

Coordinate Geometry

10th Maths - Chapter 7

This is Problem-8 from Exercise 7.1

1. If $\mathbf{Q} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is equidistant from $\mathbf{P} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} x \\ 6 \end{pmatrix}$. Find the values of x .

Solution:

The input parameters for this problem are available in Table (1)

Symbol	Value	Description
\mathbf{P}	$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$	First point
\mathbf{Q}	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	Second point
\mathbf{R}	$\begin{pmatrix} ? \\ 6 \end{pmatrix}$	Desired point

Table 1

If \mathbf{Q} is equidistant from the points \mathbf{P} and \mathbf{R} ,

$$\|\mathbf{P} - \mathbf{Q}\| = \|\mathbf{R} - \mathbf{Q}\| \quad (1)$$

$$\implies \|\mathbf{P} - \mathbf{Q}\|^2 = \|\mathbf{R} - \mathbf{Q}\|^2 \quad (2)$$

which can be expressed as

$$(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q}) = (\mathbf{R} - \mathbf{Q})^\top (\mathbf{R} - \mathbf{Q}) \quad (3)$$

$$\|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top \mathbf{P} + \|\mathbf{P}\|^2 = \|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top \mathbf{R} + \|\mathbf{R}\|^2 \quad (4)$$

which can be simplified to obtain

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{Q}^\top \mathbf{P} = \|\mathbf{R}\|^2 - 2\mathbf{Q}^\top \mathbf{R} \quad (5)$$

now substituting the \mathbf{P}, \mathbf{Q} and \mathbf{R} values in (5)

$$\|\mathbf{P}\|^2 = 34 \quad (6)$$

$$\|\mathbf{R}\|^2 = x^2 + 36 \quad (7)$$

$$2\mathbf{Q}^\top \mathbf{P} = -6 \quad (8)$$

$$2\mathbf{Q}^\top \mathbf{R} = 12 \quad (9)$$

upon substituting the values in (5)

$$x^2 + 36 - 12 = 34 + 6 \quad (10)$$

$$\implies x^2 = 16 \quad (11)$$

Then the value of $x = 4$ or -4 . Hence, the desired point is \mathbf{R} is $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

or $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

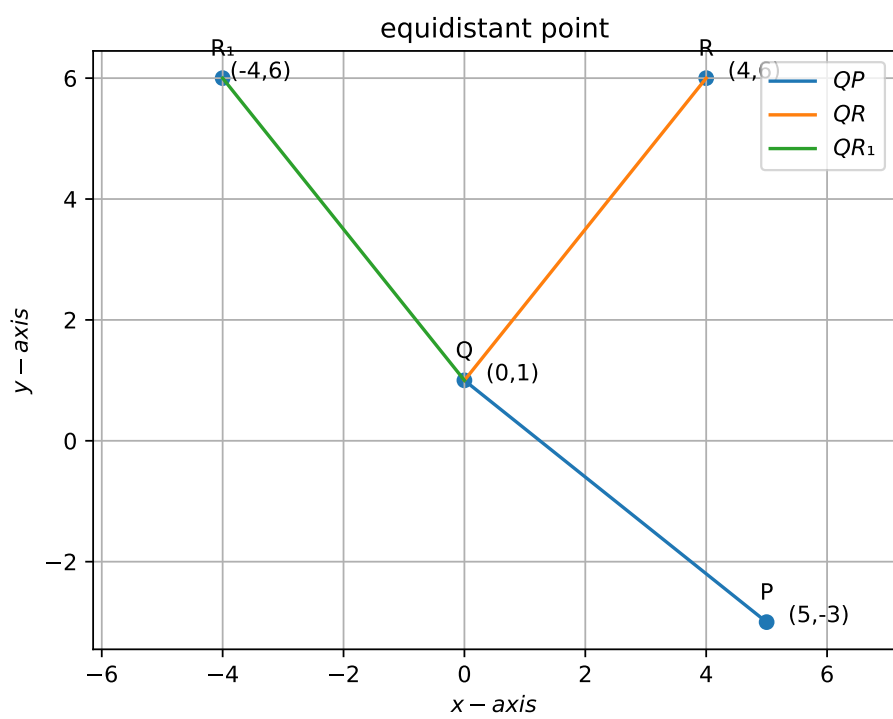


Figure 1