Coordinate Geometry

10th Maths - Chapter 7

This is Problem-8 from Exercise 7.1

1. The point on the Q= $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ which is equidistant from P= $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and R= $\begin{pmatrix} x \\ 6 \end{pmatrix}$. Find x

   Solution:
   The input parameters for this problem are available in Table

Symbol	Value	Description
P	$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$	First point
Q	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	Second point
R	$\begin{pmatrix} ? \\ 6 \end{pmatrix}$	Desired point

   Table 1

   If **Q** is equidistant from the points **P** and **R**,

$$\|\mathbf{P} - \mathbf{Q}\| = \|\mathbf{R} - \mathbf{Q}\| \tag{1}$$
$$\implies \|\mathbf{P} - \mathbf{Q}\|^2 = \|\mathbf{R} - \mathbf{Q}\|^2 \tag{2}$$

which can be expressed as

$$\left(\mathbf{P} - \mathbf{Q}\right)^{\top}\left(\mathbf{P} - \mathbf{Q}\right) = \left(\mathbf{R} - \mathbf{Q}\right)^{\top}\left(\mathbf{R} - \mathbf{Q}\right)$$
$$\implies \|\mathbf{Q}\|^2 - 2\mathbf{Q}^{\top}\mathbf{P} + \|\mathbf{P}\|^2$$
$$= \|\mathbf{Q}\|^2 - 2\mathbf{Q}^{\top}\mathbf{R} + \|\mathbf{R}\|^2 \quad (3)$$

which can be simplified to obtain

where

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{Q}^{\top}\mathbf{P} = \|\mathbf{R}\|^2 - 2\mathbf{Q}^{\top}\mathbf{R} \quad (4)$$

now substituting the P,Q and R values in (4)

$$\|\mathbf{P}\|^2 = 34 \quad (5)$$

$$\|\mathbf{R}\|^2 = x^2 + 36 \quad (6)$$

$$2\mathbf{Q}^{\top}\mathbf{P} = -6 \quad (7)$$

$$2\mathbf{Q}^{\top}\mathbf{R} = 12 \quad (8)$$

upon substituting the values in (4) the value of $x = 4$

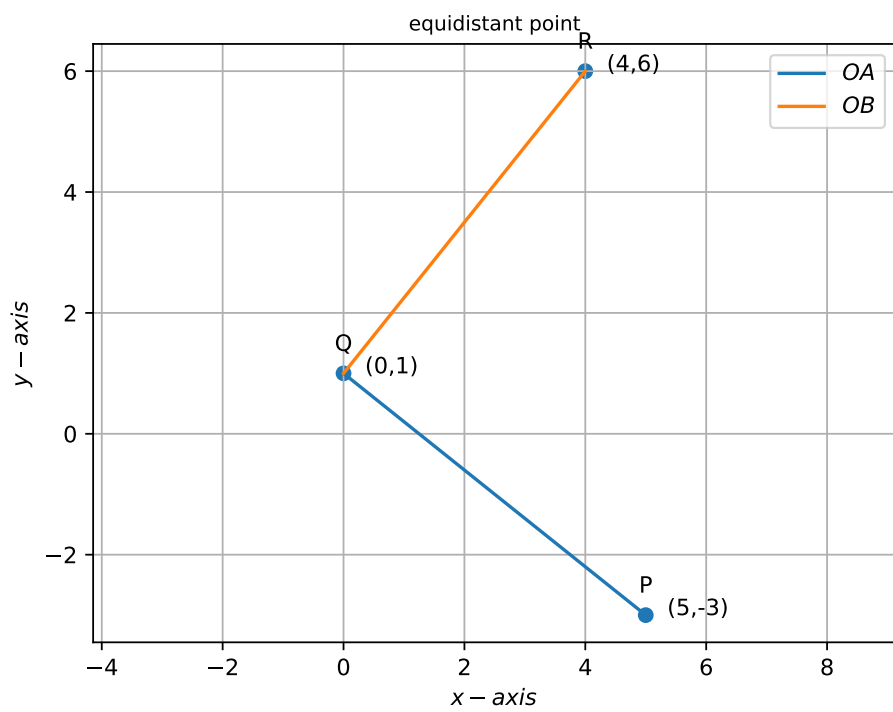Hence, the desired point is $\mathbf{R}$ is $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$.

Figure 1