0xjpar4e4

May 1, 2024

```
[1]: import matplotlib.pyplot as plt
    import pandas as pd
    import seaborn as sns
    import numpy as np
[2]: fmg_data = pd.read_csv("FMG.AX.csv")
    print("Data for ",fmg_data.shape[0]," days")
    Data for 9301 days
[3]: fmg_data.head()
[3]:
             Date
                        Open
                                                     Close
                                                            Adj Close
                                                                       Volume
                                  High
                                            Low
                                                             0.054525
    0 1988-01-29
                   0.140522 0.140522 0.140522
                                                  0.140522
                                                                          0.0
    1 1988-02-01
                   0.140522 0.140522
                                       0.140522
                                                  0.140522
                                                             0.054525
                                                                          0.0
    2 1988-02-02
                   0.140522 0.140522
                                       0.140522
                                                  0.140522
                                                             0.054525
                                                                          0.0
    3 1988-02-03
                   0.140522
                             0.140522
                                       0.140522
                                                  0.140522
                                                             0.054525
                                                                          0.0
    4 1988-02-04 0.140522 0.140522
                                       0.140522
                                                  0.140522
                                                             0.054525
                                                                          0.0
[4]: fmg_data.isna().sum(axis = 0)
[4]: Date
                   0
    Open
                  146
    High
                  146
    Low
                  146
    Close
                  146
    Adj Close
                  146
    Volume
                  146
    dtype: int64
```

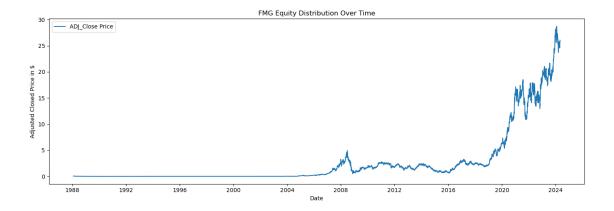
0.0.1 Drop Rows with Missing Values as it do not constitute a significant portion of our dataset (only 146 na out of 9301)

```
[5]: fmg_data.dropna(inplace=True) fmg_data.isna().sum(axis = 0)
```

```
[5]: Date
                  0
     Open
                  0
     High
                  0
     Low
                  0
     Close
                  0
     Adj Close
                   0
     Volume
                   0
     dtype: int64
[6]: fmg_data.describe()
[6]:
                    Open
                                                            Close
                                                                      Adj Close \
                                 High
                                                Low
            9155.000000
                                                                   9155.000000
                          9155.000000
                                        9155.000000
                                                     9155.000000
     count
     mean
                4.103480
                             4.164070
                                           4.038634
                                                         4.098985
                                                                       2.660072
     std
                6.421693
                                           6.334778
                                                         6.417056
                                                                       5.321295
                             6.501520
     min
                0.001756
                             0.001756
                                           0.001756
                                                         0.001756
                                                                       0.000681
     25%
               0.012759
                             0.012960
                                           0.012759
                                                         0.012759
                                                                       0.004951
     50%
               0.609000
                             0.617000
                                           0.592000
                                                         0.612000
                                                                       0.237465
     75%
               5.050000
                             5.130000
                                           4.970000
                                                         5.040000
                                                                       2.237583
              29.820000
                            29.950001
                                          29.500000
     max
                                                        29.879999
                                                                      28.707384
                  Volume
            9.155000e+03
     count
     mean
            7.247730e+06
     std
            9.931788e+06
     min
            0.000000e+00
     25%
            0.000000e+00
     50%
            3.645000e+06
     75%
            1.160044e+07
     max
            1.850619e+08
```

1 2) Plotting the Prices

```
[7]: fmg_data['Date'] = pd.to_datetime(fmg_data['Date'])
plt.figure(figsize=(14, 5))  # Adjust the figure size as needed
plt.plot(fmg_data['Date'], fmg_data['Adj Close'], label='ADJ_Close Price')
plt.xlabel('Date')
plt.ylabel('Adjusted Closed Price in $')
plt.title('FMG Equity Distribution Over Time')
plt.legend()
plt.tight_layout()  # Adjust layout to prevent overlapping labels
plt.show()
```



1.1 Comments:

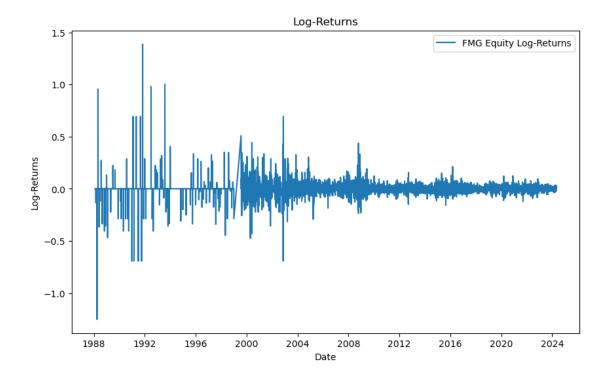
The adjusted close price is plotted as it makes comparison across time convenient Adjusted Close takes into account stock splits, dividends and other corporate actions

1.2 Observations:

It can be seen the FMG peaks in around the 2024

2 3) Plotting Log Returns

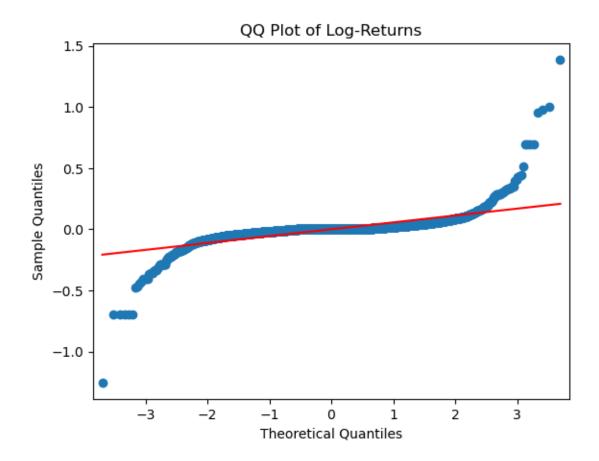
Plotting Log returns vs time help to observe volatility. Also helps observe general trend of the stock



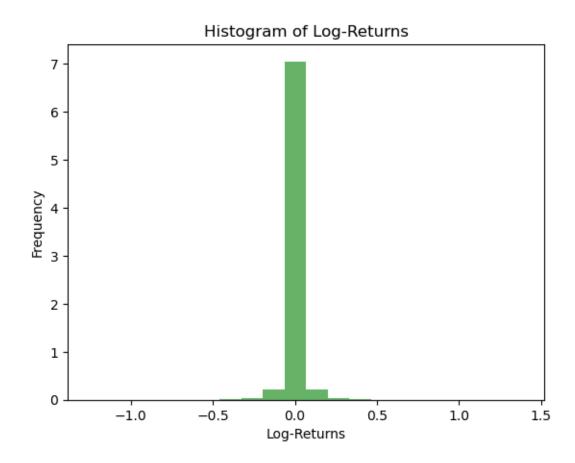
3 4) Checking Normality of Log Returns

3.1 QQ Plot

```
[9]: from statsmodels.graphics.gofplots import qqplot
# QQ Plot
qqplot(fmg_data['Log_Returns'].dropna(), line='s')
plt.title('QQ Plot of Log-Returns')
plt.show()
```



3.2 Histogram



3.3 Kolmogrov-Smirnov Test , D'Agostino-Pearson's Test, Kolmogrov-Smirnov Test

```
[11]: from scipy.stats import jarque_bera, kstest, normaltest
  import statsmodels.api as sm
  alpha = 0.05

print("Null Hypothesis: Returns are Normally Distributed")
print("")

# Jarque-Bera Test
print("Jarque-Bera Test")
  jb_stat, jb_p_value = jarque_bera(fmg_data['Log_Returns'].dropna())
  print("Jarque-Bera Test Statistic:", jb_stat)

if(jb_p_value>alpha):
    print("We accept the Null Hypothesis,Log returns are not normally______
distributed according to Jarque-Bera Test")
else:
```

```
print("We reject the Null Hypothesis, Log returns are not normally \sqcup
 ⇔distributed according to Jarque-Bera Test")
print("")
# Kolmogorov-Smirnov Test
print("Kolmogorov-Smirnov Test")
ks stat, ks p value = kstest(fmg data['Log Returns'].dropna(), 'norm')
print("Kolmogorov-Smirnov Test Statistic:", ks_stat)
if(ks_p_value>alpha):
    print("We accept the Null Hypothesis, Log returns are not normally ⊔
 ⇒distributed according to Kolmogorov-Smirnov Test")
else:
    print("We reject the Null Hypothesis, Log returns are not normally ⊔
 ⇒distributed according to Kolmogorov-Smirnov Test")
print("")
# D'Agostino-Pearson's Test
print("D'Agostino-Pearson's Test")
dap_stat, dap_p_value = normaltest(fmg_data['Log_Returns'].dropna())
print("D'Agostino-Pearson's Test Statistic:", dap_stat)
if(dap_p_value>alpha):
    print("We accept the Null Hypothesis, Log returns are not normally ⊔
 ⇔distributed according to D'Agostino-Pearson's Test")
    print("We reject the Null Hypothesis, Log returns are not normally ⊔
  →distributed according to D'Agostino-Pearson's Test")
Null Hypothesis: Returns are Normally Distributed
Jarque-Bera Test
Jarque-Bera Test Statistic: 6442987.188628093
We reject the Null Hypothesis, Log returns are not normally distributed according
to Jarque-Bera Test
Kolmogorov-Smirnov Test
Kolmogorov-Smirnov Test Statistic: 0.44516357446692667
We reject the Null Hypothesis, Log returns are not normally distributed according
to Kolmogorov-Smirnov Test
D'Agostino-Pearson's Test
D'Agostino-Pearson's Test Statistic: 6282.754909460969
```

3.4 OBSERVATIONS:

to D'Agostino-Pearson's Test

From the histogram plot, qq-plot we observed the data didn't quite follow normal distribution

We reject the Null Hypothesis, Log returns are not normally distributed according

This claim stands stronger with Jarque Bera, Kolmogrov-Smirnov, D'Agostino's tests rejecting the Null Hypothesis, and hence data does not follow normal distribution.

4 5) Estimating Historical Volatility Using Log Returns

```
[12]: # Estimate Historical Volatility

print("Estimate Historical Volatility")

print("")

daily_volatility = fmg_data['Log_Returns'].std()

annual_historical_volatility = np.std(fmg_data['Log_Returns']) * np.sqrt(252) ___

# Assuming 252 trading days in a year

print("Daily Volatility: ",daily_volatility)

print(" ")

print("Annual Historical Volatility:", annual_historical_volatility)
```

Estimate Historical Volatility

Daily Volatility: 0.056217849243989294

Annual Historical Volatility: 0.8923819433288527

5 6) Identifying Risk Free Rate for AUD

Source for RFR: CNBC'

```
[13]: rfr = 0.044210
print("")
print("Risk Free Rate for AUD=",rfr)
```

Risk Free Rate for AUD= 0.04421

6 7) Testing if Log Returns are independent/uncorrelated

```
[14]: print("Log-Returns Independence/Uncorrelated")

# Test Assumption of Log-Returns Independence/Uncorrelated
acf = sm.tsa.acf(fmg_data['Log_Returns'].dropna())

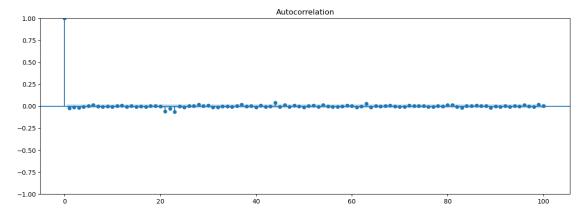
print("")
print("Finding lag with max auto-correlation ")
# Finding lag with max auto-correlation

max_arg_index = acf[1:].argmax() + 1
print("max_arg_index=",max_arg_index)
if(acf[max_arg_index] == np.max(acf[1:])):
```

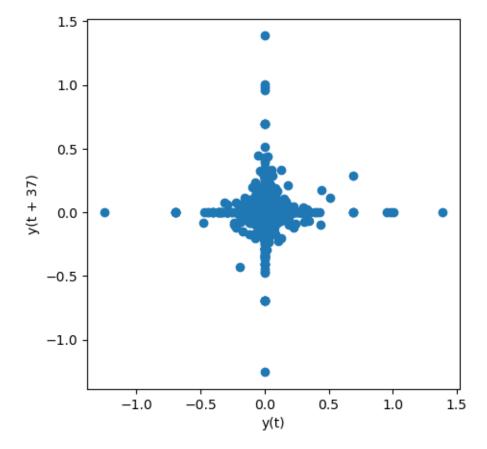
Log-Returns Independence/Uncorrelated

```
Finding lag with max auto-correlation max_arg_index= 37 max_arg= 0.01938659284483168
```

```
[15]: import statsmodels.api as sm
  from statsmodels.graphics.tsaplots import plot_acf
  plt.rc("figure", figsize = (15, 5))
  plot_acf(fmg_data['Log_Returns'].dropna(), lags = 100)
  plt.show()
```



Plot of Lag of 37, which had max auto correlation



- 7 8.In-The-Money (ITM) European call option and In-The-Money European put option for the maturity May31, 2024
- 8 AND
- 9 9. Using CRR, Black-Scholes and Simulation Methods

```
[17]: from datetime import date
import math

def nCr(n, r):
    return math.comb(n, r)
```

9.1 CRR MODEL

```
[18]: def crr_price(s0, strike, maturity_date, rfr, sigma, steps, dividend=0,__
       →option_type='call', pricing_date=date.today()):
          11 11 11
          s0: current stock price
          strike: strike price of the option
          rfr: risk-free interest rate
          sigma: volatility
          maturity_date: [day, date, year] all should be without leading zeros
          steps: number of steps in the CRR model
          dividend: dividend paid by the stock
          option type: 'call' or 'put'
          pricing\_date: [day, date, year] all should be without leading zeros, tells<sub>\sup</sub>
       \hookrightarrow when
           11 11 11
          md = date(maturity_date[2], maturity_date[1], maturity_date[0])
          pd = date(pricing_date[2], pricing_date[1], pricing_date[0])
          maturity_time = ((md - pd).days) / 365
          delta = maturity_time / steps
          u = np.exp(sigma * np.sqrt(delta))
          d = 1 / u
          if not dividend:
              p_{-} = (np.exp(rfr * delta) - d) / (u - d)
          else:
              p_{-} = (np.exp((rfr - dividend) * delta) - d) / (u - d)
          option price = 0
          for ups in range(0, steps + 1):
              downs = steps - ups
```

```
curr_stock_price = s0 * (u ** ups) * (d ** downs)
              curr_payoff = 0
              if option_type == 'call':
                  curr_payoff = max(curr_stock_price - strike, 0)
              else:
                  curr_payoff = max(strike - curr_stock_price, 0)
              option_price += nCr(steps, ups) * (p_ ** ups) * ((1 - p_) ** downs) *__
       ⇔curr_payoff
          option_price = option_price / ((1 + rfr * delta) ** steps)
          return option_price
[19]: s0 = fmg_data[fmg_data['Date'] == '2024-04-30']['Adj Close'].iloc[0]
      # Get the current stock price of FMG on April 30,2024
      # Maturity date= May 31,2024
      strike = s0 - 0.2 * s0 # Assuming the call option price is arbitrarily 20%
      ⇔less than the current stock price
      strike_put = s0 + 0.2 * s0  # Assuming the put option strike_price_is_l
      ⇒arbitrarily 20% higher than the current stock price
      s0, strike
      print("current stock price=",s0);
      print("call strike=",strike)
      print("put strike=",strike_put)
     current stock price= 26.049999
     call strike= 20.8399992
     put strike= 31.25999879999998
[20]: print("CRR Model In-The-Money (ITM) European call option= ",crr_price(s0=s0, u
       ⇔strike=strike, rfr=rfr,⊔
       ⇒sigma=annual_historical_volatility,maturity_date=[31, 5, 2024], steps=1000, 
       ⇔dividend=0.02, option_type='call', pricing_date=[30, 4, 2024]))
      print("CRR Model In-The-Money (ITM) European put option= ",crr_price(s0=s0, __
       ⇔strike=strike_put, rfr=rfr,
       ⇒sigma=annual_historical_volatility,maturity_date=[31, 5, 2024], steps=1000, 
       dividend=0.02, option_type='put', pricing_date=[30, 4, 2024]))
     CRR Model In-The-Money (ITM) European call option= 5.886638438441413
```

CRR Model In-The-Money (ITM) European Call option= 5.886636436441413

9.2 Black-Scholes-Merton Model

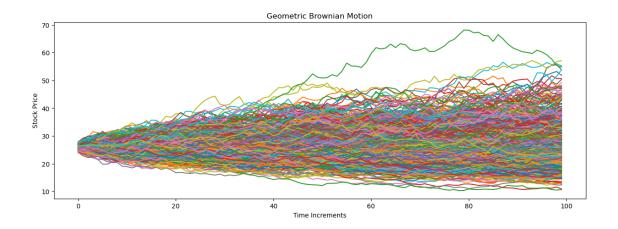
```
[21]: from scipy.stats import norm
      # Black-Scholes model
      def bsm_price(s0, strike, rfr, sigma, maturity_date, pricing_date=[5, 5, 2023],__
        →option_type='call'):
          11 11 11
          s0: current stock price
          strike: strike price of the option
          rfr: risk free interest rate
          sigma: volatility
          maturity_date: [day, date, year] all should be without leading zeros
          pricing date: [day, date, year] all should be without leading zeros, tells_{\sqcup}
        \hookrightarrow when
          option_type: 'call' or 'put'
          11 11 11
          md = date(maturity_date[2], maturity_date[1], maturity_date[0])
          pd = date(pricing_date[2], pricing_date[1], pricing_date[0])
          maturity_time = ((md - pd).days) / 365
          d1 = (np.log(s0 / strike) + (rfr + (sigma ** 2) / 2) * maturity_time) / ___

¬(sigma * np.sqrt(maturity_time))
          d2 = d1 - sigma * np.sqrt(maturity_time)
          option_price = None
          call_option_price = s0 * norm.cdf(d1) - strike * np.exp(-1 * rfr *_\text{\text{\text{t}}}
       →maturity_time) * norm.cdf(d2)
          if option_type == 'call':
               option_price = call_option_price
          else:
               option_price = call_option_price + strike * np.exp(-1 * rfr *_
        →maturity_time) - s0
          return option_price
```

Black-Scholes-Merton Model In-The-Money (ITM) European call option= 5.923530495968041 Black-Scholes-Merton Model In-The-Money (ITM) European put option= 6.170280656766813

9.3 Simulation with Geometric Brownian Motion

```
[23]: import numpy as np
      import matplotlib.pyplot as plt
      def GBM_paths(S, T, r, q, sigma, steps, N):
        Inputs
        #S = Current stock Price
        #K = Strike Price
        \#T = Time to maturity 1 year = 1, 1 months = 1/12
        \#r = risk \ free \ interest \ rate
        #q = dividend yield
        # sigma = volatility
        Output
        # [steps,N] Matrix of asset paths
        \# S(t) = S(0) * exp(mu-sigm**2/2) + sigma*B(t) GBM.
        dt = T/steps
        ST = np.log(S) + np.cumsum(((r - q - sigma**2/2)*dt + 
        sigma*np.sqrt(dt) * \
        np.random.normal(size=(steps,N))),axis=0)
        return np.exp(ST)
      # S=stock price S_{0} s0
      T = 31/365.0 \# time to maturity
      r = rfr # risk free risk in annual %
      q = 0 # annual dividend rate = N/A
      sigma = annual_historical_volatility # annual volatility in %
      steps = 100 # time steps
      N = 1000 \# number of trials
      paths = GBM_paths(s0,T,r,q,sigma,steps,N)
      plt.plot(paths);
      plt.xlabel("Time Increments")
      plt.ylabel("Stock Price")
      plt.title("Geometric Brownian Motion")
      plt.show()
```



```
payoffs = np.maximum(paths[-1]-strike, 0)
call_simulated_option_price = np.mean(payoffs)*np.exp(-r*T) #discounting t
print(" ")
print(f"Simulation Method Call option price is {call_simulated_option_price}")
print("")
payoffs = np.maximum(strike_put-paths[-1], 0)
call_simulated_option_price = np.mean(payoffs)*np.exp(-r*T) #discounting t
print(" ")
print(f"Simulation Method Put option price is {call_simulated_option_price}")
print("")
```

Simulation Method Call option price is 5.95045605928565

Simulation Method Put option price is 6.286277479401012

- 10 10. Estimate the volatility parameter other than the historical volatility
- 11 Volatility Estimation by Garch (Generalized Auto Regressive Conditional Heteroskedasticity) Method

```
[25]: [!pip install arch
```

```
Defaulting to user installation because normal site-packages is not writeable Requirement already satisfied: arch in c:\users\91991\appdata\roaming\python\python311\site-packages (7.0.0) Requirement already satisfied: numpy>=1.22.3 in c:\programdata\anaconda3\lib\site-packages (from arch) (1.24.3)
```

```
Requirement already satisfied: scipy>=1.8 in c:\programdata\anaconda3\lib\site-
packages (from arch) (1.10.1)
Requirement already satisfied: pandas>=1.4 in c:\programdata\anaconda3\lib\site-
packages (from arch) (1.5.3)
Requirement already satisfied: statsmodels>=0.12 in
c:\programdata\anaconda3\lib\site-packages (from arch) (0.14.0)
Requirement already satisfied: python-dateutil>=2.8.1 in
c:\programdata\anaconda3\lib\site-packages (from pandas>=1.4->arch) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in
c:\programdata\anaconda3\lib\site-packages (from pandas>=1.4->arch) (2022.7)
Requirement already satisfied: patsy>=0.5.2 in
c:\programdata\anaconda3\lib\site-packages (from statsmodels>=0.12->arch)
(0.5.3)
Requirement already satisfied: packaging>=21.3 in
c:\programdata\anaconda3\lib\site-packages (from statsmodels>=0.12->arch) (23.0)
Requirement already satisfied: six in c:\programdata\anaconda3\lib\site-packages
(from patsy>=0.5.2->statsmodels>=0.12->arch) (1.16.0)
```

```
[26]: from arch import arch_model
def garch_volatility(x):
    model = arch_model(y = x, vol = 'GARCH', p = 1, q = 1)
    res = model.fit(disp = 'off')
    return res.conditional_volatility[-1]

# mutliplied data by 10 due to scaling error, later divided by 10 as well
daily_gv = garch_volatility(fmg_data['Log_Returns'].dropna().values * 10)/10
annual_gv = daily_gv * np.sqrt(252) # Assuming 252 trading days in a year
print("daily garch volatality=",daily_gv)
print("annual garch volatality=",annual_gv)
print("")
```

daily garch volatality= 0.04258818721221279 annual garch volatality= 0.6760665129154574

11.1 Recalculating With Volatility from GARCH Method

11.2 CRR MODEL with GARCH Volatility

```
[27]: print("GARCH Volatility CRR Model call and put option prices:")

print(" ")

print("CRR Model In-The-Money (ITM) European call option (GARCH volatility)=

",crr_price(s0=s0, strike=strike, rfr=rfr,

sigma=annual_gv,maturity_date=[31, 5, 2024], steps=1000, dividend=0.02,

option_type='call', pricing_date=[30, 4, 2024]))
```

```
print("CRR Model In-The-Money (ITM) European put option (GARCH volatility)=_\( \) ",crr_price(s0=s0, strike=strike_put, rfr=rfr,\( \) sigma=annual_gv,maturity_date=[31, 5, 2024], steps=1000, dividend=0.02,\( \) option_type='put', pricing_date=[30, 4, 2024]))
print(" ")
```

GARCH Volatility CRR Model call and put option prices:

```
CRR Model In-The-Money (ITM) European call option (GARCH volatility)= 5.531146796969827
CRR Model In-The-Money (ITM) European put option (GARCH volatility)= 5.683513790815682
```

11.3 Black-Scholes-Merton Model with GARCH Volatility

```
print("Black-Scholes-Merton Model call and put option prices:")

print(" ")

print("Black-Scholes-Merton Model In-The-Money (ITM) European call option

GARCH Volatility)= ",bsm_price(s0=s0, strike=strike, rfr=rfr,u)

sigma=annual_gv, maturity_date=[31, 5, 2024], pricing_date=[30, 4, 2024],u

option_type='call'))

print("Black-Scholes-Merton Model In-The-Money (ITM) European put option (GARCHu

Volatility)= ",bsm_price(s0=s0, strike=strike_put, rfr=rfr, sigma=annual_gv,u

maturity_date=[31, 5, 2024], pricing_date=[30, 4, 2024], option_type='put'))

print(" ")
```

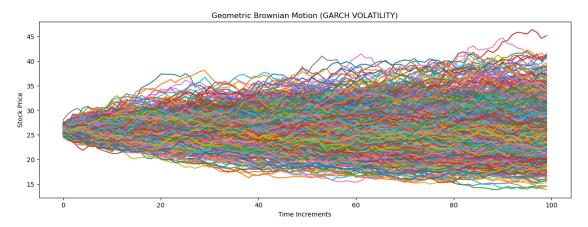
Black-Scholes-Merton Model call and put option prices:

```
Black-Scholes-Merton Model In-The-Money (ITM) European call option (GARCH Volatility) = 5.570827974635847
Black-Scholes-Merton Model In-The-Money (ITM) European put option (GARCH Volatility) = 5.648303170961917
```

11.4 Simulation with Geometric Brownian Motion with GARCH Volatility

```
[29]: import numpy as np
import matplotlib.pyplot as plt
def GBM_paths(S, T, r, q, sigma, steps, N):
    """
    Inputs
    #S = Current stock Price
    #K = Strike Price
    #T = Time to maturity 1 year = 1, 1 months = 1/12
    #r = risk free interest rate
```

```
#q = dividend yield
  # sigma = volatility
  Output
  # [steps,N] Matrix of asset paths
  \# S(t) = S(0) * exp(mu-sigm**2/2) + sigma*B(t) GBM.
  dt = T/steps
  ST = np.log(S) + np.cumsum(((r - q - sigma**2/2)*dt + 
  sigma*np.sqrt(dt) * \
 np.random.normal(size=(steps,N))),axis=0)
  return np.exp(ST)
# S=stock price S_{0} s0
T = 31/365.0 \# time to maturity
r = rfr # risk free risk in annual %
q = 0 # annual dividend rate = N/A
sigma = annual_gv # annual GARCH volatility in %
steps = 100 # time steps
N = 1000 \# number of trials
paths = GBM_paths(s0,T,r,q,sigma,steps,N)
plt.plot(paths);
plt.xlabel("Time Increments")
plt.ylabel("Stock Price")
plt.title("Geometric Brownian Motion (GARCH VOLATILITY)")
plt.show()
```



Simulation Method Call option price (GARCH Volatility) is 5.489823499861419

Simulation Method Put option price (GARCH Volatility)is 5.707664747595342

12 RESULTS:

12.1 WITH HISTORICAL VOLATILITY:

```
CRR Model In-The-Money (ITM) European call option= 5.886638438441413
CRR Model In-The-Money (ITM) European put option= 6.201445909488313
```

Black-Scholes-Merton Model In-The-Money (ITM) European call option= 5.923530495968041 Black-Scholes-Merton Model In-The-Money (ITM) European put option= 6.170280656766813

Simulation Method Call option price is 5.95045605928565 Simulation Method Put option price is 6.286277479401012

12.2 WITH GARACH VOLATILITY:

CRR Model In-The-Money (ITM) European call option (GARCH volatility) = 5.531146796969827 CRR Model In-The-Money (ITM) European put option (GARCH volatility) = 5.683513790815682

Black-Scholes-Merton Model In-The-Money (ITM) European call option (GARCH Volatility) = 5.5708 Black-Scholes-Merton Model In-The-Money (ITM) European put option (GARCH Volatility) = 5.64830

Simulation Method Call option price (GARCH Volatility) is 5.489823499861419 Simulation Method Put option price (GARCH Volatility)is 5.707664747595342

13 Observation:

As Volatility from Garch was less than previous estimate, it was expected option prices would reduce This is because high volatility options have more priced options Garch, providing a precise estimate of volatility helped us obtain a lesser option prices in BSM values and Simulated results