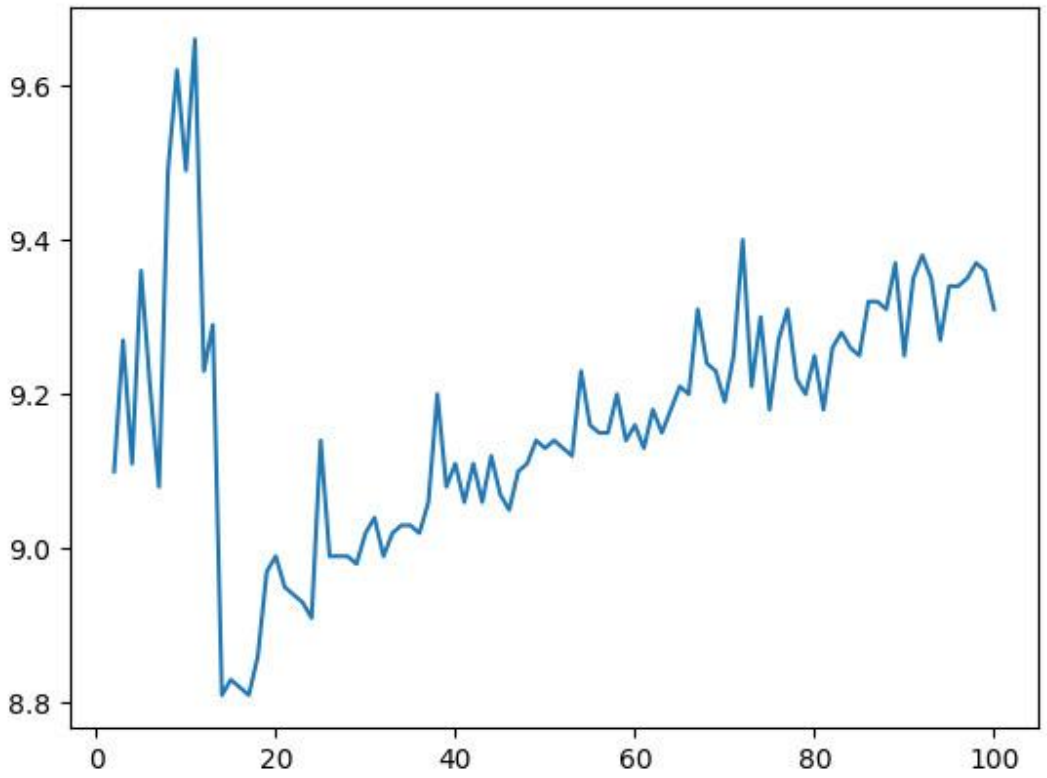
Distance Oracles

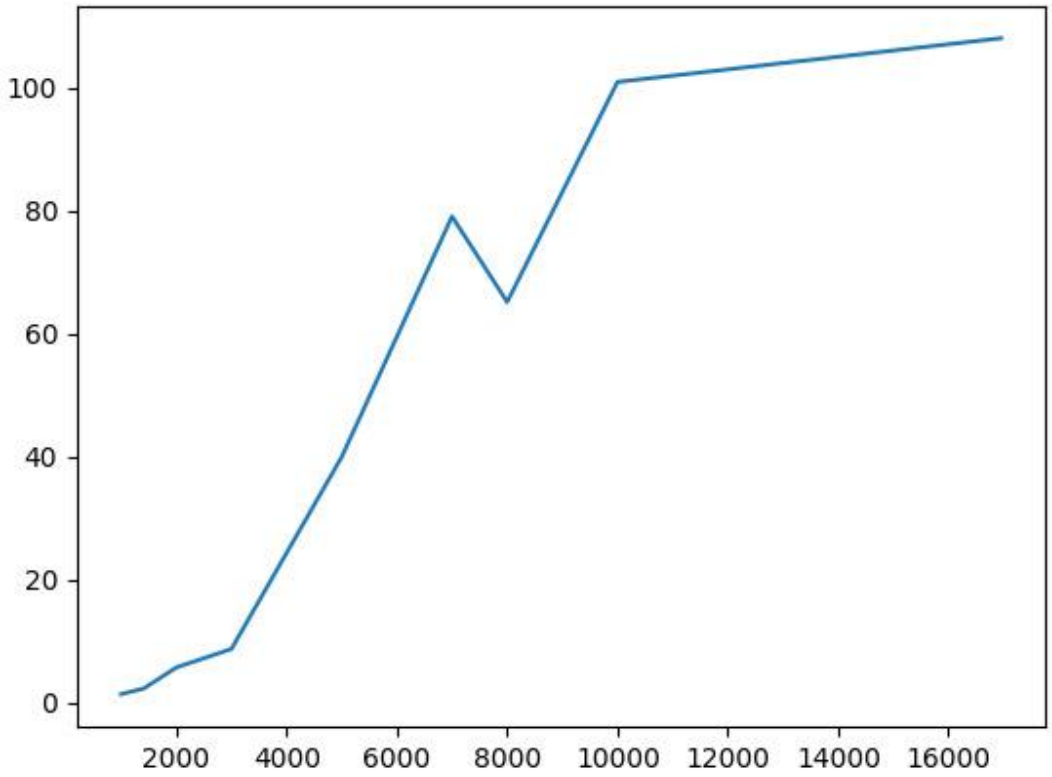
In this project we implemented the algorithm given by Thorup and Zwick to compute the approximate distance between any 2 pair of vertices in the graph. The distance given by the algorithm, dependent on the parameter ‘K’ given by the user, is at most 2*K - 1 the actual distance between the 2 nodes. The above algorithm efficiently precomputes the graph and assembles a data structure in time K * m * n^(1/k), while taking a memory of only K*n^(1 + 1/k), compared to the traditional Floyd-Warshall O(n^3) algorithm taking O(n^2) space. The algorithm after making the data structure handles queries in O(k) time which is constant time per query.

We ran our algorithm on both dense and sparse graphs and noted the time required by the algorithm as well as that required by the Floyd-Warshall algorithm.

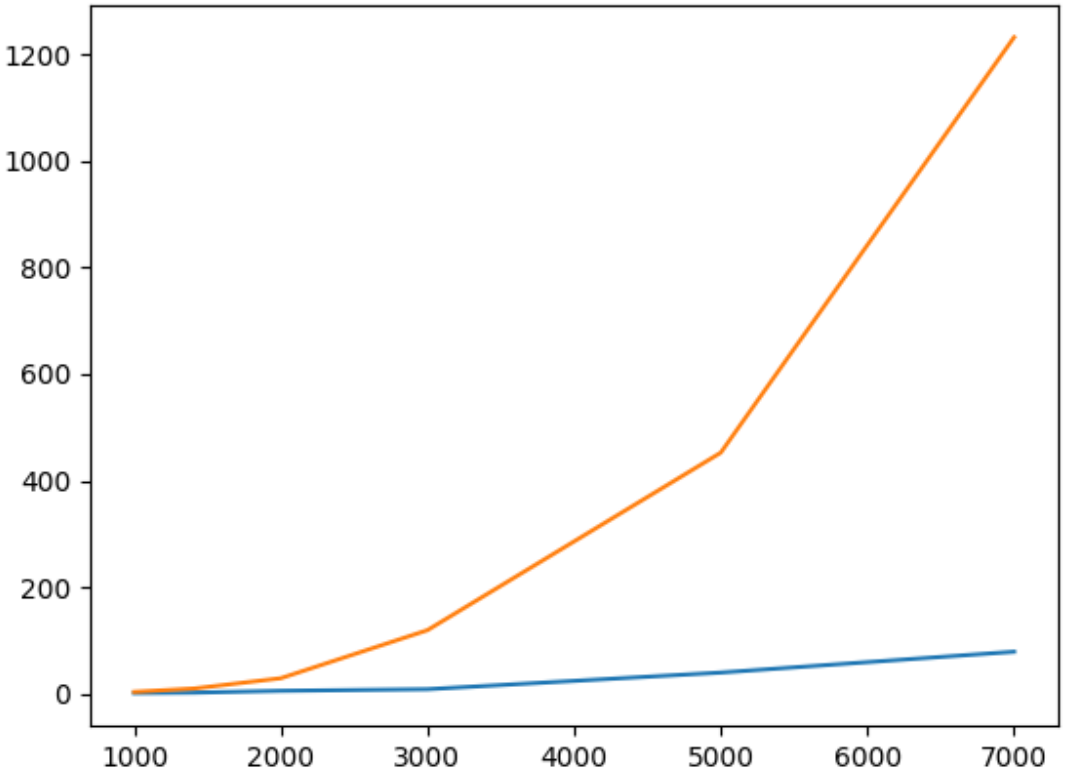
<u>For Sparse Graphs</u>



The above graph is time taken on the y axis for the different values of K observed on the x axis. We can see that the time first decreases then increases. This behaviour is attributed to the increasing and the decreasing effects of K on the time required as can be seen in the formula for the time complexity O(K * m * n^(1/k)). The above graph was drawn for n = 3000 and m = 3000.

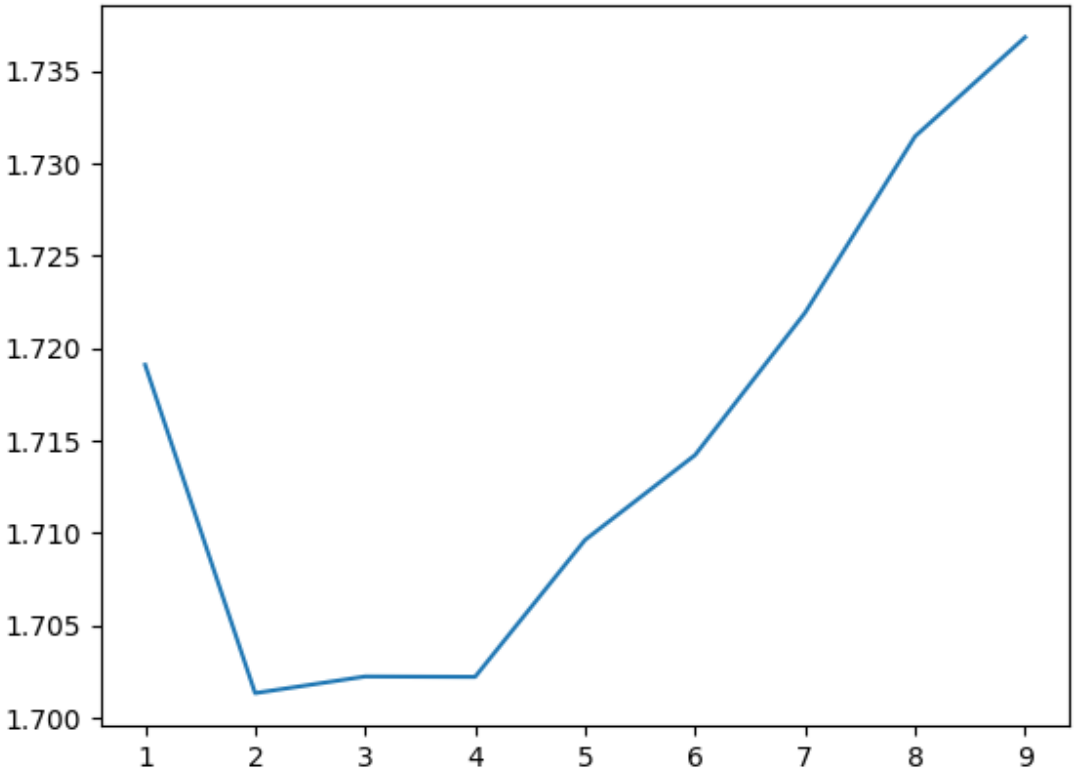


The above graph was made for time taken (in seconds) vs number of nodes in the graph. We see that the graph keeps on increasing.

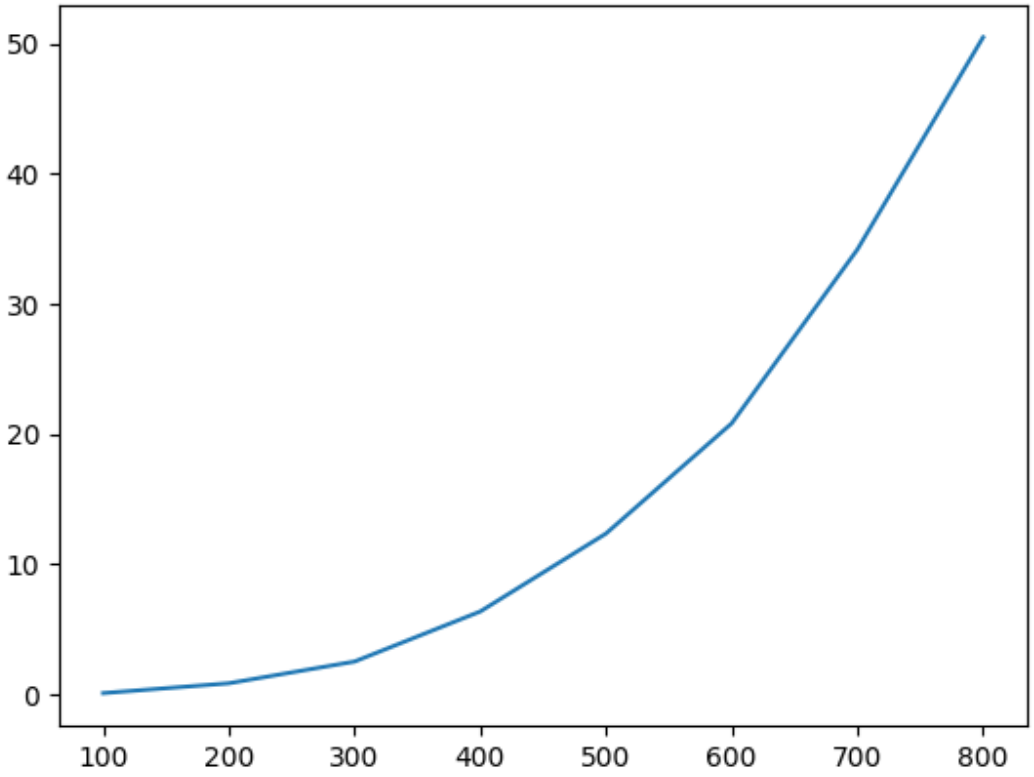


Here the graph shows the comparison between Floyd-Warshall and Thorup-Zwick algorithm. The orange line shows the Floyd-Warshall algorithm and the blue line Thorup-Zwick over the number of nodes on the x axis and the time required on the y axis. Plotted until Floyd-Warshall became computationally infeasible.

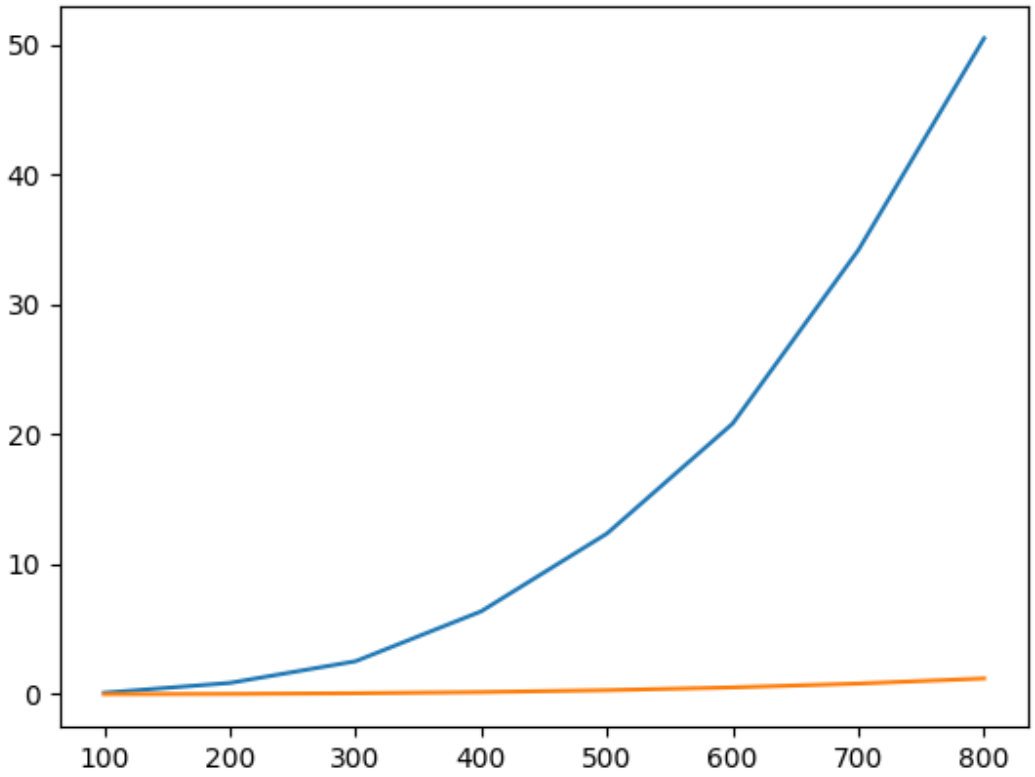
For Dense Graphs



The above graph is for time taken vs K, plotted for n = 250, m = 30,000



The above graph is plotted with time taken (in seconds) on the y axis and the number of nodes on the x axis, with the number of edges close to O(n^2).



The above graph is for comparison between Floyd-Warshall and Thorup-Zwick algorithm. The orange line shows the Floyd Warshall and the blue line shows the Throup-Zwick algorithm. Floyd-Warshall depneds only on the the number of nodes, and because we are considering dense graphs, the input size of the number of nodes is taken less (< 1000), hence the Floyd-Warshall terminates quickly, and the Thorup-Zwick which depends on the number of edges as well shows an increasing graph.