_2 = 2$

ii. Let S = {(2,3), (1,4)} and T = {(0,2), (-1,5)} be two bases for R^2, and let

$[\mathbf{v}]_S = (-2,6)$

Find $[\mathbf{v}]_T$ **(4 Marks)**

We can first find **v** in the standard basis. We have $\mathbf{v} = A_S[\mathbf{v}]_S$ where A_S is the matrix whose columns are the vectors in S. Now convert to the T basis.

$[\mathbf{v}]_T = (A_T)^{-1}\mathbf{v} = (A_T)^{-1}A_S[\mathbf{v}]_S$ or

$$[\mathbf{v}_T]=\begin{pmatrix}0 & -1\\ 2 & 5\end{pmatrix}^{-1}\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}-2\\ 6\end{pmatrix}=\begin{pmatrix}14\\ -2\end{pmatrix}$$

Q3 ATTEMPT ANY THREE **21**

A For the following matrix A **7**

i. Find all eigenvalues and corresponding eigenvectors. **(6 marks)**

ii. Find matrices P and D such that P is nonsingular and $D = P^{-1}AP$ is diagonal.**(1 marks)**

$$A=\begin{bmatrix}3 & 2\\ 3 & -2\end{bmatrix}$$

Start from forming a new matrix by subtracting λ from the diagonal entries of the given matrix:

$$\begin{bmatrix}3-\lambda & 2\\ 3 & -\lambda-2\end{bmatrix}$$

The determinant of the obtained matrix is $(\lambda-4)(\lambda+3)$

Solve the equation $(\lambda-4)(\lambda+3)=0$.

The roots are $\lambda_1 = 4, \lambda_2 = -3$

These are the eigenvalues.

Next, find the eigenvectors.

- $\lambda = 4$

$$\begin{bmatrix}3-\lambda & 2\\ 3 & -\lambda-2\end{bmatrix}=\begin{bmatrix}-1 & 2\\ 3 & -6\end{bmatrix}$$

The null space of this matrix is $\left\{\begin{bmatrix}2\\ 1\end{bmatrix}\right\}$

This is the eigenvector.

- $\lambda = -3$

$$\begin{bmatrix}3-\lambda & 2\\ 3 & -\lambda-2\end{bmatrix}=\begin{bmatrix}6 & 2\\ 3 & 1\end{bmatrix}$$

The null space of this matrix is $\left\{\begin{bmatrix}-\frac{1}{3}\\ 1\end{bmatrix}\right\}$

This is the eigenvector.

Form the matrix P, whose column i is eigenvector no. i: $P = \begin{bmatrix} 2 & -\frac{1}{3} \\ 1 & 1 \end{bmatrix}$.

Form the diagonal matrix D whose element at row i, column i is eigenvalue no. i: $D = \begin{bmatrix} 4 & 0 \\ 0 & -3 \end{bmatrix}$.

The matrices P and D are such that the initial matrix $\begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix} = PDP^{-1}$.

B Let u = (1, 3, -4, 2), v = (4,-2, 2, 1), w = (5, -1, -2, 6) in R^4. **7**

(i) Show <3u - 2v, w> = 3<u, w> - 2<v, w> **(3 marks)**

By definition.

$$\langle u, w\rangle = 5 - 3 + 8 + 12 = 22 \qquad \text{and} \qquad \langle v, w\rangle = 20 + 2 - 4 + 6 = 24$$

Note that $3u - 2v = (-5, 13, -16, 4)$. Thus,

$$\langle 3u - 2v,\ w\rangle = -25 - 13 + 32 + 24 = 18$$

(ii) Normalize u and v **(2 marks)**

By definition,

$$\|u\| = \sqrt{1 + 9 + 16 + 4} = \sqrt{30} \qquad \text{and} \qquad \|v\| = \sqrt{16 + 4 + 4 + 1} = 5$$

We normalize u and v to obtain the following unit vectors in the directions of u and v, respectively:

$$\hat{u} = \frac{1}{\|u\|}u = \left(\frac{1}{\sqrt{30}}, \frac{3}{\sqrt{30}}, \frac{-4}{\sqrt{30}}, \frac{2}{\sqrt{30}}\right) \qquad \text{and} \qquad \hat{v} = \frac{1}{\|v\|}v = \left(\frac{4}{5}, \frac{-2}{5}, \frac{2}{5}, \frac{1}{5}\right)$$

(iii) Does this vector space has positive definite property? **(2 marks)**

$\langle 1, 3, -4, 2\rangle \cdot \langle 1, 3, -4, 2\rangle = (1)\cdot(1) + (3)\cdot(3) + (-4)\cdot(-4) + (2)\cdot(2) = 30.$

<u,u> is greater than zero so this vector space has positive definite property.

C Explain the Gram–Schmidt orthogonalization process. **(Each step 1 mark)** **7**

Suppose $\{v_1, v_2, \dots, v_n\}$ is a basis of an inner product space V. One can use this basis to construct an orthogonal basis $\{w_1, w_2, \dots, w_n\}$ of V as follows. Set

$$w_1 = v_1$$

$$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle}w_1$$

$$w_3 = v_3 - \frac{\langle v_3, w_1\rangle}{\langle w_1, w_1\rangle}w_1 - \frac{\langle v_3, w_2\rangle}{\langle w_2, w_2\rangle}w_2$$

....

$$w_n = v_n - \frac{\langle v_n, w_1\rangle}{\langle w_1, w_1\rangle}w_1 - \frac{\langle v_n, w_2\rangle}{\langle w_2, w_2\rangle}w_2 - \cdots - \frac{\langle v_n, w_{n-1}\rangle}{\langle w_{n-1}, w_{n-1}\rangle}w_{n-1}$$

In other words, for $k = 2, 3, \dots, n$, we define

$$w_k = v_k - c_{k1}w_1 - c_{k2}w_2 - \cdots - c_{k,k-1}w_{k-1}$$

where $c_{ki} = \langle v_k, w_i\rangle / \langle w_i, w_i\rangle$ is the component of v_k along w_i. By Theorem 7.8, each w_k is orthogonal the preceeding w's. Thus, $w_1, w_2, \dots, w_n$ form an orthogonal basis for V as claimed. Normalizing each will then yield an orthonormal basis for V.

D Apply the Gram–Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace U of R4 spanned by v1 = (1, -2, 2, 1), v2 = (1, 3, 1, -1), v3 = (1, 1, 4, 2) **7**

Step 1

$$\vec{u_1} = \vec{v_1} = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 1 \end{bmatrix}$$

$$\vec{e_1} = \frac{\vec{u_1}}{|\vec{u_1}|} = \begin{bmatrix} \frac{\sqrt{10}}{10} \\ -\frac{\sqrt{10}}{5} \\ \frac{\sqrt{10}}{5} \\ \frac{\sqrt{10}}{10} \end{bmatrix}$$

Step 2

$$\vec{u_2} = \vec{v_2} - \mathrm{proj}_{\vec{u_1}}(\vec{v_2}) = \begin{bmatrix} \frac{7}{5} \\ \frac{11}{5} \\ \frac{9}{5} \\ -\frac{3}{5} \end{bmatrix}$$

$$\vec{e_2} = \frac{\vec{u_2}}{|\vec{u_2}|} = \begin{bmatrix} \frac{7\sqrt{65}}{130} \\ \frac{11\sqrt{65}}{130} \\ \frac{9\sqrt{65}}{130} \\ -\frac{3\sqrt{65}}{130} \end{bmatrix}$$

Step 3

$$\vec{u_3} = \vec{v_3} - \mathrm{proj}_{\vec{u_1}}(\vec{v_3}) - \mathrm{proj}_{\vec{u_2}}(\vec{v_3}) = \begin{bmatrix} -\frac{31}{26} \\ \frac{10}{13} \\ \frac{7}{13} \\ \frac{43}{26} \end{bmatrix}$$

$$\vec{e_3} = \frac{\vec{u_3}}{|\vec{u_3}|} = \begin{bmatrix} -\frac{31\sqrt{3406}}{3406} \\ \frac{10\sqrt{3406}}{1703} \\ \frac{7\sqrt{3406}}{1703} \\ \frac{43\sqrt{3406}}{3406} \end{bmatrix}$$

ANSWER

The set of the orthonormal vectors is $\left\{ \begin{bmatrix} \frac{\sqrt{10}}{10} \\ -\frac{\sqrt{10}}{5} \\ \frac{\sqrt{10}}{5} \\ \frac{\sqrt{10}}{10} \end{bmatrix}, \begin{bmatrix} \frac{7\sqrt{65}}{130} \\ \frac{11\sqrt{65}}{130} \\ \frac{9\sqrt{65}}{130} \\ -\frac{3\sqrt{65}}{130} \end{bmatrix}, \begin{bmatrix} -\frac{31\sqrt{3406}}{3406} \\ \frac{10\sqrt{3406}}{1703} \\ \frac{7\sqrt{3406}}{1703} \\ \frac{43\sqrt{3406}}{3406} \end{bmatrix} \right\} \approx$

Q4	**ATTEMPT ANY THREE**	**12**

A i. Explain linear combination of vectors. **(2 marks)** **4**

Now suppose we are given vectors $u_1, u_2, \ldots, u_m$ in $\mathbf{R}^n$ and scalars $k_1, k_2, \ldots, k_m$ in $\mathbf{R}$. We can multiply the vectors by the corresponding scalars and then add the resultant scalar products to form the vector

$$v = k_1u_1 + k_2u_2 + k_3u_3 + \cdots + k_mu_m$$

Such a vector v is called a *linear combination* of the vectors $u_1, u_2, \ldots, u_m$.

ii. Explain degenerate linear equations and its solutions.**(2 marks)**

A linear equation is said to be *degenerate* if all the coefficients are zero—that is, if it has the form

$$0x_1 + 0x_2 + \cdots + 0x_n = b \tag{3.3}$$

The solution of such an equation depends only on the value of the constant b. Specifically,

(i) If $b \neq 0$, then the equation has no solution.

(ii) If $b = 0$, then every vector $u = (k_1, k_2, \ldots, k_n)$ in K^n is a solution.

B i. What is a system of linear equations and its solutions? **(2 marks)** **4**

A system of linear equations is a list of linear equations with the same unknowns. In particular, a system of m linear equations $L_1, L_2, \ldots, L_m$ in n unknowns $x_1, x_2, \ldots, x_n$ can be put in the *standard form*

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned} \tag{3.2}$$

where the a_{ij} and b_i are constants. The number a_{ij} is the *coefficient* of the unknown x_j in the equation L_i, and the number b_i is the *constant* of the equation L_i.

ii. What is row canonical form of a matrix? **(2 marks)**

A matrix A is said to be in *row canonical form* (or *row-reduced echelon form*) if it is an echelon matrix—that is, if it satisfies the above properties (1) and (2), and if it satisfies the following additional two properties:

(3) Each pivot (leading nonzero entry) is equal to 1.

(4) Each pivot is the only nonzero entry in its column.

C i. Define inner product spaces **(2 marks)** **4**

Let V be a real vector space. Suppose to each pair of vectors $u, v \in V$ there is assigned a real number, denoted by $\langle u, v\rangle$. This function is called a *(real) inner product* on V if it satisfies the following axioms:

$[\mathrm{I}_1]$ **(*Linear Property*):** $\langle au_1 + bu_2, v\rangle = a\langle u_1, v\rangle + b\langle u_2, v\rangle$.

$[\mathrm{I}_2]$ **(*Symmetric Property*):** $\langle u, v\rangle = \langle v, u\rangle$.

$[\mathrm{I}_3]$ **(*Positive Definite Property*):** $\langle u, u\rangle \geq 0$.; and $\langle u, u\rangle = 0$ if and only if $u = 0$.

The vector space V with an inner product is called a *(real) inner product space*.

ii. What are orthogonol complements? **(2 marks)**

Let S be a subset of an inner product space V. The orthogonal complement of S, denoted by $S^{\perp}$ (read "S perp") consists of those vectors in V that are orthogonal to every vector $u \in S$; that is,

$$S^{\perp} = \{v \in V : \langle v, u \rangle = 0 \text{ for every } u \in S\}$$

D i. Describe diagonalization **(2 marks)** **4**

Let A be any n-square matrix. Then A can be represented by (or is similar to) a diagonal matrix $D = \text{diag}(k_1, k_2, \ldots, k_n)$ if and only if there exists a basis S consisting of (column) vectors $u_1, u_2, \ldots, u_n$ such that

$$\begin{aligned} Au_1 &= k_1u_1 \\ Au_2 &= k_2u_2 \\ &\ldots\ldots\ldots \\ Au_n &= k_nu_n \end{aligned}$$

In such a case, A is said to be *diagonizable*. Furthermore, $D = P^{-1}AP$, where P is the nonsingular matrix whose columns are, respectively, the basis vectors $u_1, u_2, \ldots, u_n$.

ii. Define eigenvalue and eigenvector. **(2 marks)**

Let A be any square matrix. A scalar λ is called an *eigenvalue* of A if there exists a nonzero (column) vector v such that

$$Av = \lambda v$$

Any vector satisfying this relation is called an *eigenvector* of A *belonging* to the eigenvalue λ.

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