Coordinate Geometry

10^{th} Maths - Chapter 7

This is Problem-8 from Exercise 7.1

1. If $\mathbf{Q}=\begin{pmatrix}0\\1\end{pmatrix}$ is equidistant from $\mathbf{P}=\begin{pmatrix}5\\-3\end{pmatrix}$ and $\mathbf{R}=\begin{pmatrix}x\\6\end{pmatrix}$. Find the values of x.

Solution:
The input parameters for this problem are available in Table

Symbol	Value	Description
$\mathbf{P}$	$\begin{pmatrix}5\\-3\end{pmatrix}$	First point
$\mathbf{Q}$	$\begin{pmatrix}0\\1\end{pmatrix}$	Second point
$\mathbf{R}$	$\begin{pmatrix}?\\6\end{pmatrix}$	Desired point

Table 1

If $\mathbf{Q}$ is equidistant from the points $\mathbf{P}$ and $\mathbf{R}$,

$$\|\mathbf{P}-\mathbf{Q}\| = \|\mathbf{R}-\mathbf{Q}\| \tag{1}$$
$$\implies \|\mathbf{P}-\mathbf{Q}\|^2 = \|\mathbf{R}-\mathbf{Q}\|^2 \tag{2}$$

which can be expressed as

$$(\mathbf{P}-\mathbf{Q})^{\top}(\mathbf{P}-\mathbf{Q}) = (\mathbf{R}-\mathbf{Q})^{\top}(\mathbf{R}-\mathbf{Q}) \tag{3}$$
$$= \|\mathbf{Q}\|^2 - 2\mathbf{Q}^{\top}\mathbf{P} + \|\mathbf{P}\|^2 \tag{4}$$
$$= \|\mathbf{Q}\|^2 - 2\mathbf{Q}^{\top}\mathbf{R} + \|\mathbf{R}\|^2 \tag{5}$$

which can be simplified to obtain
where

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{Q}^{\top}\mathbf{P} = \|\mathbf{R}\|^2 - 2\mathbf{Q}^{\top}\mathbf{R} \tag{6}$$

now substituting the P,Q and R values in (6)

$$\|\mathbf{P}\|^2 = 34 \tag{7}$$
$$\|\mathbf{R}\|^2 = x^2 + 36 \tag{8}$$
$$2\mathbf{Q}^{\top}\mathbf{P} = -6 \tag{9}$$
$$2\mathbf{Q}^{\top}\mathbf{R} = 12 \tag{10}$$

upon substituting the values in (6)

$$x^2 + 36 - 12 = 34 + 6 \tag{11}$$
$$\implies x^2 = 16 \tag{12}$$

Then the value of $x = 4$ or -4. Hence, the desired point is $\mathbf{R}$ is $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

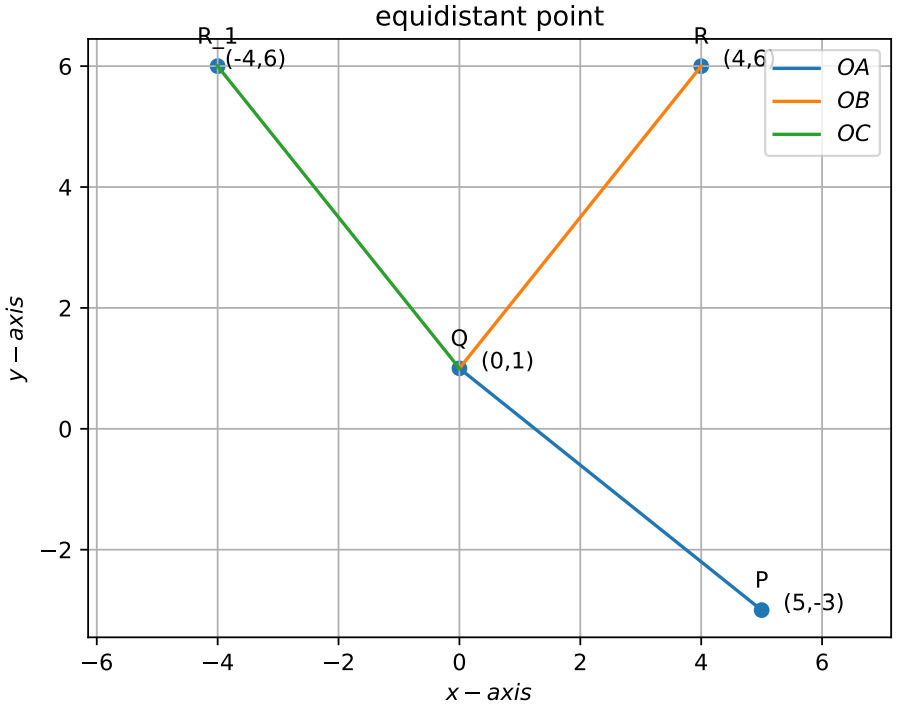
equidistant point
R_1
(-4,6)
R
(4,6
OA
OB
OC
Q
(0,1)
P
(5,-3)
6
4
2
0
−2
−6
−4
−2
0
2
4
6
y − axis
x − axis

Figure 1