Properties of vectors

12^{th} Maths - Exercise 10.2.1

1. Compute the magnitude of the following vectors

$$\overrightarrow{d} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{b} = 2\hat{i} - 7\hat{j} + 3\hat{k} \text{ and } \overrightarrow{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}.$$

Solution:

Let
$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\mathbf{b} = \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$ (1)

let us assume magnitudes of a, b, c are x,y,z respectively

$$x = \|\mathbf{a}\|, y = \|\mathbf{b}\|, z = \|\mathbf{c}\|$$
 (2)

so

$$x = \|\mathbf{a}\| = \mathbf{a}^{\mathsf{T}}\mathbf{a},\tag{3}$$

$$y = \|\mathbf{b}\| = \mathbf{b}^{\mathsf{T}}\mathbf{b},\tag{4}$$

$$z = \|\mathbf{c}\| = \mathbf{c}^{\mathsf{T}}\mathbf{c} \tag{5}$$

now substituting values of (1) in (3),(4) and (5) respectively

$$x = 1.732 \tag{6}$$

$$y = 7.874 \tag{7}$$

$$z = 1 \tag{8}$$

the magnitudes of $\mathbf{a} = \sqrt{3}$, $\mathbf{b} = \sqrt{62}$, $\mathbf{c} = 1$