

Artificial Intelligence Uncertainty - 13.

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- Rational Goals / Decisions : Depend on relative importance of various goals & the likelihood that, & degree to which, they will be achieved.
- Probability provides a way of summarizing the uncertainty that comes from our laziness, ignorance.
- Degree of belief is not equal to degree of truth.
- Degree of truth is subject to fuzzy logic.
- Beliefs are based on percepts that are received to date.
- These percepts constitute the evidence on which probability assertions are based.
- Prior / Unconditional probability: Probability before evidence.
- Posterior / Unconditional probability: " After evidence,
 → + Refers to degree of belief, in absence of any other info.

Uncertainty & rational decisions

- An agent must have preferences betw diff possible outcomes from various outcomes.
 - Outcomes is completely specified state.
 - Utility : (In AI) is quality of being useful.
 - Utility Theory: Says that every state has usefulness, or utility, to an agent & that an agent will prefer state with higher utility.
 - Utility is relative to the agent.
 - Decision theory = probability theory + utility theory.
 - Maximum ~~utility~~ ^{Expected} Utility: An agent is rational iff it chooses an action that yields highest expected utility, averaged over all the possible outcomes of the action. (But actions are repeated multiple times; can have diff utility each time)
 - Belief state: Reflect the history of percepts to date.
- Because the utilities may differ each time, avg. is taken out to find utility of an action.

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- Agent's belief state not just represent the possibilities of world but there probabilities.
 - In probability theory, set of all possible worlds is called sample space. The possible worlds are mutually exhaustive - two possible worlds simultaneously not possible.
 Ω refers to sample space & ω refers to elements of the space (possible worlds)
 - A fully specified probability model associates a numerical probability $P(\omega)$ with every possible world & sum is 1.
 $0 \leq P(\omega) \leq 1$ for each ω & $\sum_{\omega \in \Omega} P(\omega) = 1$.
 - In probability theory, set of possible worlds are refer to events. Sets are always described by proposition in a formal language.
 - ~~For~~ A proposition contains possible worlds in which proposition holds (true).
 + Its probability is the probability sum of probabilities of individual possible worlds.
 - Variables in probability theory are called random variables specified with names with first char capital & values in small case.
 - A proposition of random variable can be in small case e.g. $A = \text{true}$ can be written as 'a' & $A = \text{false}$ as '-a'. Also just value can be used instead of proposition.
 - Probabilities $P(\text{weather} = \text{sunny}) = 0.6$, $P(\text{weather} = \text{rain}) = 0.1$, & $P(\text{weather} = \text{snow}) = 0.01$ & can be written as $P(\text{weather}) = \langle 0.6, 0.1, 0.01 \rangle$ where P is vector.
 Here P defines probability distribution of Weather, we can define that a random variable takes on some value x as parameterized fn of x .
 e.g. $P(\text{MoonTemp} = x) = \text{Uniform}[18C, 26C](x)$
 expresses the belief that moon-temp is betw 18 to 26C.
 We call this a probability distribution function (PDF)

+ PDF should be interpreted as:
 Eg the probability of temp bet^w $18^{\circ} - 26^{\circ}$ is 100%, 13-37
 that in $18^{\circ} - 22^{\circ}$ is 50%, $18^{\circ} - 20^{\circ}$ is 25% & so on.

+ PDF of X at ' x ' is written as $P(x)$; is the probability that X falls within arbitrarily small region beginning at x , divided by width of region.

Joint Probability ~~prob~~ distribution: Probability distⁿ on 2 or more variables. Eg $P(\text{Weather, Carity})$

- Full Joint Probability ~~Function~~ ^{Distribution}: JPD for all the random variables of the world.

- $P(-a) = 1 - P(a)$.

- Inclusion-Exclusion principle:

$$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$$

- Marginal probability = Conditional probability
 + The process is called marginalization / summing out.
 because we sum of the probabilities of each possible value of the other variables. thereby taking them out of the eqⁿ. Written as

$$P(Y) = \sum_{z \in Z} P(Y, z) \quad \text{Eg } P(\text{Carity}) = \sum_{z \in \{\text{Catch, Toostache}\}} P(\text{Carity}, z)$$

- Conditional probability

$$P(Y) = \sum_x P(Y|x) P(x)$$

Baye's rule & its use

- Bayes' rule $P(b|a) = \frac{P(a|b)P(b)}{P(a)}$

- General form of Bayes' rule

$$P(Y|X) = \alpha P(X|Y)P(Y)$$

- Conditional independence

$$P(X, Y|Z) = P(X|Z)P(Y|Z)$$

$$P(X|Y, Z) = P(X|Z) \text{ \& } P(Y|X, Z) = P(Y|Z)$$

- Full Joint distribution

$$P(\text{Cause}, \text{Effect}_1, \dots, \text{Effect}_n) = P(\text{Cause}) \prod_i P(\text{Effect}_i | \text{Cause})$$

* Also called, Naive Bayes Model.

Efficient Representation of Conditional Distribution

- Canonical distribution: Consists of complete CPT with some standard pattern but with fewer parameters.
- Deterministic nodes: - Node whose value is exactly specified by its parents, & with no uncertainty.
- Uncertain relⁿ can be characterized by so-called noisy relⁿ.

- Leak node: covers miscellaneous causes.

* - The inhibition probability can be used to generate 0 for CPT/full CPT. Only 'K' ^{rows} ~~parameters~~ are required instead of 2^K

- Ashish R. Gavande