

Coordinate Geometry

10th Maths - Chapter 7

This is Problem-8 from Exercise 7.1

1. The Point $\mathbf{Q} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is equidistant from $\mathbf{P} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} x \\ 6 \end{pmatrix}$. Find the value of x

Solution:

The input parameters for this problem are available in Table

Symbol	Value	Description
\mathbf{P}	$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$	First point
\mathbf{Q}	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	Second point
\mathbf{R}	$\begin{pmatrix} ? \\ 6 \end{pmatrix}$	Desired point

Table 1

If \mathbf{Q} is equidistant from the points \mathbf{P} and \mathbf{R} ,

$$\|\mathbf{P} - \mathbf{Q}\| = \|\mathbf{R} - \mathbf{Q}\| \quad (1)$$

$$\implies \|\mathbf{P} - \mathbf{Q}\|^2 = \|\mathbf{R} - \mathbf{Q}\|^2 \quad (2)$$

which can be expressed as

$$\begin{aligned}
(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q}) &= (\mathbf{R} - \mathbf{Q})^\top (\mathbf{R} - \mathbf{Q}) \\
\implies \|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top \mathbf{P} + \|\mathbf{P}\|^2 &= \|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top \mathbf{R} + \|\mathbf{R}\|^2 \quad (3)
\end{aligned}$$

which can be simplified to obtain

where

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{Q}^\top \mathbf{P} = \|\mathbf{R}\|^2 - 2\mathbf{Q}^\top \mathbf{R} \quad (4)$$

now substituting the P,Q and R values in (4)

$$\|\mathbf{P}\|^2 = 34 \quad (5)$$

$$\|\mathbf{R}\|^2 = x^2 + 36 \quad (6)$$

$$2\mathbf{Q}^\top \mathbf{P} = -6 \quad (7)$$

$$2\mathbf{Q}^\top \mathbf{R} = 12 \quad (8)$$

upon substituting the values in (4) we get a quadratic equation

$$x^2 - 16 = 0 \quad (9)$$

therefore the the value of x = 4

Hence, the desired point is \mathbf{R} is $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$.

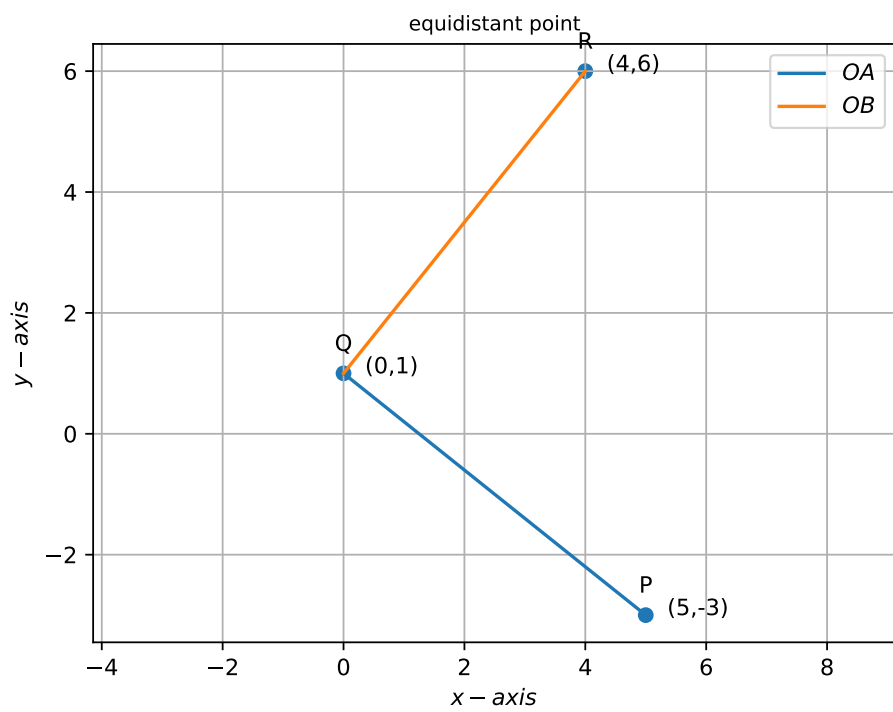


Figure 1