

Vectors

1 12th Maths - Chapter 10

This is Problem-4 from Exercise 10.4

1. Show that $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 2(\vec{a} \cdot \vec{b})$

2 Solution

Let us assume that

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad (1)$$

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \quad (2)$$

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \quad (3)$$

by using vector product

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} -4 \\ 8 \\ -4 \end{pmatrix} \quad (4)$$

$$2(\mathbf{a} \times \mathbf{b}) = \begin{pmatrix} -8 \\ 16 \\ -8 \end{pmatrix} \quad (5)$$

$$(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) = \begin{pmatrix} -8 \\ 16 \\ -8 \end{pmatrix} \quad (6)$$

we can clearly see that (5) is equal to (6)

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$$