Coordinate Geometry

10^{th} Maths - Chapter 7

This is Problem-8 from Exercise 7.1

1. The Point Q= $\begin{pmatrix}0\\1\end{pmatrix}$ is equidistant from P=$\begin{pmatrix}5\\-3\end{pmatrix}$ and R=$\begin{pmatrix}x\\6\end{pmatrix}$. Find the value of x

Solution:
The input parameters for this problem are available in Table

Symbol	Value	Description
P	$\begin{pmatrix}5\\-3\end{pmatrix}$	First point
Q	$\begin{pmatrix}0\\1\end{pmatrix}$	Second point
R	$\begin{pmatrix}?\\6\end{pmatrix}$	Desired point

Table 1

If **Q** is equidistant from the points **P** and **R**,

$$\|\mathbf{P}-\mathbf{Q}\| = \|\mathbf{R}-\mathbf{Q}\| \tag{1}$$

$$\implies \|\mathbf{P}-\mathbf{Q}\|^2 = \|\mathbf{R}-\mathbf{Q}\|^2 \tag{2}$$

which can be expressed as

$$\begin{aligned}(\mathbf{P}-\mathbf{Q})^{\top}(\mathbf{P}-\mathbf{Q}) &= (\mathbf{R}-\mathbf{Q})^{\top}(\mathbf{R}-\mathbf{Q}) \\ \implies \|\mathbf{Q}\|^2 - 2\mathbf{Q}^{\top}\mathbf{P} + \|\mathbf{P}\|^2 &= \|\mathbf{Q}\|^2 - 2\mathbf{Q}^{\top}\mathbf{R} + \|\mathbf{R}\|^2 \quad (3)\end{aligned}$$

which can be simplified to obtain
where

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{Q}^{\top}\mathbf{P} = \|\mathbf{R}\|^2 - 2\mathbf{Q}^{\top}\mathbf{R} \quad (4)$$

now substituting the P,Q and R values in (4)

$$\|\mathbf{P}\|^2 = 34 \quad (5)$$

$$\|\mathbf{R}\|^2 = x^2 + 36 \quad (6)$$

$$2\mathbf{Q}^{\top}\mathbf{P} = -6 \quad (7)$$

$$2\mathbf{Q}^{\top}\mathbf{R} = 12 \quad (8)$$

upon substituting the values in (4) we get a quadratic equation

$$x^2 - 16 = 0 \quad (9)$$

therefore the the value of $\|x\| = 4$
Hence, the desired point is $\mathbf{R}$ is $\begin{pmatrix}4\\6\end{pmatrix}$.

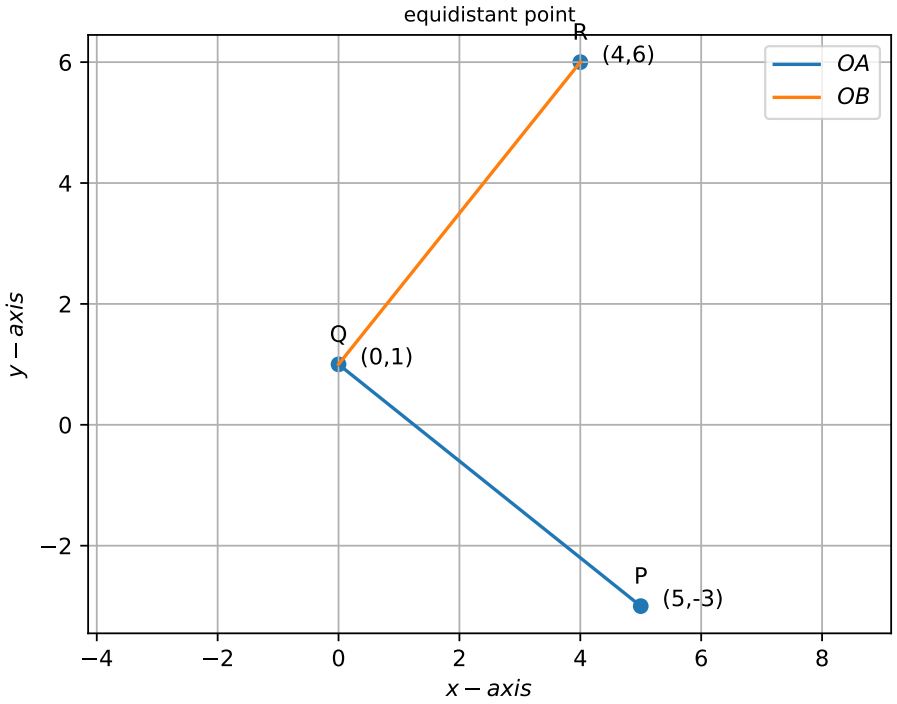
equidistant point
R
(4,6)
OA
OB
Q
(0,1)
P
(5,-3)
6
4
2
0
−2
−4
−2
0
2
4
6
8
y − axis
x − axis

Figure 1