AI Constrain Satisfaction problem (CSP)

- CSP is defined by a set of varibles, $X_1, X_2, X_3 \ldots X_n$ & a set of constraints, $C_1, C_2, C_3, \ldots C_m$.
- Each X_i has domain D_i of possible values.
- C_i has some subset of variables & specifies the allowable combinations of values for that subset.
- A state of problem is defined by an assignment of values to some or all variables.
- An assignment that does not violate any constraints is called consistent all legal assignment.
- Complete assignment : Every variable is mentioned
- Solution : Consistent & Complete assignment.
- Some CSPs also require a solⁿ the maximizes / minimizes an objective fⁿ.
- CSP can be viewed as constraint graph.
- Path by which solⁿ is reached is irrelevant
- General Purpose CSP ~~pr~~ algos can solve problems order of magnitude larger than then solvable via general purpose search algos.
- Domains of values of CSP variables can be infinite. As it not possible to enumerate all values here, instead, constraint language be used. eg.
  Start Job$_1$ < Start Job$_2$.
- Domains can be made finite by bounding values of all the variables.
- Type of constraints
  + ~~Single~~ Unary : ~~Puts~~ Restriction on a single variable.
  + Binary : Restriction on two variables simultaneously / & applies to both the variables.

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- Types of constraints (another dimension)
  + Absolute: Violation of absolute constraint rules out a potential soln
  + Preference: Indicate which soln is preferred; failing which we still have a soln.
- All CSP algos generate successors by considering possible assignments for only a single variable at each node of a tree; the order is not imp or the reln betw variables is not imp. as ~~the~~ the assignment ~~problem~~ is commutative
  + A commutative problem is not affected by the assignment order.
- Backtracking search for CSPs :- Uses DFS that chooses values for one variable at a time & backtracks when a variable has no legal values left to assign.
  + Min remaining value heuristic / Most constrained variable heuristic / Fail-first heuristic : Chooses the variable with fewest "legal" values for assignment. It is the variable that is most likely to cause ~~an~~ failure soon; therby pruning search tree. Is not useful at initial point.
  + Degree heuristic :- Select a variable that ~~has~~ is present or involved in largest no. of constraints with other unassigned variables; thereby reduces the ~~br~~ branching factor. MRV is more poowerful but Degree can be tie-breaker.
  + Least-constraining value heuristic :
    * used to decide value once a variable is selected
    * It selects a valuer that rules out fewest choices for the neighbouring variables.

Propagating info through constraints

- By looking ahead for constraints, we can drastically reduced the search space.
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- Forward Checking
  + Whenever X is assigned a value, FC will looks at each unassigned variable connected with X by a constraint & delete's from the later's domain each value assigned to X.
  + Efficient way to incrementally compute the info that the MRV heuristic needs to do its job.

- Constraint Propagation
  + FC does not look far ahead & ∴ can lead to inconsistency.
  + CP propagates implication of a constraint on one variable onto other variables.
  + Arc consistency provides a fast method of constrain propagation that is substantially stronger than FC.
    * For given domains of X & Y, there is arc consistency bet^w constrain-connected ~~adjacent~~ variables X & Y if value 'x' given to X is consistent with 'y' to Y.
    * Can be applied as a pre-processing step or as a propagation step (like FC). The latter is called Maintaining arc consistency.

- k-consisteny
  + A CSP is k-consistent if for any set of k-1 variables and for any consistent assignment to those variables a consistent value can always be assigned to any k^{th} variable.
  + A strongly kconsistent graph is a graph if it is k-consistent and is also (k-1) consistent, (k-2) consistent...... all the way down to k-consistent

- Intelligent backtracking
  + If back search fails, simple backtracking would go to the preceding variable & a try a diff. value. This may not solve the problem.
  + A more intelligent approcch is try conflict set & backtracking using this is called conflict-directed ~~backtracking~~. back jumping

+ A conflict set for X is set of previously assigned variables that are connected to X by constraints. A backtracking ~~to~~ would backtrack to the most recent variable in conflict set.
+ ~~B~~ This method is called backjumping.

- Min-conflicts heuristic: Selects the value that is minimum conflicts with other variables

Structure of problems

- Problem can be simplified by decomposing
- Independent subproblems can be identified by looking for connected components. Each component corresponds to subproblem in CSP.
- If S_i is soln for CSP_i then $\bigcup_i S_i$ is soln for $\bigcup_i CSP_i$.
- These reduces time required for sol^n.
- Any tree-structured CSP can be solved in linear time of variables' number.
- A general constraint graph can be reduced to trees in 2 ways
  + Assigning value to a variable such that remaining variables form a tree.
  + Tree decomposition: Decompose constraint graph into sub-~~probl~~ components.

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