

Coordinate Geometry

10th Maths - Chapter 7

This is Problem-8 from Exercise 7.1

1. The point on the line $Q = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ which is equidistant from $P = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $R = \begin{pmatrix} x \\ 6 \end{pmatrix}$. Find x

Solution:

The input parameters for this problem are available in Table

Symbol	Value	Description
P	$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$	First point
Q	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	Second point
R	$\begin{pmatrix} ? \\ 6 \end{pmatrix}$	Desired point

Table 1

If **Q** is equidistant from the points **P** and **R**,

$$\|\mathbf{P} - \mathbf{Q}\| = \|\mathbf{R} - \mathbf{Q}\| \quad (1)$$

$$\implies \|\mathbf{P} - \mathbf{Q}\|^2 = \|\mathbf{R} - \mathbf{Q}\|^2 \quad (2)$$

which can be expressed as

$$\begin{aligned}
 (\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q}) &= (\mathbf{R} - \mathbf{Q})^\top (\mathbf{R} - \mathbf{Q}) \\
 \implies \|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top \mathbf{P} + \|\mathbf{P}\|^2 &= \|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top \mathbf{R} + \|\mathbf{R}\|^2 \quad (3)
 \end{aligned}$$

which can be simplified to obtain

where

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{Q}^\top \mathbf{P} = \|\mathbf{R}\|^2 - 2\mathbf{Q}^\top \mathbf{R} \quad (4)$$

now substituting the P,Q and R values in (4)

$$\|\mathbf{P}\|^2 = 34 \quad (5)$$

$$\|\mathbf{R}\|^2 = x^2 + 36 \quad (6)$$

$$2\mathbf{Q}^\top \mathbf{P} = -6 \quad (7)$$

$$2\mathbf{Q}^\top \mathbf{R} = 12 \quad (8)$$

upon substituting the values in (4) the value of $x = 4$

Hence, the desired point is \mathbf{R} is $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$.

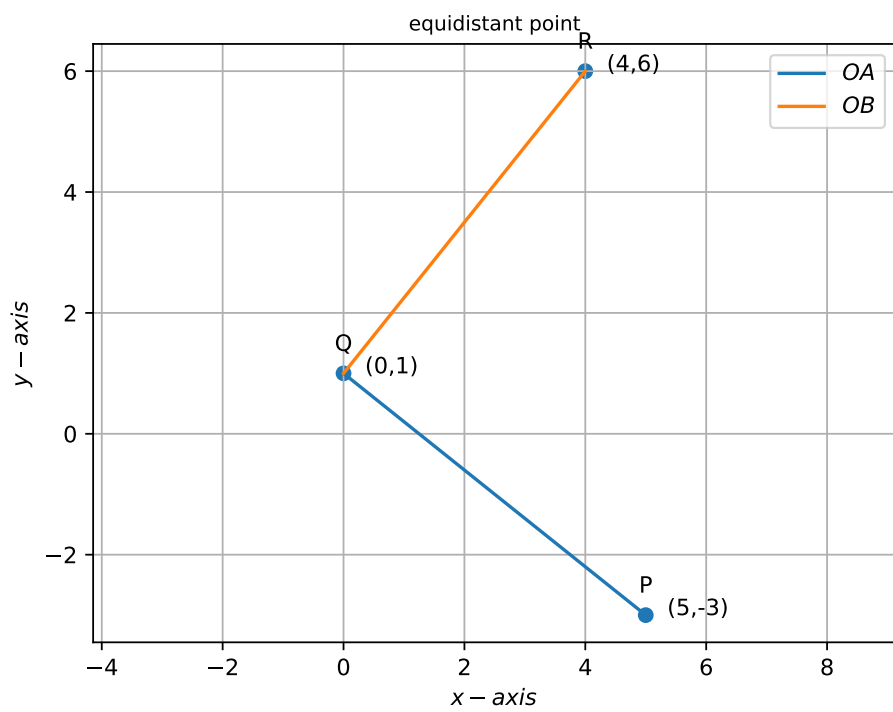


Figure 1