Vectors

1   12th Maths - Chapter 10

This is Problem-6 from Exercise 10.5 (Miscellaneous Exercise)

1. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\overrightarrow{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\overrightarrow{b} = \hat{i} - 2\hat{j} + \hat{k}$.

2   Solution

Let us assume that required vector as **x**
Given **a** and **b** are

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{2}$$

now assume that

$$\mathbf{a} + \mathbf{b} = \mathbf{c} \tag{3}$$

$$\mathbf{c} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \tag{5}$$

so our resultant vector is $\mathbf{c} = \begin{pmatrix} 3 & 2 & 0 \end{pmatrix}$

$$\frac{\mathbf{c}}{\|\mathbf{c}\|} = \frac{1}{\sqrt{10}} \times \begin{pmatrix} 3 & 2 & 0 \end{pmatrix} \tag{6}$$

so $\frac{1}{\sqrt{10}} \times \begin{pmatrix} 3 & 2 & 0 \end{pmatrix}$ is a unit vector which is in the direction of resultant vector of $\mathbf{a}$ and $\mathbf{b}$ but we want the $\mathbf{x}$ which magnitude is 5 so

$$\mathbf{x} = 5 \times \frac{1}{\sqrt{10}} \times \begin{pmatrix} 3 & 2 & 0 \end{pmatrix} \tag{7}$$

$$\mathbf{x} = \begin{pmatrix} \frac{3\sqrt{10}}{2} & \frac{\sqrt{10}}{2} \end{pmatrix} \tag{8}$$

Hence the required vector $\mathbf{x}$ is $\begin{pmatrix} \frac{3\sqrt{10}}{2} & \frac{\sqrt{10}}{2} \end{pmatrix}$