

## A1 - Making Simple Decisions

16-1

- Utility function: - Assigns a single no. to express the desirability of a state. Denoted as  $U(s)$  for state  $s$ .  
+ maps state to real nos.
- States give complete snapshots of world. Similar to situations.
- Nondeterministic action: - An action that will have possible outcome states  $\text{Result}_i(A)$ ; where the index  $i$  ranges over diff. outcomes.
- Expected Utility  
$$EU(A|\epsilon) = \sum_i P(\text{Result}_i(A) | D_0(A), \epsilon) U(\text{Result}_i(A))$$
  
where  
 $A$  - Action,  $\epsilon$  - All evidence,  $\text{Result}_i(A)$  -  $U$ -Utility.  
 $D_0(A)$  - Proposition that  $A$  is executed.
- Maximum Expected Utility: A rational agent should choose action that maximizes expected utility.
- If an agent maximizes, ~~the~~ utility  $f^n$ , that correctly reflects the performance measure by which its behaviour its been judge, then it will achieve highest possible performance by score if we avg. over environments in which the agent could be placed.

### Basis of utility Theory

Lottery - Is a probability distribution over a set of actual outcomes.

+ A lottery 'L' with possible outcomes  $C_1, \dots, C_n$  occurring with prob.  $p_1, \dots, p_n$  is written as:

$$L = [p_1, C_1; p_2, C_2; \dots p_n, C_n]$$

- X - The primary ~~utility~~ <sup>issue</sup> for utility theory is to understand how preferences betw complex lotteries is related to underlying states in those lotteries. i.e., how ~~the~~ a state(s) in a complex lottery gives more utility/preference to its lottery compare to state(s) ~~for~~ <sup>of</sup> other ~~complex~~ <sup>complex</sup> lottery.

- Q. 1. Lotteries are of 2 types.
- + Simple: The participants are ~~not~~ <sup>contains only</sup> there is a processes where ~~the p is~~ each process is independent of previous process, and the participants are rewarded by chance.
  - + Complex: contain series of processes, where only in the first process of series the participants are rewarded only by chance, ~~then~~ In subsequent processes, the rewarding is based on chance & outcome of previous process.
- There are 6 ~~preference~~ constraints in Utility theory.

### - Utility Principle:-

If an agent's preferences obey the axioms of utility, then there exists a real value  $U$  that operate on states such that  $U(A) > U(B)$  iff. A is preferred <sub>to</sub> B, &  $U(A) = U(B)$ , if the agent is indifferent <sub>to</sub> A & B.

- Maximum Expected Utility Principle. (2nd defn)  
The utility of a lottery is the sum of probability of each outcome times the utility of that outcome.

$$U(Lp_1, S_1; \dots; p_n, S_n) = \sum_i p_i U(S_i)$$

- Monotonic preferences: Preferences made only on the basis of a particular <sup>basic</sup> thought/perspective/reasoning.
- Risk-averse agent: They prefer a sure thing with a pay-off that is less than expected monetary value of a gamble.
- Certainty Equivalent: The value that an agent can accept in lieu of a lottery.
- Insurance premium: Diff <sup>get</sup> <sup>w</sup> expected monetary value of a lottery & its certainty equivalent.



- Value  $f^n$  / Ordinal Utility  $f^n$ : An agent in a deterministic env has a value  $f^n$  / ordinal Utility  $f^n$ .  
 † Provides ranking of states rather than numerical values
- $u_T$ : Utility with  $p$  best possible prize.
- $u_L$ : Utility with worst possible catastrophe
- Normalized utilities: Utilities with scale  $u_T=1$  &  $u_L=0$ .
- Standard lottery:  $[u_T:p; u_L:(1-p)]$ .
- Utilities of intermediate outcome are assessed by asking the agent to indicate a preference betw the given outcome state  $S$  & a standard lottery.
- Multi-attribute utility theory: Outcomes are characterized by 2 or more attributes.
- Dominance: If utility of one outcome <sup>of an action</sup> is greater than other outcome, then former dominates later
- Strict dominance: Dominates over all attributes.
- Stochastic Dominance: (used for continuous range of outcomes for of an attribute): If utility
- Cumulative Distribution: Measures the prob. that value is less than or equal to any ~~val~~ "established" value, integrates original distribution
- If  $A_1$  stochastically dominates  $A_2$ , then the expected utility of  $A_1$  is at least as high of  $A_2$ , for monotonically non-decreasing function.
- Hence  $A_2$  can be discarded.
- $X^+$  can be used to make rational decisions -

### Preference structure & multi-attribute utility

- Representation Theorems: Used to identify regularities in the preference behaviour

## Mutual Preferential Independence (MPI) -

- Each attribute may be important, it does not affect the way in which one trades off the other attributes against each other.
- If attributes  $X_1, \dots, X_n$  are mutually preferentially independent, then the agent's preference behaviour can be desc<sup>d</sup> as maximizing the  $f^n V(x_1, \dots, x_n = \sum_i V_i(x_i)$  where each  $V_i$  is a value  $f^n$  referring only to the attrib  $X_i$

## Decision Networks

Combine Bayesian  $M^w$  with additional node types for actions & utilities. Returns the action with highest utility

## Value of Information

- Information value Theory: - Enables an agent to choose what info to acquire.
- Sensing actions acquire info.
  - + Affect internal state of an agent
  - + Evaluate sensing action by their effect on the agent's subsequent "real" actions
- X + Done using Information value theory which involves form of a sequential decision making.

- Ashish R. Gavande