M III

Unit I - Higher order linear differential equations.

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = Q(x)$$

$$D^r = \frac{d^r}{dx^r}$$

$$(a_0 D^n) + (a_1 D^{n-1} + \cdots + a_{n-1} D + a_n) y = Q(x)$$

$$F(D) y = Q(x)$$

General Solution

y = complimentary Function (y_c) + Particular integral (y_p)

CF (y_c) is defined solution of $F(D)y = 0$

$$PI\ (y_p) = \frac{1}{F(D)} Q(x)$$

Type I - Method of solving $F(D)y = 0$

where $F(D) = a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_{n-1} D + a_n$

Step I - Write the differential eq^n in the operator form $F(D)y = 0$

Step II - Write the characteristic eq^n $F(m) = 0$ $(D \to m)$

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Step III - Solve and find roots

Step IV -

- Real & Distinct $- c_1 e^{m_1 y} + c_2 e^{m_2 x}$
- Real & repeated $m_1 m_2$ $(c_1 + c_2 x) e^{ax}$
- Complex conjugate $e^{ax}(c_2 \cos bx + c_3 \sin bx)$
- Complex & repeated $e^{ax}((A_1 + A_2 x)\cos bx + (B_1 + B_2 x)\sin bx))$

Q Solve $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$

Step 1: $(D^2 - 7D + 12)y = 0$

Step 2: Characteristic eq^n

Step 3:

$$m^2 - 7m + 12 = 0$$
$$(m-4)(m-3) = 0$$
$$m = 4 \quad , m = 3$$

Step 4: Roots are real and distinct

$$y_c = c_1 e^{4x} + c_2 e^{3x}$$

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Q $y''' + 6y'' + 11y' + 6y = 0$

Step1- $(D^3 + 6D^2 + 11D + 6)y = 0$

Step2- Characteristic eq^n

Step3- $m^3 + 6m^2 + 11m + 6 = 0$

$m = -3, -1, -2.$
$m = -1, -2, -3.$

Step4- Roots are real and distinct

$y_c = c_1 e^{-1x} + c_2 e^{-2x} + c_3 e^{-3x}$

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$(a+b)^3$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

CASE 2: characteristic roots are real & repeated

Let m_1 be the root reapeated 'r' times

$$Y_c = (c_1 + c_2x + c_3x^2 + c_4x^3 + \cdots + c_rx^{r-1})e^{mx}$$

Q $\boxed{\text{Solve } \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0}$

Step 1: Write the differential equation in operator form

$$F(D)y = 0$$
$$(D^2 - 6D + 9)y = 0$$

Step 2: Characteristic equation:

$$F(m) = 0 \Rightarrow m^2 - 6m + 9 = 0$$

Step 3: Find characteristic roots:

$$(m-3)^2 = 0 \quad , m = 3, 3$$

Step 4: Real & Repeated roots

$$y = (c_1 + c_2x)e^{3x}$$

Q $\boxed{\text{Solve } (D^3 + 3D^2 + 3D + 1)y = 0}$

Step 1: $F(D)y = 0$
$(D^3 + 3D^2 + 3D + 1)y = 0$

Step 2: Characteristic equation: $m^3 + 3m^2 + 3m + 1 = 0$

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quadratic equation $\begin{cases} ax^2+bx+c=0 \\ x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \end{cases}$

Step 3: characteristic roots:

$(m+1)^3 = 0$

$m = -1, -1, -1$

Step 4: Roots are real & repeated

$y = (c_1 + c_2 x + c_3 x^2) e^{-1x}$

Q $\boxed{2y'' - y' - 10y = 0}$

Step 1: $(2D^2 - D - 10)y = 0$

Step 2: characteristic eq:

$2m^2 - m - 10 = 0$

Step 3: $2m^2 - m - 10 = 0$

$m = \dfrac{1 \pm \sqrt{1+80}}{4} = \dfrac{1 \pm 9}{4} = \dfrac{5}{2}, -2$

Step 4: Roots are real & distinct

$y_c = c_1 e^{\frac{5x}{2}} + c_2 e^{-2x}$

Q $\boxed{(D^4 - 2D^3 + D^2)y = 0}$

Step 1: $(D^4 - 2D^3 + D^2)y = 0$

Step 2: $(m^4 - 2m^3 + m^2) = 0$

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$e^{i\theta} = \cos\theta + i\sin\theta$ $\quad y' = \frac{dy}{dx}$

$\sqrt{-1} = i$ $\quad y'' = \frac{d^2y}{dx^2}$

Step 3: $m^2(m^2 - 2m + 1) = 0$

$m^2 (m-1)^2 = 0$

$m = 0, 0, 1, 1$

Step 4: Roots are real & repeated

$y_c = (c_1 + c_2 x)e^{0x} + (c_3 + c_4 x)e^{1x}$

CASE 3: Roots are complex conjugates of the form

$\alpha \pm i\beta$

$y_c = e^{\alpha x}[A\cos\beta x + B\sin\beta x]$

Q $\boxed{y'' - y' + y = 0}$

(I) $(D^2 - D + 1)y = 0$

(II) $m^2 - m + 1 = 0$

(III) $m = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2} = \underset{\alpha}{\frac{1}{2}} \pm i\underset{\beta}{\frac{\sqrt{3}}{2}}$

(IV) Roots are complex conjugates

$y_c = e^{\frac{1}{2}x}\left[A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right]$

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Q $\boxed{4\frac{d^2y}{dx} - 8\frac{dy}{dx} + 7y = 0}$

operating of y (will saying we say)

(I) $(4D^2 - 8D + 7)y = 0$

(II) $4m^2 - 8m + 7 = 0$

(III) $m = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(7)}}{2(4)}$

$= \frac{8 \pm \sqrt{64 - 16(7)}}{8}$

$= \frac{8 \pm \sqrt{64 - 112}}{8}$

$= \frac{8 \pm \sqrt{-48}}{8}$

$= \frac{8 \pm i\sqrt{16 \times 3}}{8}$

$= \frac{8 \pm 4i\sqrt{3}}{8} = \frac{8}{8} \pm \frac{4i\sqrt{3}}{8} = 1 \pm \frac{i\sqrt{3}}{2}$

$m = 1 \pm \frac{i\sqrt{3}}{2}$

(IV) Complex conjugate

$y_c = e^x \left[A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x \right]$

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Q $\boxed{(D^4+4D^2+4)y=0}$

(I) $(D^4+4D^2+4)y=0$

(II) $m^4+4m^2+4=0$

(III) $(m^2+2)^2=0$

$m^2+2=0$, $m^2+2=0$

$m^2=-2$, $m^2=-2$

$m=\pm\sqrt{-2}$, $m=\pm\sqrt{-2}$

$m=\pm i\sqrt{2}$, $m=\pm i\sqrt{2}$

$m=\pm i\sqrt{2}, \pm i\sqrt{2}$

$\alpha=0, \beta=\sqrt{2}$

(IV) Complex conjugate & repeated roots

$$y_c=e^{0x}\left[(A_1+A_2x)\cos\sqrt{2}x+(B_1+B_2x)\sin\sqrt{2}x\right]$$

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Q $\boxed{\text{Solve } (D^3+D^2-2D+12)y=0}$

Step 2 $m^3+m^2-2m+12=0$

Step 3 By calculator,

$m=-3,\ 1\pm\sqrt{3}i$;

Manually how to solve :-

-3	1	1	-2	12	← coefficients
	$+0$	$+$ -3	$+$ -6	$+$ -12	add up and down value
	1	-2	4	e	
		-3×1	-3×-2	-3×4	

$1 \quad -2 \quad 4$

$1m^2+(-2)m+4=0$

$(m+3)(m^2-2m+4)=0$

$m=-3$, $m=\dfrac{2\pm\sqrt{4-4(1)(4)}}{2(1)}$

$=\dfrac{2\pm\sqrt{-12}}{2}=\dfrac{2\pm i2\sqrt{3}}{2}=1\pm i\sqrt{3}$

$\therefore m=-3,\ 1\pm i\sqrt{3}$

$Y_c = A_1e^{-3x}+e^{x}(A_2\cos\sqrt{3}x+A_3\sin\sqrt{3}x)$

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Q $\boxed{\text{Solve } (D^4 - 2D^3 + D^2)y = 0}$

Step2:- $m^4 - 2m^3 + m^2 = 0$

Step3:- $m^2(m^2 - 2m + 1) = 0$

$m^2 = 0 \quad , \quad m^2 - 2m + 1 = 0 \rightarrow (m-1)^2$

$m = 0, 0 \quad , \quad m = 1, 1$

$m = 0, 0, 1, 1$

Step4:- Roots are Real & repeated

$(C_1 + C_2 x)e^{0x} + (C_3 + C_4 x)e^{1x}$

Q $\boxed{\text{Solve } \frac{d^4y}{dx^4} - 16y = 0}$

Step 1:- $(D^4 - 16)y = 0$

Step 2:- $m^4 - 16 = 0$

Step 3:- $(m^2 - 4)(m^2 + 4) = 0$

$m = \pm 2, \; m = \pm 2i$

$m = 2, -2, 0 \pm i2$

$C_1 e^{2x} + C_2 e^{-2x} + e^{0x}(C_3 \cos 2x + C_4 \sin 2x)$

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Q $\boxed{\text{Solve } \frac{d^4y}{dx^4} - 13\frac{d^2y}{dx^2} + 36y = 0}$

Step 1:- $(D^4 - 13D^2 + 36)y = 0$

sum = 13
product = 36

Step 2:- $m^4 - 13m^2 + 36 = 0$

$9 \times 4 = 36$
$9 + 4 = 13$

Step 3:- $(m^2 - 9)(m^2 - 4) = 0$

$m^2 = 9$, $m^2 = 4$
$m = \pm 3$, $m = \pm 2$

$m = +3, -3, +2, -2$

Step 4:- Roots are real. ~~& repeated~~ & distint.

$y_c = C_1 e^{3x} + C_2 e^{-3x} + C_3 e^{2x} + C_4 e^{-2x}$

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Q $\boxed{\frac{d^4y}{dx^4} + 25\frac{d^2y}{dx^2} + 144y = 0}$

Step 1:- $(D^4 + 25D^2 + 144)y = 0$

sum = 25
product = 144

Step 2:- $m^4 + 25m^2 + 144 = 0$

$16 \times 9 = 144$
$16 + 9 = 25$

Step 3:- $(m^2 + 16)(m^2 + 9) = 0$

$m^2 = -16$, $m^2 = -9$
$m = \pm i\sqrt{16}$, $m = \pm i\sqrt{9}$
$m = \pm i4$, $m = \pm i3$

$m = \pm i4, \pm i3 \longrightarrow 0 \pm i4, 0 \pm i3$

Step :- $Y_c = e^{0x}(C_1\cos 3x + C_2\sin 3x) + e^{0x}(C_3\cos 4x + C_4\sin 4$

→ Splitting the midterm

Q $\boxed{\text{Solve } (D^4 + 6D^2 + 25)y = 0}$

Step 2:- $m^4 + 6m^2 + 25 = 0$

Step 3:- $m^4 + \underline{10m^2 - 4m^2} + 25 = 0$

↳ midterm splitted

$(m^4 + 10m^2 + 25) - (4m^2) = 0$

$(m^2 + 5)^2 - (2m)^2 = 0$

$A^2 - B^2 = (A+B)(A-B)$ //FORMULA

$(m^2 + 5 + 2m)(m^2 + 5 - 2m) = 0$

$m = \dfrac{-2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$

$m = \dfrac{-2 \pm \sqrt{4 - 20}}{2}$

$m = \dfrac{-2 \pm i\sqrt{16}}{2}$

$m = \dfrac{-2 \pm 4i}{2}$

$m = \dfrac{2 \pm \sqrt{4 - 20}}{2(1)}$

$m = \dfrac{2 \pm \sqrt{-16}}{2}$

$m = \dfrac{2 \pm i\sqrt{16}}{2}$

$m = \dfrac{2 \pm 4i}{2}$

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$$m = \frac{-2}{2} \pm \frac{4i}{2}$$

$$m = -1 \pm 2i$$

$$m = \frac{2}{2} \pm \frac{4i}{2}$$

$$m = 1 \pm 2i$$

$$\therefore m = -1 \pm 2i,\ 1 \pm 2i$$

Step 4:-

$$y_c = e^{-1x}(C_1\cos 2x + C_2\sin 2x) + e^{1x}(C_3\cos 2x + C_4\sin 2x)$$

Q $\boxed{(D^4 + a^4)y = 0}$

Step 1:- $m^4 + a^4 = 0$

Step 3:- $m^4 + \underline{2m^2a^2 - 2m^2a^2} + a^4 = 0$

$$(m^4 + 2m^2a^2 + a^4) - (2m^2a^2) = 0$$

$$(m^2 + a^2)^2 - (\sqrt{2}ma)^2 = 0 \quad \ldots (A^2 - B^2 \text{ formula})$$

$$(m^2 + a^2 + \sqrt{2}ma)(m^2 + a^2 - \sqrt{2}ma) = 0$$

$$m^2 + a^2 + \sqrt{2}ma = 0$$

$$m = \frac{-\sqrt{2}a \pm \sqrt{2a^2 - 4a^2}}{2(1)}$$

$$m = \frac{-\sqrt{2}a \pm i\sqrt{2}a}{2}$$

$$m^2 + a^2 - \sqrt{2}ma = 0$$

$$m = \frac{\sqrt{2}a \pm \sqrt{2a^2 - 4a^2}}{2(1)}$$

$$m = \frac{\sqrt{2}a \pm i\sqrt{2}a}{2}$$

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$$= -\frac{\sqrt{2}a}{2} \pm \frac{i\sqrt{2}a}{2}$$

$$= -\frac{\cancel{\sqrt{2}}a}{\cancel{\sqrt{2}}\times\sqrt{2}} \pm \frac{i\cancel{\sqrt{2}}a}{\cancel{\sqrt{2}}\sqrt{2}}$$

$$= \frac{-a}{\sqrt{2}} \pm \frac{ia}{\sqrt{2}}$$

$$m = \frac{a}{\sqrt{2}}(-1 \pm i)$$

$$= \frac{\sqrt{2}a}{2} \pm \frac{i\sqrt{2}a}{2}$$

$$= \frac{a}{\sqrt{2}} \pm \frac{ia}{\sqrt{2}}$$

$$m = \frac{a}{\sqrt{2}}(1 \pm i)$$

Step 4:-

roots are

$$m = -\frac{a}{\sqrt{2}} \pm \frac{ia}{\sqrt{2}}, \quad \frac{a}{\sqrt{2}}(1 \pm i)$$

$$m = -\frac{a}{\sqrt{2}} \pm \frac{ia}{\sqrt{2}}; \quad \frac{a}{\sqrt{2}} \pm \frac{ia}{\sqrt{2}}$$

Step 5:-

$$y_c = e^{-\frac{a}{\sqrt{2}}x}\left[C_1\cos\frac{ax}{\sqrt{2}} + C_2\sin\frac{ax}{\sqrt{2}}\right] +$$

$$e^{\frac{ax}{\sqrt{2}}}\left[C_3\cos\frac{ax}{\sqrt{2}} + C_4\sin\frac{ax}{\sqrt{2}}\right]$$

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Q $\boxed{(D^2+1)^3\,(D^2+D+1)^2 y = 0}$

step2: $(m^2+1)^3\,(m^2+m+1)^2 = 0$

step3: $(m^2+1)^3 = 0$

$m^2 = -1, -1, -1$

$m = \pm i, \pm i, \pm i$

$(m^2+m+1)^2 = 0$

$m = \dfrac{-1 \pm \sqrt{1-4(1)(1)}}{2(1)}$

$m = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$m = \dfrac{-1 \pm \sqrt{-3}}{2}$

$m = \dfrac{-1 \pm i\sqrt{3}}{2}, \dfrac{-1 \pm i\sqrt{3}}{2}$

step4:

roots are $m = \underbrace{\pm i, \pm i, \pm i}_{\substack{\alpha = 0 \\ \beta = 1}}, \overbrace{\dfrac{-1}{2} \pm \dfrac{i\sqrt{3}}{2}, \dfrac{-1}{2} \pm \dfrac{i\sqrt{3}}{2}}^{\substack{\alpha = -1/2 \\ \beta = \sqrt{3}/2}}$

$$y_c = e^{0x}\left((A_1 + A_2x + A_3x^2)\cos x + (B_1 + B_2x + B_3x^2)\sin x\right) + e^{-\frac{x}{2}}\left((A_4 + A_5x)\cos\frac{\sqrt{3}}{2}x + (B_4 + B_5x)\sin\frac{\sqrt{3}}{2}x\right)$$

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Q $4\dfrac{d^2s}{dt^2} = -9s$

Q E.I $\dfrac{d^2y}{dx} + Py = 0$

$y = 0, x = 0$
$y = a, x = l/2$
find y?

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1) $(D^4 - 1)y = 0$

$\rightarrow$ $(D^2-1)(D^2+1) = 0$

$(m^2-1)(m^2+1) = 0$

$m = \pm 1 \quad , m = \pm i$

$m = +1, -1, \pm i$

$y_c = c_1 e^{x} + c_2 e^{-x} + e^{0x}(c_3 \cos x + c_4 \sin x)$

2) $(D^3 - 7D - 6)y = 0$

$\rightarrow$ $m^3 - 7m - 6 = 0$

$m = 3, -1, -2$

$y_c = c_1 e^{3x} + c_2 e^{-x} + c_3 e^{-2x}$

3) $(D^2 + 6D + 9)y$

$\rightarrow$ $m^2 + 6m + 9 = 0$

$m^2 + 3m + 3m + 9 = 0$

$m(m+3) + 3(m+3) = 0$

$(m+3)(m+3) = 0$

$m = -3, -3$

$y_c = (c_1 + c_2 x)e^{-3x}$

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4) $(D^2+9)^2 y = 0$

$\rightarrow$ $(D^2+9)(D^2+9)=0$

$m^2+9=0$ and $m^2+9=0$

$m=\pm 3i$ $\quad$ $m=\pm 3i$

$y_c = e^{0x}((A_1+A_2x)\cos 3x + (B_1+B_2x)\sin 3x)$

5) $(D^2+1)^2 (D-3)(D^2+9)y = 0$

$\rightarrow$ $(m^2+1)^2 (m-3)(m^2+9)=0$

$(m^2+1)(m^2+1)(m-3)(m^2+9)=0$

$m^2+1=0$

$m=\pm i, \pm i$

$m=3$

$m=\pm 3i$

$y_c = e^{0x}((c_1+c_2x)\cos x + (c_3+c_4x)\sin x) + c_5 e^{3x}$

$+ e^{0x}(c_6\cos 3x + c_7 \sin 3x)$

6) $(D^4+2D^2+1)y=0$

$\rightarrow$ $m^4+2m^2+1=0$

$m^2+2m+1=0$

$m^2+m+m+1=0$

$m(m+1)+1(m+1)=0$

$(m+1)(m+1)=0$

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$$m = \pm i, \pm i$$

$$y_c = e^{0x}\left((c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x\right)$$

7) $(D^3 - 5D^2 + 8D - 4) = 0$

$\rightarrow$ $m^3 - 5m^2 + 8m - 4 = 0$

$$m = 1, 2, 2$$

$$y_c = c_1 e^{x} + (c_2 + c_3 x)e^{2x}$$

8) $(D^6 + 6D^4 + 9D^2)y = 0$

$\rightarrow$ $(m^6 + 6m^4 + 9m^2) = 0$

$$m^2(m^4 + 6m^2 + 9) = 0$$

$$m^2(m^4 + 3m^2 + 3m^2 + 9) = 0$$

$$m^2(m^2(m^2+3) + 3(m^2+3)) = 0$$

$$m^2((m^2+3)(m^2+3)) = 0$$

$$m = 0, 0$$

$$m^2 = -3$$

$$m = \pm i\sqrt{3}, \pm i\sqrt{3}$$

$$y_c = e^{0x}(c_1 + c_2 x) + e^{0x}\left((c_3 + c_4 x)\cos\sqrt{3}x + (c_5 + c_6 x)\sin\sqrt{3}x\right)$$

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9) $(D^4+1)y = 0$

$\rightarrow$ $m^4+1=0$

$m^4+1=0$

$\Rightarrow m^4+1+2m^2-2m^2=0$

$\Rightarrow (m^2+1)^2-2m^2=0$

$\Rightarrow (m^2+1)^2-(\sqrt{2}m)^2=0$

$\Rightarrow (m^2+1+\sqrt{2}m)(m^2+1-\sqrt{2}m)=0$

$m = \dfrac{-\sqrt{2} \pm \sqrt{2-4}}{2}$ and $m = \dfrac{\sqrt{2} \pm \sqrt{2-4}}{2}$

$= \dfrac{-\sqrt{2} \pm \sqrt{-2}}{2}$ $\qquad = \dfrac{\sqrt{2} \pm \sqrt{-2}}{2}$

$= -\dfrac{\sqrt{2}}{2} \pm \dfrac{i\sqrt{2}}{2}$ $\qquad = \dfrac{\sqrt{2}}{2} \pm \dfrac{i\sqrt{2}}{2}$

$= \dfrac{-\sqrt{2}}{\sqrt{2}\sqrt{2}} \pm \dfrac{i\sqrt{2}}{\sqrt{2}\sqrt{2}}$ $\qquad = \dfrac{\sqrt{2}}{\sqrt{2}\sqrt{2}} \pm \dfrac{i\sqrt{2}}{\sqrt{2}\sqrt{2}}$

$m = -\dfrac{1}{\sqrt{2}} \pm \dfrac{i}{\sqrt{2}}, \quad \dfrac{1}{\sqrt{2}} \pm \dfrac{i}{\sqrt{2}}$

$y_c = e^{-1/\sqrt{2}\,x}\left(A_1\cos\frac{1}{\sqrt{2}}x + B_1\sin\frac{1}{\sqrt{2}}x\right) + e^{1/\sqrt{2}\,x}\left(A_2\cos\frac{1}{\sqrt{2}}x + B_2\sin\frac{1}{\sqrt{2}}x\right)$

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$Q(x)$: e^{ax}, $\sin ax$ or $\cos ax$, x^n

→ **Particular Integral**

→ **General Method**

$$\frac{dy}{dx} + my = f(x) \qquad \frac{d}{dx} = D \qquad [D+m]y = f(x)$$

$$I.F = e^{\int p\,dx} = e^{\int m\,dx} = e^{mx}$$

G.S

$$y\,I.F = \int f(x) \cdot I.F + C$$

$$y\,e^{mx} = \int f(x)\,e^{mx} + C$$

$$y = e^{-mx} \int f(x) \cdot e^{mx} + C \cdot e^{-mx}$$

$$y_p = \frac{1}{\phi(D)} f(x)$$

FORMULAE

→ $$y_p = \frac{1}{D+m} f(x) = e^{-mx} \int f(x)\,e^{mx}\,dx$$

→ $$\frac{1}{D-m} f(x) = e^{mx} \int f(x)\,e^{-mx}\,dx$$

→ $$\frac{1}{(D+m_1)(D+m_2)} f(x) = \frac{1}{D+m_1}\left[\frac{1}{D+m_2} f(x)\right]$$

$$= \frac{1}{D+m_1}\left[e^{-m_2 x} \int e^{m_2 x} f(x)\,dx\right]$$

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Beauty of exponential { In exponential whether we differentiate or integrate we get the same function.

$$= e^{-m_1 x}\int e^{m_1 x}\left\{ e^{-m_2 x}\int e^{m_2 x} f(x)\,dx \right\} dx$$

$$\frac{1}{(D+m_1)(D+m_2)} = \frac{1}{m_2-m_1}\left[\frac{1}{D+m_1} - \frac{1}{D+m_2}\right] f(x)$$

2-3 marks questions (MCQ)

Q1 $\boxed{[D+1]y = e^{e^x}}$

$m = -1$

$y_c = c_1 e^{-x}$

$$y_p = \frac{1}{\phi(D)} f(x) = \frac{1}{D+1} e^{e^x}$$

$$= e^{-x}\int e^{x} e^{e^x} dx$$

$\ldots$ $\left(\text{put } e^x = t,\ e^x dx = dt\right)$

$$= e^{-x}\int e^{t} dt = e^{-x} e^{t} = e^{-x} e^{e^x}$$

$$y = y_c + y_p = c_1 e^{-x} + e^{-x} e^{e^x}$$

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Q2 $\boxed{(D+1)y = \sin e^x}$

$m = -1$

$y_c = c_1 e^{-x}$

$$y_p = \frac{1}{\phi(D)} f(x) = \frac{1}{D+1} \sin(e^x)$$

$$= e^{-x} \int e^x \sin(e^x)\, dx$$

put $e^x = t$

$e^x dx = dt$

$$= e^{-x} \int \sin t \, dt$$

$$= e^{-x}(-\cos t)$$

$$= e^{-x}(-\cos(e^x))$$

$$y = c_1 e^{-x} - e^{-x}\cos(e^x)$$

Q3 $\boxed{(D+2)y = e^{-x}\cos(e^x)}$

$m = -2$

$y_c = c_1 e^{-2x}$

$$y_p = \frac{1}{\phi(D)} f(x) = \frac{1}{D+2} e^{-x}\cos(e^x)$$

$$= e^{-2x} \int e^{2x} e^{-x} \cos(e^x)\, dx$$

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$$= e^{-2x} \int e^{x} \cos(e^{x})\,dx$$

$$\cancel{= e^{-2x} \int e^{x} \cos(e^{x})\,dx}$$

$$\cdots \left(\begin{array}{l} \text{put } e^{x} = t \\ e^{x}dx = dt \end{array} \right)$$

$$= e^{-2x} \int \cos(t)\,dt$$

$$= e^{-2x} \sin t$$

$$y = c_1 e^{-2x} + e^{2x} \sin(e^{x})$$

Q4 $\boxed{(D+2)y = e^{-2x} \sec^2 x\,(1+\tan x)}$

$$y_c = c_1 e^{-2x}$$

$$y_p = \frac{1}{D+2} e^{-2x} \sec^2 x\,(1+\tan x)$$

$$= e^{-2x} \int e^{2x} \cdot e^{-2x} \sec^2 x\,(1+\tan x)\,dx$$

$$= e^{-2x} \int e^{0} \sec^2 x\,(1+\tan x)\,dx$$

$$= e^{-2x} \int \sec^2 x\,(1+\tan x)\,dx$$

$$\text{put } (1+\tan x) = t$$
$$\sec^2 x\,dx = dt$$

$$= e^{-2x} \int (t)\,dt = e^{-2x} \frac{t^2}{2} = \frac{e^{-2x}(1+\tan x)^2}{2}$$

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pto

$$y = ce^{-2x} + e^{-2x}\frac{(1+\tan x)^2}{2}$$

Q5

$$\boxed{(D+1)y = \frac{1}{1+e^x}}$$

$$y_c = c_1 e^{-x}$$

$$y_p = \frac{1}{D+1} \cdot \frac{1}{1+e^x} = e^{x}\int \frac{e^x}{1+e^x}\,dx$$

put $1+e^x = t$

$e^x dx = dt$

$$= e^{-x}\int \frac{dt}{x} = e^{-x}\log t$$

$$= e^{-x}\log(1+e^x)$$

$$y = c_1 e^{-x} + e^{-x}\log(1+e^x)$$

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UV Rule

5 marks questions (Theory questions)

Q1 $\boxed{(D^2+3D+2)y = e^{e^x}}$

A.E $m^2+3m+2=0$

$m=-2,-1$

$y_c = c_1 e^{-2x} + c_2 e^{-x}$

$$y_p = \frac{1}{D^2+3D+2} e^{e^x} = \frac{1}{(D+1)(D+2)} e^{e^x} = \underbrace{\frac{1}{D+2}\left[\frac{1}{D+1} e^{e^x}\right]}_{***}$$

$$= \frac{1}{D+2} e^{-x} \int e^x e^{e^x} dx$$

$$= \frac{1}{D+2} e^{-x} e^{e^x}$$

$$= e^{-2x} \int e^{2x} e^{-x} e^{e^x} dx$$

$$= e^{-2x} \int e^x \cdot e^{e^x} dx$$

$$= e^{-2x} e^{e^x}$$

$$y = c_1 e^{-2x} + c_2 e^{-x} + e^{-2x} e^{e^x}$$

Here we can take $\frac{1}{D+2}$ first, like

$$\frac{1}{D+1}\left[\frac{1}{D+2} e^{e^x}\right]$$

upon solving,

$$\frac{1}{D+1} e^{-2x} \int e^{2x} e^{e^x} dx$$

$$\frac{1}{D+1} e^{-2x} \int e^x e^x e^{e^x} dx$$

put $e^x = t$

$e^x dx = dt$

$$\frac{1}{D+1} e^{-2x} \int \underbrace{t e^t}_{\downarrow} dt$$

Here we will have to use UV Rule & the answer will become lengthy.

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Q2 $$\boxed{(D^2+5D+6)y = e^{-2x}\sec^2 x\,(1+2\tan x)}$$

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Shortcut Method

$f(x) = e^{ax}$

$\sin(ax+b)$ / $\cos(ax+b)$

x^m, m is +ve polynomial

$e^{ax} V$, V is $f(x)$

xV, V is $f(x)$

$\phi(D)y = Q(x)$

$$y_p = \frac{1}{\phi(D)} Q(x)$$

→ **Type 1:**

$Q(x) = e^{ax}$

1) $\frac{1}{\phi(D)} e^{ax} = \frac{1}{\phi(a)} e^{ax}$, $\phi(a) \neq 0$

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Q1 $\boxed{(D^2-1)y = e^{3x}}$

$m = 1, -1$

$y_c = c_1 e^{x} + c_2 e^{-x}$

$$y_p = \left(\frac{1}{D^2-1}\right) e^{3x}$$

$$D \to 3 \quad = \frac{1}{3^2-1} e^{3x} = \frac{1}{8} e^{3x}$$

$$y = y_c + y_p = c_1 e^{x} + c_2 e^{-x} + \frac{1}{8} e^{3x}$$

Q2 $\boxed{(D^2-1)=e^x}$

$$y_p = \frac{1}{D^2-1} e^x$$

$$D \to 1 \quad = \frac{1}{1^2-1} e^x$$

first multiply by x.
then differenciate

→ **Type 2:**

If $\phi(a) = 0 \to$ case of failure

$$\frac{1}{\phi(D)} e^{ax} = \frac{x}{\phi'(a)} e^{ax}, \quad \phi(a) \neq 0$$

$$y_p = \frac{x}{2D} e^x$$

$$D \to 1 \quad = \frac{x}{2} e^x$$

Q3 $\boxed{(D^2-7D+6)y=e^{2x}}$

$$\phi(D)y=0$$

$$(D^2-7D+6)y=0$$

$$D \to m$$

$$m^2-7m+6=0$$

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$m = 6, 1$

Roots are real & distinct

$y_c = c_1 e^{6x} + c_2 e^{x}$

$y_p = \frac{1}{D^2 - 7D + 6} e^{2x}$

$D \to a \to 2$

$= \frac{1}{2^2 - 7 \times 2 + 6} e^{2x}$

$= \frac{1}{-4} e^{2x}$

$y = y_c + y_p = c_1 e^{6x} + c_2 e^{x} - \frac{1}{4} e^{2x}$

Q4 $\boxed{(D^2 + 3D + 2)y = e^{-4x}}$

$y_p = \frac{1}{\phi(D)} \cdot Q(x) = \frac{1}{D^2 + 3D + 2} e^{-4x}$

$D \to a \to -4$

$y_p = \frac{1}{(-4)^2 + 3(-4) + 2} e^{-4x} = \frac{1}{6} e^{-4x}$

$m = 1, 1$

$y_c = (c_1 + c_2 x) e^{x}$

$y = (c_1 + c_2 x) e^{x} + \frac{1}{6} e^{-4x}$

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Q5 $\boxed{(D^2-2D+1)y = e^{2x}}$

$$(m^2-2m+1)y = e^{2x}$$

$$m^2-2m+1=0$$
$$(m-1)^2=0$$
$$m=1,1 \qquad y_c = (c_1+c_2x)e^x$$

$$y_p = \frac{1}{D^2-2D+1}e^{2x} = \frac{1}{2^2-2(2)+1}e^{2x} = \frac{1}{4-4+1}$$

$$y_p = e^{2x}$$

$$y = (c_1+c_2x)e^x + e^{2x}$$

Q6 $\boxed{(D^2-9)y = e^{3x}}$

$$y_p = \frac{1}{\phi(D)}Q(x) = \frac{1}{(D^2-9)}e^{3x}$$

$$D \Rightarrow a \Rightarrow 3$$

$$= \frac{1}{(3^2-9)}e^{3x} = \frac{1}{0}e^{3x} \leftarrow \text{Case of failure}$$

$$= \frac{x}{2D-0}e^{3x} \Rightarrow D \to 3$$

$$= \frac{x}{2\times 3}e^{3x} = \frac{x}{6}e^{3x}$$

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$$y_c = c_1 e^{3x} + c_2 e^{-3x}$$

$$y = c_1 e^{3x} + c_2 e^{-3x} + \frac{x}{6} e^{3x}$$

Q7 $\boxed{(D^2+5D+4) = e^{-4x}}$

$$y_p = \frac{1}{D^2+5D+4} e^{-4x}$$

$$D \to a \to -4$$

$$= \frac{1}{(-4)^2+5(-4)+4} e^{-4x} = \frac{1}{0} e^{-4x}$$ ← Case of failure

$$= \frac{x}{2D+5} e^{-4x}$$

$$= \frac{x}{-8+5} e^{-4x} = \frac{-x}{3} e^{-4x}$$

$y_c \Rightarrow m^2+5m+4=0$

$m^2+4m+1m+4=0$

$m(m+4)+1(m+4)=0$

$(m+4)(m+1)=0$

$m=-4, -1$

$$y_c = c_1 e^{-4x} + c_2 e^{-x}$$

$$y = c_1 e^{-4x} + c_2 e^{-x} - \frac{x}{3} e^{-4x}$$

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whenever there is a contant K then replace D by zero (0).

Q8)

① $\frac{1}{\phi(D)} ke^{ax} = \frac{1}{\phi(0)} e^{0x} \cdot K$; $\underbrace{\phi(0) \neq 0}_{\text{condition}}$

② $\frac{1}{\phi(D)} a^{x} = \frac{1}{\phi(\log a)} a^{x}$; $\underbrace{\phi(\log a) \neq 0}_{\text{condition}}$

③ $\frac{1}{\phi(D)} a^{-x} = \frac{1}{\phi(-\log a)} a^{-x}$; $\underbrace{\phi(-\log a) \neq 0}_{\text{condition}}$

④ $\underbrace{\sin(hx)}_{\text{sin hyperbolic}} = \frac{e^{x} - e^{-x}}{2}$

⑤ $\underbrace{\cos(hx)}_{\text{cos hyperbolic}} = \frac{e^{x} + e^{-x}}{2}$

Q $\boxed{(D^2-4)y = e^{2x}+2}$

Auxilary Equation ← A.E $m^2 - 4 = 0$

$m = \pm 2$

$y_c = c_1 e^{2x} + c_2 e^{-2x}$

$y_p = \frac{1}{D^2-4}(e^{2x}+2)$

$= \frac{1}{D^2-4} e^{2x} + \frac{1}{D^2-4} \times 2$

$D \to a \to 2$ $\quad D \to 0$

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$$= \frac{1}{2^2-4} e^{2x} + \frac{1}{0-4} \times 2$$

$\Rightarrow$ Case of failure

$$= \frac{x}{2D} e^{2x} - \frac{1}{2}$$

$D \to 2$

$$y_p = \frac{x}{4} e^{2x} - \frac{1}{2}$$

$$y = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{4} e^{2x} - \frac{1}{2}$$

Q $\boxed{(D-2)^3 = e^{2x} + 3^x}$

$(m-2)^3 = 0$

$m = 2, 2, 2$

$y_c = (c_1 + c_2 x + c_3 x^2) e^{2x}$ $\longrightarrow$ we can also write as $(c_1 x^2 + c_2 x + c_3) e^{2x}$ as c_1, c_2, c_3 are constants

$$y_p = \frac{1}{(D-2)^3} (e^{2x} + 3^x)$$

$$= \frac{1}{(D-2)^3} \times e^{2x} + \frac{1}{(D-2)^3} \times 3^x$$

$\Rightarrow$ $D \to 2$ $\qquad$ $D \to \log 3$

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Case of failure

$$= \frac{x}{3(D-2)^2} e^{2x} + \underbrace{\frac{1}{(\log 3 - 2)^3} 3^x}$$

No need to calculate log, write as it is!

$$y_p = \frac{x^2}{6(D-2)} e^{2x}$$

Case of failure

$$y_p = \frac{x^3}{6} e^{2x} + \frac{1}{(\log 3 - 2)^3} 3^x$$

$$y = y_c + y_p = (c_1 + c_2 x + c_3 x^2) e^{2x} + \frac{x^3}{6} e^{2x} + \frac{1}{(\log 3 - 2)^3} 3^x$$

Q $\boxed{(D^2-4)y = (1+e^x)^2 + 3}$

$$y_c = c_1 e^{2x} + c_2 e^{-2x}$$

$$y_p = \frac{1}{(D^2-4)} \left\{ (1+e^x)^2 + 3 \right\}$$

$$\cancel{= \frac{1}{(D^2-4)} (1+e^x)^2 + \frac{1}{(D^2-4)} \times 3}$$

$$= \frac{1}{(D^2-4)} \left\{ 1 + e^{2x} + 2e^x + 3 \right\}$$

$$= \frac{1}{(D^2-4)}\left(e^{2x}+2e^{x}+4\right)$$

$$= \frac{1}{(D^2-4)}e^{2x} + \frac{1}{(D^2-4)}2e^{x} + \frac{1}{(D^2-4)}4e^{0x}$$

$D \to 2$ $\quad$ $D \to 1$ $\quad$ $D \to 0$

Don't forget here it is $e^{0}x$

$$= \frac{1}{(2^2-4)}e^{2x} + \frac{1}{(1^2-4)}2e^{x} + \frac{1}{(0^2-4)}4e^{0x}$$

$\Rightarrow$ Case of failure

$$= \frac{x\,e^{2x}}{2D}$$

$\Rightarrow$ $D \to 2$

$$= \frac{x}{4}e^{2x} + \frac{1}{-3}2e^{x} + \frac{1}{-4}4e^{0x}$$

$$y_p = \frac{xe^{2x}}{4} - \frac{2e^{x}}{3} - 1e^{0x}$$

$$y = c_1e^{2x} + c_2e^{-2x} + \frac{x}{4}e^{2x} - \frac{2e^{x}}{3} - 1e^{0x}$$

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Q $\boxed{(D^2+6D+9)y = \sinh 3x}$

A.E $m^2+6m+9=0$

$m = -3, -3$

$y_c = (c_1 + c_2 x)e^{-3x}$

$$y_p = \frac{1}{D^2+6D+9} \sinh 3x$$

$$= \frac{1}{D^2+6D+9} \frac{e^{3x}-e^{-3x}}{2}$$

$$= \frac{1}{D^2+6D+9} \frac{e^{3x}}{2} - \frac{1}{D^2+6D+9} \frac{e^{-3x}}{2}$$

$D \to 3$ $\qquad$ $D \to -3$

$$= \frac{1}{36} \frac{e^{3x}}{2} - \frac{1}{9-18+9} \frac{e^{-3x}}{2}$$

Case of failure

$$y_p = \frac{1}{72} e^{3x} - \frac{x}{(2D+6)} \frac{e^{-3x}}{2}$$

Case of failure

$$y_p = \frac{1}{72} e^{3x} - \frac{x^2}{2} \frac{e^{-3x}}{2}$$

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$$y_p = \frac{1}{72} e^{3x} - \frac{x^2 e^{-3x}}{4}$$

$$y = (c_1 + c_2 x) e^{-3x} + \frac{1}{72} e^{3x} - \frac{x^2 e^{-3x}}{4}$$

Homework

Q1 $(D^2 + 4D + 4) y = e^{-2x}$

Q2 $(D^2 - 25) y = e^{5x}$

Q3 $(D^2 - D + 1) y = e^{x} + 5$

Q4 $(D^2 - 4D + 4) y = \cosh 2x$

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Q1) $(D^2 + 4D + 4) y = e^{-2x}$

$\rightarrow$ $m^2 + 4m + 4 = 0$

$m^2 + 2m + 2m + 4 = 0$

$(m+2)(m+2) = 0$

$m = -2, -2$

$$y_c = (c_1 + c_2 x) e^{-2x}$$

$$y_p = \frac{1}{D^2 + 4D + 4} e^{-2x}$$

$$= \frac{1}{(-2)^2 + 4(-2) + 4} e^{-2x} = \frac{1}{4 + (-8) + 4} e^{-2x} = \frac{1}{0} e^{-2x}$$

case of failure

$$= \frac{x}{2D + 4} e^{-2x} = \frac{x}{-4 + 4} e^{-2x}$$

case of failure

$$y_p = \frac{x^2}{2} e^{-2x}$$

$$y = (c_1 + c_2 x) e^{-2x} + \frac{x^2}{2} e^{-2x}$$

Q2) $(D^2 - 25) y = e^{5x}$

→ $m^2 - 25 = 0$

$m^2 = 25$

$m = \pm 5$ $\qquad y_c = c_1 e^{5x} + c_2 e^{-5x}$

$$y_p = \frac{1}{D^2 - 25} e^{5x} \qquad D \to a \to 5$$

$$= \frac{1}{25 - 25} e^{5x}$$

case of failure

$$= \frac{x}{2D} e^{5x} = \frac{x}{10} e^{5x}$$

$$y = c_1 e^{5x} + c_2 e^{-5x} + \frac{x}{10} e^{5x}$$

Q3) $(D^2 - D + 1) y = e^x + 5$

→ $m^2 - m + 1 = 0$

$$m = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2} = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

$$y_c = e^{1/2 x} \left(A_1 \cos \frac{\sqrt{3}}{2} x + A_2 \sin \frac{\sqrt{3}}{2} x \right)$$

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$$y_p = \frac{1}{D^2-D+1}(e^x+5) = \frac{1}{D^2-D+1}e^x + \frac{1}{D^2-D+1}(5)$$

$$= \frac{1}{1^2-1+1}(e^x) + \frac{1}{D^2-D+1}(5)e^{0x} \qquad \boxed{e^{0x}=1}$$

$$y_p = e^x + 5$$

$$y = e^x + 5 + e^{\frac{1}{2}x}\left(A_1\cos\frac{\sqrt{3}}{2}x + B_1\sin\frac{\sqrt{3}}{2}x\right)$$

Q4) $(D^2-4D+4)y = \cosh 2x$

→ $m^2-4m+4=0$

$m=2,2$

$y_c = (c_1+c_2x)e^{2x}$

$$y_p = \frac{1}{D^2-4D+4}\cosh 2x$$

$$= \frac{1}{D^2-4D+4}\frac{e^{2x}+e^{-2x}}{2} = \frac{1}{D^2-4D+4}\frac{e^{2x}}{2} + \frac{1}{D^2-4D+4}\frac{e^{-2x}}{2}$$

$$= \frac{1}{4-8+4}\frac{e^{2x}}{2} + \frac{1}{4+8+4}\frac{e^{-2x}}{2}$$

$$= \frac{1}{0}\frac{e^{2x}}{2} + \frac{1}{16}\frac{e^{-2x}}{2}$$

case of failure

$$= \frac{x}{2D-4}\frac{e^{2x}}{2} + \frac{1}{16}\frac{e^{-2x}}{2} = \frac{x^2}{2}\frac{e^{2x}}{2} + \frac{1}{16}\frac{e^{-2x}}{2}$$

case of failure

$$y_p = \frac{x^2e^{2x}}{4} + \frac{e^{-2x}}{32}$$

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$D(\sin ax) = a\cos ax$ ✗
$D^2(\sin ax) = -a^2 \sin ax$ ✓
$D^3(\sin ax) = +a^3 \cos ax$ ✗
$D^4(\sin ax) = -a^4 \sin ax$ ✓
⋮

we can conclude, $D^2 = -a^2$

$$(D^2)^n = (-a^2)^n$$

Type II

$$Q(x) = \sin(ax+b) \text{ or } \cos(ax+b)$$

Case I: $F(D)$ contains only even order terms in D. such as

$$D^2, D^4, D^6, \ldots$$

$$y_p = \frac{1}{F(D^2)} \sin(ax+b) \text{ or } \cos(ax+b) = \frac{1}{F(-a^2)} \sin(ax+b) \text{ or } \cos(ax+b)$$

$$F(-a^2) \neq 0$$

Q Find P.I of the diff. eqn.

$$(D^6 + 3D^4 + 2D^2 + 9)y = \sin 2x$$

$$y_p = \frac{1}{F(D)} Q(x) = \frac{1}{(D^6 + 3D^4 + 2D^2 + 9)} (\sin 2x)$$

$Q(x) = \sin 2x$, $F(D)$ contains only even order terms in D.

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$$y_p = \frac{1}{F(D^2)} \sin ax = \frac{1}{F(-a^2)} \sin ax \quad , F(-a^2) \neq 0$$

step 1

step2 $= \dfrac{1}{(-4)^3 + 3(-4)^2 + 2(-4) + 9} \sin 2x = \dfrac{\sin 2x}{-64+48-8+9}$

Replace, $(D^2)^n \rightarrow (-a^2)^n = (-4)^2$ $\quad = \dfrac{\sin 2x}{-15}$

$$= \frac{1}{(D^2)^3 + 3(D^2)^2 + 2D^2 + 9} \sin ax$$

$\rightarrow (-a^2)^3 + 3(-a^2)^2 + 2(-a^2) + 9$

$= (-2^2)^3 + 3(-2^2)^2 + 2(-2^2) + 9$

Q $(D^4 + 10D^2 + 9)y = \cos 4x$

$$y_p = \frac{1}{F(D)} Q(x) = \frac{1}{D^4 + 10D^2 + 9} (\cos 4x)$$

$Q(x) = \cos 4x$; $F(D)$ contains only even order terms in D.

$\cos(ax+b)$

$$y_p = \frac{1}{F(D^2)} \cos ax \qquad (D^2)^n \rightarrow (-a^2)^n \rightarrow (-4^2)^n \rightarrow (-16)^n$$

$$= \frac{1}{(D^2)^2 + 10D^2 + 9} \cos 4x = \frac{1}{(-16)^2 + 10(-16) + 9} \cos 4x$$

$$= \frac{1}{105} \cos 4x = \frac{\cos 4x}{109}$$

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Q Find General Solution

$$(D^2+9)y = \cos(2x-1) - \cos^2 x/2$$

$$y_p = \frac{1}{F(D)} Q(x)$$

$$C.F = m^2+9=0$$
$$m^2 = -9$$
$$m = \pm 3i$$

Roots are complex conjugate.

$$y_c = e^{0x}(A_1\cos 3x + A_2 \sin 3x)$$

$$y_p = \frac{1}{D^2+9}\left[\cos(2x-1) - \cos^2 x/2\right]$$

$$= \frac{1}{D^2+9}\cos(2x-1) - \frac{1}{D^2+9}\cos^2 x/2$$

$$y_{p_1} - y_{p_2}$$

$$y_{p_1} = \frac{1}{D^2+9}\cos(2x-1)$$

$$(D^2)^n \longrightarrow (-a^2)^n \longrightarrow (-2^2)^n \longrightarrow (-4)^n$$

$$= \frac{1}{-4+9}\cos(2x-1)$$

$$= \frac{\cos(2x-1)}{5}$$

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$$y_{p_2} = \frac{+1}{D^2+9} \cos^2 x/2$$

$$(D^2)^n \to (-a^2)^n \to \left(-\frac{1}{2}^2\right)^n \to \left(-\frac{1}{4}\right)^n \quad \cdots \left(\cos^2\theta = \frac{1+\cos 2\theta}{2}\right)$$

$$= \frac{1}{D^2+9}\left(\frac{1}{2}\left(1+\cos\frac{2x}{2}\right)\right)$$

$$= \frac{1}{2}\,\frac{1}{D^2+9}\left(1+\cos x\right)$$

$$= \frac{1}{2}\left[\underbrace{\frac{1}{D^2+9}(1)}_{D\to 0} + \underbrace{\frac{1}{D^2+9}(\cos x)}_{D^2\to -1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{9} + \frac{1}{8}\cos x\right]$$

$$y = y_c + y_p$$

$$y = A\cos 3x + B\sin 3x + \frac{\cos(2x-1)}{5} - \frac{1}{2}\left[\frac{1}{9} + \frac{1}{8}\cos x\right]$$

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Case II $F(D)$ contains even as well as odd order terms of D.

even order terms

1) Replace $(D^2)^n \longrightarrow (-a^2)^n$ & odd order terms of type D^{2n+1}

$\hookrightarrow D^{2n+1} = D^{2n} D = (-a^2)^n . D$

2) Substitute these values, $\frac{1}{F(D)} \sin(ax+b)$ or $\cos(ax+b)$
Adding all terms

reduces to $= \frac{1}{LD+m} \sin(ax+b)$ or $\cos(ax+b)$

3) Multiply & divide by conjugate

$$\frac{LD-m}{(LD+m)(LD-m)} \sin(ax+b) \text{ or } \cos(ax+b)$$

$$\frac{LD-m}{\underbrace{L^2D^2-m^2}} \sin(ax+b) \text{ or } \cos(ax+b)$$

$\longrightarrow ((LD)^2-m^2)$ ... $(a^2-m^2) = (a+m)(a-m)$

operator $Q(x)$ with $(LD-m)$ & substitute $D^2 = -a^2$ in denominator.

Q. $(D^3 - 3D^2 + 3D - 1)y = \sin 2x$

$$y_p = \frac{1}{D^3-3D^2+3D-1} \sin 2x$$

$D^2 = -4$, $D^3 = D^2 . D$
$= -4D$

$$= \frac{1}{-4D+12+3D-1}$$

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$$y_p = \frac{1}{-D+11} \sin 2x = -\frac{1}{D-11} \sin 2x$$

$$= (-1) \frac{1}{(D-11)} \frac{(D+11)}{(D+11)} \sin 2x$$

Here we have to take derivative

$$= (-1) \frac{(D+11)}{D^2 - 11^2} \sin 2x = (-1) \frac{(D+11)}{D^2 - 121} \sin 2x$$

$$y_p = \frac{-1}{D^2 - 121} (2\cos 2x + 11 \sin 2x)$$

$$D^2 = -4$$

$$= \frac{-1}{-4-121} (2\cos 2x + 11 \sin 2x)$$

$$= \frac{1}{125} (2\cos 2x + 11 \sin 2x)$$

Q Solve $(D^2 - 2D + 5)y = \cos x$

$$y_p = \frac{1}{D^2 - 2D + 5} \cos x$$

$$D^2 \rightarrow -a^2 \rightarrow -1^2 \rightarrow -1$$

Step 1 General Solution = C.F + P.I

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C.F

Auxi eq^n = $m^2 - 2m + 5 = 0$

$m = 1 \pm 2i$

$y_c = e^{1x}(A\cos 2x + B\sin 2x)$

Step II

$y_p = \dfrac{1}{D^2 - 2D + 5} \cos x$

$D^2 = -1$

$y_p = \dfrac{1}{-1 - 2D + 5} \cos x = \dfrac{1}{4 - 2D} \cos x$

$y_p = \dfrac{1}{(4-2D)} \dfrac{(4+2D)}{(4+2D)} \cos x = \dfrac{4+2D}{16 - 4D^2} \cos x$

$y_p = \dfrac{4+2D}{16-4D^2} \cos x$ ⇒ Here we have to take derivative

$y_p = \dfrac{1}{16+4}(4\cos x - 2\sin x)$

$= \dfrac{1}{20}(4\cos x - 2\sin x) = \dfrac{2\cos x - \sin x}{10}$

$y = y_c + y_p$

$y = e^x(A\cos 2x + B\sin 2x) + \dfrac{2\cos x - \sin x}{10}$

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Q $(D^3+D)y = \cos 2x$

Step 1 $G.S = y_c + y_p$

$C.F = m^3 + m = 0$

$\Rightarrow m(m^2+1) = 0$

$m = 0, \; m^2 = 1$

$m = 0, \; m = \pm i$

$C.F = y_c = e^{0x}(A\cos x + B\sin x) + c_1 e^{0x}$

$y_p = \dfrac{1}{D^3+D} \cos 2x$

$D^2 = -4$

$y_p = \dfrac{1}{(D^2)(D)+D} \cos 2x$

$= \dfrac{1}{-4D+D} \cos 2x = \dfrac{-1}{3D} \cos 2x$

$y_p = \dfrac{1}{(-3D)} \dfrac{(3D)}{(3D)} \cos 2x$

$= \dfrac{3D}{-9D^2} \cos 2x = \dfrac{3D}{36} \cos 2x$

$= -\dfrac{6\sin 2x}{36} = -\dfrac{\sin 2x}{6} = y_p$

$y = y_c + y_p$

$y = c_1 e^{0x} + e^{0x}(A\cos x + B\sin x) - \dfrac{\sin 2x}{6}$

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Case III

$F(D) = 0$

$f(-a^2) = 0$; Denominator becomes 0

$$y_p = \frac{1}{(D^2+a^2)^r} \sin(ax+b)$$

$$= \left(\frac{-1}{2a}\right)^r \left(\frac{x^r}{r!}\right) \sin\left(ax+b+r\frac{\pi}{2}\right)$$

Q $(D^2+9)^2 y = \sin 2x$

$$y_p = \frac{1}{(D^2+9)^{2\to r}} \sin \overset{\to a}{3x}$$

$(D^2)^n \to (-a^2)^n \to (-3^2)^n \to (-9)^n$

$D^2 \to -9$, $y_p = \frac{1}{(-9+9)^2} \sin 3x$ Method fail

$a = 3, r = 2$

$$y_p = \left(\frac{-1}{2a}\right)^r \cdot \frac{x^r}{r!} \sin\left(ax+b+r\frac{\pi}{2}\right)$$

$$= \left(-\frac{1}{6}\right)^2 \cdot \frac{x^2}{2!} \sin\left(3x+0+\frac{2\pi}{2}\right)$$

$$= \left(-\frac{1}{6}\right)^2 \cdot \frac{x^2}{2!} \sin\left(3x+\frac{2\pi}{2}\right)$$

$$= \frac{x^2}{72} \sin(3x+\pi) = \frac{-x^2}{72} \sin 3x$$

$\ldots (\sin(\pi + 3x) =$

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Q $(D^3+D)y = \sin x$

$$y_p = \frac{1}{D^3+D}\sin x$$

$\Rightarrow D^2 = -1$

$$y_p = \frac{1}{-D+D}\sin x = \frac{1}{0}\sin x$$

$$y_p = -\frac{1}{0}\sin x$$ Case of failure

$$y_p = \frac{1}{D^3+D}\sin x$$

multiply by x take derivative

$$y_p = \frac{x}{3D^2+1}\sin x = \frac{x}{-3+1}\sin x = \frac{-x}{2}\sin x$$

OR

$$y_p = \frac{1}{D(D^2+1)}\sin x = \frac{1}{D^2+1}\,\frac{1}{D}\sin x$$

$$= \frac{1}{D^2+1}(-\cos x) = (-1)\frac{1}{(D^2+1)}\cos x$$

$a=1, r=1$

$$= (-1)\left(\frac{-1}{2}\right)^1 \frac{x^1}{1!}\cos(x+\pi/2)$$

$$= \frac{-x}{2}\sin x$$

$\sin 3x + \cos\pi + \cos 3x \sin\pi)$

$-\sin 3x + \cos 3x \times (0) = -\sin 3x$

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Type III

$F(x) = x^m$, m is +ve integer

$$\frac{1}{\phi(D)} x^m = [\phi(D)]^{-1} x^m$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \cdots$$
$$(x-1)^{-1} = 1 + x^2 + x^3 + x^4 + \cdots$$

Q $(D^2+1)y = x^4$

$y_c \Rightarrow$ Auxi eq^n

$m^2 + 1 = 0$
$m^2 = -1$
$m = 0 \pm i$

$$y_c = e^{0x}(A\cos x + B\sin x)$$

$$y_p = \frac{1}{D^2+1} \cdot x^4 = [1+D^2]^{-1} \cdot x^4$$

$$= [1 - D^2 + (D^2)^2 - (D^2)^3 + \cdots] x^4$$

$$= [1 - D^2 + D^4] x^4$$

$$= x^4 - D^2(x^4) + D^4(x^4)$$

$$y_p = x^4 - 12x^2 + 24$$
$$y = y_c + y_p$$

$$y = e^{0x}(A\cos x + B\sin x) + x^4 - 12x^2 + 24$$

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Q $(D^3+8)y = x^4+2x+1$

$m^3+8=0$

$m^3=-8$

$m=-2,\ 1\pm\sqrt{3}i$

$y_c = c_1e^{-2x} + e^{1x}(A\cos\sqrt{3}x + B\sin\sqrt{3}x)$

$y_p = \dfrac{1}{D^3+8}(x^4+2x+1)$

$= [D^3+8]^{-1}(x^4+2x+1)$

$= [8+D^3]^{-1}(x^4+2x+1)$

$= \dfrac{1}{8}\left[1+\dfrac{D^3}{8}\right]^{-1}(x^4+2x+1)$

$= \dfrac{1}{8}\left[1-\dfrac{D^3}{8}+\left(\dfrac{D^3}{8}\right)^2-\dots\right](x^4+2x+1)$

$y_p = \dfrac{1}{8}\left[1-\dfrac{D^3}{8}\right](x^4+2x+1)$

$= \dfrac{1}{8}\left[x^4+2x+1-\dfrac{1}{8}D^3(x^4+2x+1)\right]$

$= \dfrac{1}{8}\left[x^4+2x+1-\dfrac{1}{8}D^2(4x^3+2)\right]$

$= \dfrac{1}{8}\left[x^4+2x+1-\dfrac{1}{8}D(12x^2)\right]$

$D=4x^3+2+0$

$D^2=12x^2+0$

$D^3=24x$

$= \dfrac{1}{8}\left[x^4+2x+1-\dfrac{1}{8}(24x)\right]$

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$$y_p = \frac{1}{8}\left[x^4 + 2x + 1 - 3x\right]$$

$$= \frac{1}{8}\left(x^4 - x + 1\right)$$

$$y = y_c + y_p$$

$$y = c_1 e^{-2x} + e^{1x}\left(A\cos\sqrt{3}\,x + B\sin\sqrt{3}\,x\right) + \frac{1}{8}\left(x^4 - x + 1\right)$$

Q $(D^4 + D^2 + 1)y = 53x^2 + 17$

$$y_c = m^4 + m^2 + 1 = 0$$

$$(m^2)^2 + 2m^2 + 1 - m^2 = 0$$

$$(m^2+1)^2 - (m)^2 = 0 \quad \ldots (a^2 - b^2) = (a+b)(a-b)$$

$$(m^2 + 1 + m)(m^2 + 1 - m) = 0$$

$$(m^2 + m + 1)\ (m^2 - m + 1) = 0$$

$$m = \frac{-1 \pm \sqrt{1^2 - 4}}{2} \qquad m = \frac{1 \pm \sqrt{3}\,i}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2} \qquad = \frac{1}{2} \pm \frac{\sqrt{3}\,i}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$= \frac{-1}{2} \pm \frac{\sqrt{3}}{2}i$$

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$$y_c = e^{-\frac{1}{2}x}\left(c_1\cos\frac{\sqrt{3}}{2}x + c_2\sin\frac{\sqrt{3}}{2}x\right) + e^{1/2x}\left(c_3\cos\frac{\sqrt{3}}{2}x + c_4\sin\frac{\sqrt{3}}{2}x\right)$$

$$y_p = \frac{1}{D^4+D^2+1}(53x^2+17)$$

$$y_p = [1+(D^4+D^2)]^{-1}(53x^2+17)$$

$$= \left[1-(D^4+D^2)+(D^4+D^2)^2 - \cdots\right](53x^2+17)$$

$$= [1-D^4-D^2](53x^2+17)$$

This will become zero

$D^4(-53x^2-17)$

$D^3(-53\times 2x)$

$D^2(-106)$

$D(0) = 0$

$$= [1-D^2](53x^2+17)$$

$$= (53x^2+17) - D^2(53x^2+17)$$

$$= 53x^2+17-106$$

$$y_p = 53x^2-89$$

$$y = y_c + y_p$$

$$y = e^{-1/2x}\left[c_1\cos\frac{\sqrt{3}}{2}x + c_2\sin\frac{\sqrt{3}}{2}x\right] + e^{1/2x}\left[c_3\cos\frac{\sqrt{3}}{2}x + c_4\sin\frac{\sqrt{3}}{2}x\right] + 53x^2 - 89$$

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Q $(D^2-D+1)y = 3x^2-1$

$m^2-m+1=0$

$m = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$y_c = e^{1/2 x}\left[c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right]$

$y_p = \frac{1}{1+(D^2-D)}(3x^2-1)$

$= [1+(D^2-D)]^{-1}(3x^2-1)$

$= [1-(D^2-D)+(D^2-D)^2-(D^2-D)^3+\dots](3x^2-1)$

$y_p = [1-(D^2-D)+D^2](3x^2-1)$

$= [1-\cancel{D^2}+D+\cancel{D^2}](3x^2-1)$

$= [1+D](3x^2-1)$

$= 3x^2-1+D(3x^2-1)$

$y_p = 3x^2-1+6x$

Q $(D^4+25)y = x^4+x^2+1$

$m^4+25=0$

$y_c = (m^2)^2+10m^2+25-10m^2$

$$(m^2+5)^2 = (\sqrt{10}\,m)^2$$

$$y_c = (m^2+5+\sqrt{10}\,m)(m^2+5-\sqrt{10}\,m)$$

$$= (m^2+\sqrt{10}\,m+5)(m^2-\sqrt{10}\,m+5)$$

$$= \frac{-\sqrt{10} \pm \sqrt{(\sqrt{10})^2 - 4\times1\times5}}{2(1)}$$

$$= \frac{-\sqrt{10} \pm \sqrt{10-20}}{2}$$

$$= \frac{-\sqrt{10} \pm \sqrt{-10}}{2}$$

$$= \frac{-\sqrt{10} \pm i\sqrt{10}}{2} \quad \text{Similarly} = \frac{\sqrt{10} \pm i\sqrt{10}}{2}$$

$$m = -\frac{\sqrt{10}}{2} \pm \frac{i\sqrt{10}}{2}, \frac{\sqrt{10}}{2} \pm i\frac{\sqrt{10}}{2}$$

$$y_c = e^{-\frac{\sqrt{10}x}{2}}\left(A_1\cos\frac{\sqrt{10}}{2}x + B_1\sin\frac{\sqrt{10}}{2}x\right) + e^{+\frac{\sqrt{10}x}{2}}\left(A_2\cos\frac{\sqrt{10}}{2}x + B_2\sin\frac{\sqrt{10}}{2}x\right)$$

$$y_p = \frac{1}{D^4+25}(x^4+x^2+1)$$

$$= \frac{1}{25}\left(1+\frac{D^4}{25}\right)^{-1}(x^4+x^2+1)$$

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$$= \frac{1}{25}\left(1 - \frac{D^4}{25}\right)(x^4 + x^2 + 1)$$

$$= \frac{1}{25}\left(x^4 + x^2 + 1 - \frac{1 \times 24}{25}\right)$$

$$= \frac{1}{25}\left(x^4 + x^2 + \frac{25-24}{25}\right)$$

$$= \frac{1}{25}\left(x^4 + x^2 + \frac{1}{25}\right)$$

$$y = y_c + y_p$$

$$y = e^{-\frac{\sqrt{10}x}{2}}\left(A_1 \cos\frac{\sqrt{10}x}{2} + B_1 \sin\frac{\sqrt{10}x}{2}\right) + e^{\frac{\sqrt{10}x}{2}}\left(A_2 \cos\frac{\sqrt{10}x}{2} + B_2 \sin\frac{\sqrt{10}x}{2}\right)$$

$$+ \frac{1}{25}\left(x^4 + x^2 + \frac{1}{25}\right)$$

Q $(D^2 - 2D + 5)y = 25x^2$

$$y_c = m^2 - 2m + 5 = 0 \longrightarrow \frac{2 \pm \sqrt{4 - 4 \times 1 \times 5}}{2(1)}$$

$$m = 1 \pm 2i \qquad \frac{2 \pm i4}{2} = 1 \pm 2i$$

$$y_c = e^{1x}(A \cos 2x + B \sin 2x)$$

$$y_p = \frac{1}{D^2 - 2D + 5} 25x^2$$

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$$= \frac{1}{5}\left[1+\left(\frac{D^2-2D}{5}\right)\right]^{-1}(25x^2)$$

$$= \frac{1}{5}\left[1-\left(\frac{D^2-2D}{5}\right)+\left(\frac{D^2-2D}{5}\right)^2-\cdots\right](25x^2)$$

$$= \frac{1}{5}\left[1-\frac{D^2}{5}+\frac{2D}{5}+\left(\frac{D^4+4D^2+4D^3}{25}\right)\right](25x^2)$$

$$= \frac{1}{5}\left[1-\frac{D^2}{5}+\frac{2D}{5}+\cancel{\frac{D^4}{25}}^{\,0}+\frac{4D^2}{25}+\cancel{\frac{4D^3}{25}}^{\,0}\right](25x^2)$$

$$= \frac{1}{5}\left[1-\frac{D^2}{5}+\frac{2D}{5}+\frac{4D^2}{25}\right](25x^2)$$

$$= \frac{1}{5}\left[1+\frac{2D}{5}\left(-\frac{D^2}{5}+\frac{4D^2}{25}\right)\right](25x^2)$$

$$= \frac{1}{5}\left[1+\frac{2D}{5}-\frac{5D^2}{125}\right](25x^2)$$

$$= \frac{1}{5}\left[25x^2+\frac{2D}{5}(25x^2)-\frac{5D^2}{125}(25x^2)\right]$$

$$= \frac{1}{5}\left[25x^2+\frac{2}{5}(25\times 2x)-\frac{5}{125}(25\times 2)\right]$$

$$= \frac{1}{5}\left[25x^2+20x-2\right]$$

$$y_p = 5x^2+4x-\frac{2}{5}$$

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$$y = y_c + y_p$$

$$y = e^{3x}(A\cos 2x + B\sin 2x) + 5x^2 + 4x - \frac{2}{5}$$

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Type IV : $Q(x) = e^{ax} V(x)$

$$y_p = \frac{1}{F(D)} \{ e^{ax} V(x) \}$$

$$= e^{ax} \left\{ \frac{1}{F(D+a)} V(x) \right\}$$

Q Solve
find y_p of differential eq^n $(D^2-4)y = e^{3x} x^2$

$$y_p = \frac{1}{D^2-4} \left[e^{3x} x^2 \right]$$

$$= e^{3x} \left[\frac{1}{(D+3)^2-4} x^2 \right]$$

$$= e^{3x} \left[\frac{1}{D^2+6D+5} x^2 \right]$$

$$= \frac{e^{3x}}{5} \left[\frac{1}{1 + \frac{6D+D^2}{5}} x^2 \right]$$

$$= \frac{e^{3x}}{5} \left\{ \left[\left(1 + \frac{6D+D^2}{5} \right) \right]^{-1} x^2 \right\}$$

$$= \frac{e^{3x}}{5} \left\{ 1 - \left(\frac{6D+D^2}{5} \right) + \left(\frac{6D+D^2}{5} \right)^2 - \dots \right\} x^2$$

$$= \frac{e^{3x}}{5} \left[1 - \left(\frac{6D+D^2}{5} \right) + \left(\frac{36D^2}{25} \right) \right] x^2$$

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$$= \frac{e^{3x}}{5}\left[1 - \frac{6D}{5} - \frac{D^2}{5} + \frac{36D^2}{25}\right]x^2$$

$$= \frac{e^{3x}}{5}\left[1 - \frac{6D}{5} + \underbrace{\left(\frac{36D^2}{25} - \frac{D^2}{5}\right)}_{\frac{31}{25}D^2}\right]x^2$$

$$= \frac{e^{3x}}{5}\left[1 - \frac{6D}{5} + \frac{31D^2}{25}\right]x^2$$

$$= \frac{e^{3x}}{5}\left[x^2 - \frac{6D(x^2)}{5} + \frac{31}{25}D^2(x^2)\right]$$

$$= \frac{e^{3x}}{5}\left[x^2 - \frac{6(2x)}{5} + \frac{31}{25}(2)\right]$$

$$= \frac{e^{3x}}{5}\left[x^2 - \frac{12x}{5} + \frac{62}{25}\right]$$

Q Find y_p, $(D^2-4)y = e^{3x}\sin 2x$

$$y_p = \frac{1}{D^2-4}\left\{e^{3x}\sin 2x\right\}$$

$$= e^{3x}\left\{\frac{1}{(D+3)^2-4}\sin 2x\right\}$$

$$= e^{3x}\left\{\frac{1}{D^2+6D+5}\sin 2x\right\}$$

$$D^2 = -4$$

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$$= e^{3x}\left\{\frac{1}{-4+6D+5}\sin 2x\right\} = e^{3x}\left\{\frac{1}{6D+1}\sin 2x\right\}$$

$$= e^{3x}\left\{\left[\frac{1\times(6D-1)}{(6D+1)(6D-1)}\sin 2x\right]\right\}$$

$$= e^{3x}\left\{\frac{6D-1}{36D^2-1}\sin 2x\right\}$$

$$= \frac{e^{3x}}{36D^2-1}\left[12\cos 2x - \sin 2x\right]$$

$D^2 = -4$

$$= \frac{e^{3x}}{36(-4)-1}\left[12\cos 2x - \sin 2x\right]$$

$$= \frac{e^{3x}}{-144-1}\left[12\cos 2x - \sin 2x\right]$$

$$= \frac{e^{3x}}{-145}\left[12\cos 2x - \sin 2x\right]$$

Q Find y_p of the differential eq^n

$$(D^2+2D+1)y = e^{-x}\log x$$

$$y_p = \frac{1}{D^2+2D+1}e^{-x}\log x$$

This is of the form $e^{ax}(V(x))$

$$\frac{1}{F(D)}e^{ax}V(x) = e^{ax}\left\{\frac{1}{F(D+a)}V(x)\right\}$$

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$$y_p = e^{-x}\left\{\frac{1}{(D-1)^2+2(D-1)+1}\log x\right\}$$

$$= e^{-x}\left\{\frac{1}{D^2-2D+1+2D-2+1}\log x\right\}$$

$$= e^{-x}\left\{\frac{1}{D^2}\log x\right\}$$

$$= e^{-x}\iint \log x\,dx$$

$$= e^{-x}\int (x\log x - x)\,dx$$

$$= e^{-x}\left[\left\{(\log x)\frac{x^2}{2} - \int \frac{1}{x}\frac{x^2}{2}dx\right\} - \frac{x^2}{2}\right]$$

$$\left\{\int uv\,dx = u\int v\,dx - \int \frac{du}{dx}\int v\,dx\right\}$$

$$= e^{-x}\left[\frac{x^2}{2}\log x - \frac{x^2}{4} - \frac{x^2}{2}\right]$$

$$= e^{-x}\left[\frac{x^2}{2}\log x - \frac{3x^2}{4}\right]$$

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Q Solve the D.E $(D^2+2D+1)y = \frac{e^{-x}}{x+2}$

$y = y_c + y_p$

$y_c \Rightarrow m^2+2m+1=0$

$m^2+m+m+1=0$

$m(m+1)+1(m+1)=0$

$(m+1)(m+1)=0$

$m=-1, \quad m=-1$

$y_c = (c_1+c_2x)e^{-x}$

$y_p = \frac{1}{D^2+2D+1} \frac{e^{-x}}{x+2}$

$= e^{-x} \frac{1}{(D-1)^2+2(D-1)+1} \frac{1}{x+2}$

$= e^{-x} \frac{1}{D^2} \frac{1}{x+2}$

$y_p = e^{-x} \iint \frac{1}{x+2} dx$

$= e^{-x} \int \log(x+2) dx$

$= e^{-x}\left[\log(x+2)x - \int \frac{1}{x+2} x\, dx\right]$

$= e^{-x}(x\log(x+2) - \int \frac{x+2-2}{x+2} dx$

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$$= e^{-x}\left[\left(x\log(x+2)\right) - \int\left(1 - \frac{2}{x+2}\right)dx\right]$$

$$= e^{-x}\left[x\log(x+2) - x + 2\log(x+2)\right]$$

Q Solve the D.E $(D^2 - 7D + 6)y = e^{-2x}\left[(1+x^2) + \cos 2x\right]$

$y_c \ni m^2 - 7m + 6 = 0$

$m^2 - 6m - m + 6 = 0$

$m(m-6) - 1(m-6) = 0$

$(m-6)(m-1) = 0$

$m = 6, \quad m = 1$

$y_c = c_1 e^{6x} + c_2 e^{x}$

$$y_p = \frac{1}{D^2 - 7D + 6} e^{-2x}\left[(1+x^2) + \cos 2x\right]$$

$$= e^{-2x}\left\{\frac{1}{D^2 - 7D + 6}\left[(1+x^2) + \cos 2x\right]\right\}$$

$$= e^{-2x}\left\{\frac{1}{(D-2)^2 - 7(D-2) + 6}\left[(1+x^2) + \cos 2x\right]\right\}$$

$$= e^{-2x}\left\{\frac{1}{D^2 + 4 - 4D - 7D + 14 + 6}\left[(1+x^2) + \cos 2x\right]\right\}$$

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$$= e^{-2x}\left\{\frac{1}{D^2-11D+24}\left[(1+x^2)+\cos 2x\right]\right\}$$

$$= \frac{e^{-2x}}{24}\,\frac{1}{\frac{D^2-11D}{24}+1}\left[\underset{①}{(1+x^2)}+\underset{②}{\cos 2x}\right]$$

$$y_{p_1} = \frac{e^{-2x}}{24}\left[\frac{D^2-11D}{24}+1\right]^{-1}(1+x^2)$$

$$= \frac{e^{-2x}}{24}\left[1-\left(\frac{11D-D^2}{24}\right)\right]^{-1}(1+x^2)$$

$$= \frac{e^{-2x}}{24}\left[1+\left(\frac{11D-D^2}{24}\right)+\left(\frac{11D-D^2}{24}\right)^2+\cdots\right](1+x^2)$$

$$= \frac{e^{-2x}}{24}\left[1+\left(\frac{11D-D^2}{24}\right)+\frac{121}{576}D^2\right](1+x^2)$$

$$= \frac{e^{-2x}}{24}\left[1+\frac{11D}{24}-\frac{D^2}{24}+\frac{121}{576}D^2\right](1+x^2)$$

$$= \frac{e^{-2x}}{24}\left[1+\frac{11D}{24}+\frac{97}{576}D^2\right](1+x^2)$$

$$= \frac{e^{-2x}}{24}\left[1+x^2+\frac{11D}{24}(1+x^2)+\frac{97}{576}D^2(1+x^2)\right]$$

$$= \frac{e^{-2x}}{24}\left[1+x^2+\frac{11(2)x}{24}+\frac{97(2)}{576}\right]$$

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$$= \frac{e^{-2x}}{24}\left[x^2 + \frac{11x}{12} + \frac{97}{288} + 1\right]$$

$$y_{p_1} = \frac{e^{-2x}}{24}\left[x^2 + \frac{11x}{12} + \frac{385}{288}\right]$$

$$y_{p_2} = e^{-2x}\frac{1}{D^2 - 11D + 24}\cos 2x$$

$$D^2 = -a^2 = -4$$

$$= e^{-2x}\frac{1}{-4 - 11D + 24}\cos 2x$$

$$= e^{-2x}\frac{1}{20 - 11D}\cos 2x$$

$$= -e^{-2x}\frac{1}{(11D - 20)}\cos 2x$$

$$= -e^{-2x}\frac{1 \times (11D + 20)}{(11D - 20)(11D + 20)}\cos 2x$$

$$= -e^{-2x}\frac{(11D + 20)}{121D^2 - 400}\cos 2x \qquad D^2 = -4$$

$$= -e^{-2x}\frac{(11D + 20)}{121(-4) - 400}\cos 2x$$

$$= -e^{-2x}\frac{(11D + 20)}{-884}\cos 2x$$

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$$= \frac{e^{-2x}}{884} (11D + 20) \cos 2x$$

$$= \frac{e^{-2x}}{884} \left(11D(\cos 2x) + 20(\cos 2x) \right)$$

$$y_{p_2} = \frac{e^{-2x}}{884} \left(-22 \sin 2x + 20 \cos 2x \right)$$

Type V: if $Q(x) = x\,V$

where $V = \sin ax / \cos ax / e^{ax}$

$$y_p = \frac{1}{F(D)} (x\,V(x))$$

$$y_p = \left\{ x - \frac{1}{F(D)} F'(D) \right\} \left\{ \frac{1}{F(D)} V(x) \right\}$$

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Q Find y_p of diff eq^n, $(D^2+1)y = x \cos 2x$

$$y_p = \left\{ x - \frac{1}{F(D)} F'(D) \right\} \left\{ \frac{1}{F(D)} V(x) \right\}$$

$$= \left\{ x - \frac{1}{D^2+1} (2D) \right\} \left\{ \frac{1}{D^2+1} \cos 2x \right\}$$

$D^2 = -4$

$$= \left\{ x - \frac{1}{D^2+1} (2D) \right\} \left\{ \frac{1}{-4+1} \cos 2x \right\}$$

$$= \left\{ x - \frac{1}{D^2+1}(2D) \right\} \left\{ \frac{\cos 2x}{-3} \right\}$$

$$= \frac{x\cos 2x}{-3} + \left(\frac{2}{3} \frac{1}{D^2+1} \right) \left(D(\cos 2x) \right)$$

$$= -\frac{1}{3} x\cos 2x + \frac{2}{3} \frac{1}{D^2+1} (-2\sin 2x)$$

$D^2 = -4$

$$= \frac{-1}{3} x\cos 2x + \frac{4}{9} \sin 2x$$

Method of variation of parameters

P.I
- general method
- shortcut method
- method of variation of parameter

Step I Find C.F

Step II Identify y_1 and y_2

Step III Find W, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$

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Step IV Assume $y_p = A(x)y_1 + B(x)y_2$

Step V where, $A(x) = \int -\frac{Q}{W} y_2 \, dx$

$$B(x) = \int \frac{Q y_1}{W} \, dx$$

Step VI Find G.S

Q $(D^2+1)y = \tan x$

S1: $m^2+1=0$
$m^2=-1$
$m=\pm i$

$y_c = c_1\cos x + c_2 \sin x$

S2: $y_1 = \cos x$ $\quad y_2 = \sin x$

S3: $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1 \neq 0$

S4: $y_p = A(x)y_1 + B(x)y_2$, Assume

S5: $A(x) = \int -\frac{Q}{W} y_2 \, dx$ | $B(x) = \int \frac{Q}{W} y_1 \, dx$

$= -\int \frac{\tan x \sin x}{1} dx$ | $= \int \frac{\tan x \cos x}{1} dx$

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$$= -\int \frac{\sin^2 x}{\cos x}\, dx$$

$$= -\int \frac{(1-\cos^2 x)}{\cos x}\, dx$$

$$= -\int \left(\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}\right) dx$$

$$= -\int (\sec x - \cos x)\, dx$$

$$= \int -\sec x + \cos x\, dx$$

$$= -\log(\sec x + \tan x) + \sin x$$

$$A(x) = \sin x - \log(\sec x + \tan x)$$

$$= \int \frac{\sin x}{\cancel{\cos x}} \cancel{\cos x}\, dx$$

$$= \int \sin x\, dx$$

$$B(x) = -\cos x$$

$$y_p = A(x) y_1 + B(x) y_2$$

$$y_p = [\sin x - \log(\sec x + \tan x)] \cos x + (-\cos x)(\sin x)$$

$$= [\sin x - \log(\sec x + \tan x)] \cos x - \cos x . \sin x$$

$$= \cancel{\sin x \cos x} - \log(\sec x + \tan x) \cos x - \cancel{\cos x \sin x}$$

$$y_p = -\log(\sec x + \tan x) \cos x$$

$$y = G.S = c_1 \cos x + c_2 \sin x - \log(\sec x + \tan x) \cos x$$

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Q $(D^2+4)y = \sec 2x$

$\rightarrow$ $CF = m^2+4$

$m = \pm 2i$

$y_c = c_1\cos 2x + c_2 \sin 2x$

$y_1 = \cos 2x$ and $y_2 = \sin 2x$

$y_p = A(x)y_1 + B(x)y_2$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$= 2(\cos^2 2x + \sin^2 2x) = 2 \neq 0$

$$A(x) = \int \frac{-Q y_2}{W}\, dx$$

$$= \int \frac{-\sin 2x \sec 2x}{2}$$

$$= -\frac{1}{2}\int \tan 2x\, dx$$

$$= \frac{1}{4}\log(\cos 2x)$$

$$B(x) = \int \frac{Q y_1}{W}\, dx$$

$$= \int \frac{\cos 2x \sec 2x\, dx}{2}$$

$$= \frac{1}{2}\int dx = \frac{1}{2}x$$

$y_p = A(x)y_1 + B(x)y_2$

$$= \left[\frac{1}{4}\log(\cos 2x)\right]\cos 2x + \left[\frac{1}{2}x\right]\sin 2x$$

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General solution is

$$y = y_c + y_p$$

$$y = c_1\cos 2x + c_2\sin 2x + \frac{1}{4}\cos 2x \log(\cos 2x) + \frac{1}{2}x\sin$$

Q $(D^2+4)y = \cot 2x$

$\rightarrow$ $m^2+4=0$

$m^2=-4$

$m = \pm 2i$

$$y_c = c_1\cos 2x + c_2\sin 2x$$

$y_1 = \cos 2x$ $\quad$ $y_2 = \sin 2x$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2 \neq 0$$

$$y_p = A(x)y_1 + B(x)y_2$$

$$A(x) = -\int \frac{Qy_2}{W}\,dx$$

$$= -\int \frac{\cot 2x \overset{\sin}{\cancel{\cos}} 2x}{2}\,dx$$

$$= -\frac{1}{2}\int \cos 2x\,dx$$

$$= -\frac{1}{2}\left(\frac{\sin 2x}{2}\right)$$

$$B(x) = \int \frac{Qy_1}{W}\,dx$$

$$= \int \frac{\cot 2x \cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int \frac{\cos 2x}{\sin 2x}\cdot\cos 2x\,dx$$

$$= \frac{1}{2}\int \frac{(1-\sin^2 2x)}{\sin 2x}$$

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$$= -\frac{\sin 2x}{4}$$

$$A(x) = -\frac{\sin 2x}{4}$$

$$= \frac{1}{2}\left(\int \operatorname{cosec} 2x\,dx - \int \sin 2x\,dx\right)$$

$$B(x) = \frac{1}{2}\left(\frac{\log|\operatorname{cosec} 2x - \cot 2x|}{2} + \frac{\cos 2x}{2}\right)$$

$$y = y_c + y_p$$

$$y_p = \left[-\frac{\sin 2x}{4}\right]\cos 2x + \left[\frac{\log(\operatorname{cosec} 2x - \cot 2x)}{4} + \frac{\cos 2x}{4}\right]\sin 2x$$

General solution is,

$$y = y_c + y_p$$

$$y = c_1\cos 2x + c_2\sin 2x - \frac{\sin 2x\cos 2x}{4} + \left[\frac{\log(\operatorname{cosec} 2x - \cot 2x)}{4} + \frac{\cos 2x}{4}\right]\sin 2x$$

$$= c_1\cos 2x + c_2\sin 2x - \cancel{\frac{\sin 2x\cos 2x}{4}} + \cancel{\frac{\sin 2x\cos 2x}{4}} + \frac{\log(\operatorname{cosec} 2x - \cot 2x)}{4}\sin 2x$$

$$y = c_1\cos 2x + c_2\sin 2x + \frac{\log|\operatorname{cosec} 2x - \cot 2x|}{4}\sin 2x$$

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Q $(D^2+1)y = \sec x \tan x$

→ $m^2+1=0$

$m = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$y_1 = \cos x \qquad y_2 = \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1 \neq 0$$

$y_p = A(x)y_1 + B(x)y_2$

$$A(x) = \int -\frac{Qy_2}{W}\,dx = -\int \frac{\sec x \tan x \cdot \sin x}{1}\,dx$$

$$= -\int \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\cos x}\,dx = -\int \tan^2 x\,dx$$

$$= -\int (\sec^2 x - 1)\,dx$$

$$= -\tan x + x$$

$$B(x) = \int \frac{Qy_1}{W}\,dx = \int \frac{\sec x \tan x \cdot \cos x}{1}\,dx$$

$$= \int \tan x\,dx = \log|\sec x|$$

$$y_p = (-\tan x + x)\cos x + \sin x \log|\sec x|$$

$$= -\sin x + x\cos x + \sin x \log|\sec x|$$

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$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 \cdots$$

$$v_2 = \int v_{i-1}\,dx$$

$$y = y_c + y_p$$

$$y = c_1\cos x + c_2\sin x - \sin x + x\cos x + \sin x \log|\sec x|$$

Q $(D^2+1)y = 3x + 8\cot x$

→ $m^2+1=0$

$m = \pm i$, $y_c = c_1\cos x + c_2\sin x$

$y_1 = \cos x$, $y_2 = \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1 \neq 0$$

$$y_p = A(x)y_1 + B(x)y_2$$

$$A(x) = -\int \frac{Qy_2}{W}\,dx = -\int (3x - 8\cot x)\sin x\,dx$$

$$= -\int 3x\sin x - 8\cot x\sin x\,dx$$

$$= -\int 3x\sin x - 8\cos x\,dx$$

$$= -3\left[x(-\cos x) - (1) - \sin x\right] + 8\int \cos x\,dx$$

$$= 8\sin x + 3x\cos x - 3\sin x$$

$$A(x) = 5\sin x + 3x\cos x$$

$$B(x) = \int \frac{Qy_1}{W}\,dx = \int \frac{(3x - 8\cot x)\cos x}{1}\,dx$$

$$= \int (3x\cos x - 8\cot x\cos x)\,dx$$

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$$= \int 3x\cos x\,dx - \int 8\frac{\cos x}{\sin x}\cos x\,dx$$

$$= 3[x(\sin x)-(1)(-\cos x)] - 8\int \frac{\cos^2 x}{\sin x}\,dx$$

$$= 3x\sin x + 3\cos x - 8\int \frac{(1-\sin^2 x)}{\sin x}\,dx$$

$$= 3x\sin x + 3\cos x - 8\int \operatorname{cosec} x + 8\int \sin x\,dx$$

$$= 3x\sin x + 3\cos x - 8\log|\operatorname{cosec} x - \cot x| - 8\cos x$$

$$= 3x\sin x + 5\cos x - 8\log|\operatorname{cosec} x - \cot x|$$

$$y_p = (5\sin x + 3x\cos x) + (3x\sin x - 5\cos x - 8\log|\operatorname{cosec} x - \cot x|)\sin x$$

$$= \cancel{5\sin x\cos x} + 3x\cos^2 x + 3x\sin^2 x - \cancel{5\cos x\sin x} - 8\log|\operatorname{cosec} x - \cot x|\sin x$$

$$= 3x\cos^2 x + 3x\sin^2 x - 8\log|\operatorname{cosec} x - \cot x|\sin x$$

$$= 3x(\cos^2 x + \sin^2 x) - 8\log|\operatorname{cosec} x - \cot x|\sin x$$

$$y = 3x - 8\log|\operatorname{cosec} x - \cot x|\sin x$$

Q $(D^2+4)y = \dfrac{1}{1+\cos 2x}$

Q $(D^2-2D+2)y = e^x \tan x$ solve using MVP

Q $(D^2-1)y = (1+e^{-x})^{-2}$

*** Q $(D^2-1)y = e^{-x}\sin(e^{-x}) + \cos(e^{-x})$

Q $(D^2+3D+2)y = e^{e^x}$

Q $(D^2-4D+4)y = e^{2x}\sec^2 x$

$\int_x \sin x$

$\sin x$

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Simultaneous linear differential equation

Simultaneous linear diff eq^n occur's when the diff eq^n contain's more number of equation with respect to several dependent variable corresponding to an independent variable.

1) Write the diff eq^n in operator form
2) Arrange the variable in such a way that x below x and y variable below y. either apply cramer's rule or elimination to decoupling the equation.
3) Solve for x, solve for y.

Q Equation of motion of particle is given by

$$\frac{dx}{dt} + \omega y = 0$$

$$\frac{dy}{dt} - \omega x = 0$$

Find path of the particle.

→

$$Dx + \omega y = 0 \quad \text{—} (1)$$

$$-\omega x + Dy = 0 \quad \text{—} (2)$$

$(1) \times \omega$

$(2) \times D$

$$\omega Dx + \omega^2 y = 0$$

$$-\omega Dx + D^2 y = 0$$

$$(\omega^2 + D^2) y = 0$$

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$$y = c_1\cos\omega t + c_2\sin\omega t$$

Substitute y in eq ②, we get,

$$\omega x = Dy$$

$$x = \frac{1}{\omega}Dy = \frac{1}{\omega}\frac{dy}{dt}$$

$$= \frac{1}{\omega}(-c_1\omega\sin\omega t + c_2\omega\cos\omega t)$$

$$x = -c_1\sin\omega t + c_2\cos\omega t$$

Q Solve $\frac{dx}{dt} + y = e^{t}$, $\frac{dy}{dt} - x = e^{-t}$

→ Step 1:

$$Dx + y = e^{t}$$
$$Dy - x = e^{-t}$$

Step 2:

$$Dx + y = e^{t}$$
$$-x + Dy = e^{-t}$$

$$Dx + y - e^{t} = 0$$
$$-x + Dy - e^{-t} = 0$$

$$\frac{x}{\begin{vmatrix} 1 & -e^{t} \\ D & -e^{-t} \end{vmatrix}} = \frac{y}{\begin{vmatrix} +D & -e^{t} \\ -1 & -e^{-t} \end{vmatrix}} = \frac{1}{\begin{vmatrix} D & 1 \\ -1 & D \end{vmatrix}}$$

$$\frac{x}{1\times(-e^{-t}) + De^{t}} = \frac{y}{-De^{-t} - e^{t}} = \frac{1}{D^2 + 1}$$

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$$x(D^2+1) = De^{t} - e^{-t}$$
$$y(D^2+1) = -De^{-t} - e^{t}$$

$$x(D^2+1) = e^{t} - e^{-t}$$
$$y(D^2+1) = e^{-t} - e^{t}$$

$$m^2+1=0$$
$$m = \pm i$$

$$x_c = c_1\cos t + c_2\sin t$$

$$x_p = \frac{1}{D^2+1}(e^{t}-e^{-t}) = \underset{D\to 1}{\frac{e^{t}}{D^2+1}} - \underset{D\to -1}{\frac{e^{-t}}{D^2+1}} = \frac{e^{t}}{2} - \frac{e^{-t}}{2}$$

$$x_p = \frac{1}{2}(e^{t}-e^{-t})$$

$$x = c_1\cos t + c_2\sin t + \frac{1}{2}(e^{t}-e^{-t})$$

$$y_c = c_3\cos t + c_4\sin t$$

$$y_p = \frac{1}{D^2+1}(e^{-t}-e^{t}) = \underset{D\to -1}{\frac{e^{-t}}{D^2+1}} - \underset{D\to 1}{\frac{e^{t}}{D^2+1}} = \frac{e^{-t}}{2} - \frac{e^{t}}{2}$$

$$= \frac{1}{2}(e^{-t}-e^{t}) = -\frac{1}{2}(e^{t}-e^{-t})$$

$$y = c_3\cos t + c_4\sin t - \frac{1}{2}(e^{t}-e^{-t})$$

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Q Solve diff eq^n $\frac{dx}{dt} - wy = a\cos pt$

$\frac{dy}{dt} + wx = a\sin pt$

→ $Dw - wy = a\cos pt$ — ①

$Dy - wx = a\sin pt$ — ②

$Dx - wy - a\cos pt = 0$

$wx + Dy - a\sin pt = 0$

$$\frac{x}{\begin{vmatrix} -w & -a\cos pt \\ D & -a\sin pt \end{vmatrix}} = \frac{y}{\begin{vmatrix} -a\cos pt & D \\ -a\sin pt & w \end{vmatrix}} = \frac{1}{\begin{vmatrix} D & -w \\ w & D \end{vmatrix}}$$

$$\frac{x}{wa\sin pt + Da\cos pt} = \frac{y}{-wa\cos pt + Da\sin pt} = \frac{1}{D^2+w^2}$$

$$\frac{x}{wa\sin pt + ap(-\sin pt)} = \frac{y}{-wa\cos pt + ap\cos pt} = \frac{1}{D^2+w^2}$$

$x(D^2+w^2) = wa\sin pt - ap\sin pt$

$y(D^2+w^2) = -wa\cos pt + ap\cos pt$

$x_c = c_1\cos wt + c_2\sin wt$

$$x_p = \frac{1}{D^2+w^2} wa\sin pt - \frac{1}{D^2+w^2} ap\sin pt$$

$D^2 \to -p^2$

$$= \frac{1}{-p^2+w^2} wa\sin pt - \frac{1}{-p^2+w^2} ap\sin pt$$

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$$= \frac{a(\omega - p)\sin pt}{-p^2+\omega^2}$$

$$y_c = c_3\cos\omega t + c_4\sin\omega t$$

$$y_p = \frac{a(p-\omega)}{D^2+\omega^2}\cos pt$$

$$= \frac{a(p-\omega)}{-p^2+\omega^2}\cos pt$$

$$x = c_1\cos\omega t + c_2\sin\omega t + \frac{a\cancel{(\omega-p)}\sin pt}{(-p\cancel{+\omega})(\omega+p)}$$

$$x = c_1\cos\omega t + c_2\sin\omega t + \frac{a\sin pt}{(\omega+p)}$$

$$y = c_3\cos\omega t + c_4\sin\omega t + \frac{a(p-\omega)\cos pt}{(-p+\omega)(\omega+p)}$$

$$= c_3\cos\omega t + c_4\sin\omega t - \frac{a\cancel{(\omega-p)}\cos pt}{(-p\cancel{+\omega})(\omega+p)}$$

$$y = c_3\cos\omega t + c_4\sin\omega t - \frac{a\cos pt}{(\omega+p)}$$

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Q Solve $\frac{dx}{dt} + 2x - 3y = t$, $\frac{dy}{dt} - 3x + 2y = e^{2t}$

→ $Dx + 2x - 3y = t$
$Dy - 3x + 2y = e^{2t}$

$(D+2)x - 3y - t = 0$
$-3x + (D+2)y - e^{2t} = 0$

$$\frac{x}{\begin{vmatrix} -3 & -t \\ D+2 & -e^{2t} \end{vmatrix}} = \frac{y}{\begin{vmatrix} -t & D+2 \\ -e^{2t} & -3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} D+2 & -3 \\ -3 & D+2 \end{vmatrix}}$$

$$\frac{x}{3e^{2t} + t(D+2)} = \frac{y}{3t + (D+2)e^{2t}} = \frac{1}{D^2+4D-5}$$

$x(D^2+4D-5) = 3e^{2t} + 1 + 2t$
$y(D^2+4D-5) = 3t + 4e^{2t}$

$$\frac{x}{3e^{2t} + 1 + 2t} = \frac{y}{3t + 2e^{2t} + 2e^{2t}} = \frac{1}{D^2+4D-5}$$

$D^2 + 4D - 5$
$m^2 + 4m - 5$
$m = -5, 1$

$x_c = c_1 e^{-5t} + c_2 e^{t}$

$$x_p = \frac{1}{D^2+4D-5} \; 3e^{2t} + 1 + 2t$$

$$= \underset{D \to 2}{\frac{1}{(D+5)(D-1)}} \; 3e^{2t} + \underset{D \to 0}{\frac{1}{(D^2+4D-5)}} + \frac{1}{D^2+4D-5} \; 2t$$

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$$x_p = \frac{1}{4+8-5} 3e^{2t} + \frac{1}{(-5)} + \frac{1}{5} \frac{1}{\frac{D^2+4D}{5} - 1} 2t$$

$$= \frac{1}{7} 3e^{2t} + \frac{1}{(-5)} + \frac{1}{(-5)} \left(1 - \left(\frac{D^2+4D}{5}\right)\right)^{-1} 2t$$

$$= \frac{1}{7} 3e^{2t} + \frac{1}{(-5)} + \frac{1}{(-5)} \left(2t + \frac{4}{5} \times 2\right)$$

$$\cdots \left(\frac{1}{(-5)} \left(1 + \frac{D^2+4D}{5}\right) 2t\right)$$

$$= \frac{3}{7} e^{2t} - \frac{1}{5} - \frac{1}{5} \left(2t + \frac{8}{5}\right)$$

$$= \frac{3}{7} e^{2t} - \frac{1}{5} - \frac{2t}{5} - \frac{8}{25}$$

$$= \frac{3}{7} e^{2t} - \frac{2t}{5} - \frac{13}{25}$$

$$x = c_1 e^{-5t} + c_2 e^{t} + \frac{3}{7} e^{2t} - \frac{2t}{5} - \frac{13}{25}$$

$$y_c = c_1 e^{-5t} + c_2 e^{t}$$

$$y_p = \frac{1}{D^2+4D-5} (3t + 4e^{2t})$$

$$= \frac{1}{D^2+4D-5} (3t) + \frac{1}{D^2+4D-5} (4e^{2t})$$

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$$= \frac{1}{-5} \frac{1}{\left(1-\left(\frac{D^2+4D}{5}\right)\right)} (3t) + \frac{1}{4+8-5} 4e^{2t}$$

$$= \frac{-1}{5} \left(1 + \frac{D^2+4D}{5}\right) 3t + \frac{1}{7} 4e^{2t}$$

$$= \frac{-1}{5} \left(3t + \frac{12}{5}\right) + \frac{1}{7} 4e^{2t}$$

$$= \frac{-3t}{5} - \frac{12}{25} + \frac{1}{7} 4e^{2t}$$

$$y = c_1 e^{-5t} + c_2 e^{t} - \frac{3t}{5} - \frac{12}{25} + \frac{1}{7} 4e^{2t}$$

Q In a heat exchange the temp u & v of two liquids statisfies the eq^n $4\frac{du}{dt} = v-u = 2\frac{dv}{dt}$. solve the eq^n for u and v given that $u=20, v=100$ when $t=0$.

→ $4\frac{du}{dt} = v-u \Rightarrow 4\frac{du}{dt} - v + u = 0$

$2\frac{dv}{dt} = v-u \Rightarrow 2\frac{dv}{dt} - v + u = 0$

$4Du - v + u = 0$

$2Dv - v + u = 0$

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$$u(4D+1) - v = 0 \quad \text{—} \; (1)$$
$$u + (2D-1)v = 0 \quad \text{—} \; (2)$$

eq^n (1) $v = (4D+1)u$

substitute in (2)

$$u + (2D-1)(4D+1)u = 0$$

$$u + (8D^2 + 2D - 4D - 1)u = 0$$

$$(\cancel{1} + 8D^2 - 2D - \cancel{1})u = 0$$

$$(8D^2 - 2D)u = 0$$

$$u_c \Rightarrow \quad 8m^2 - 2m = 0$$
$$2m(4m-1) = 0$$
$$m = 0, \; m = 1/4$$

$$u_c = c_1 e^{0t} + c_2 e^{1/4 t}$$

$$v = (4D+1)u = \left(4\frac{d}{dt} + 1\right)\left(c_1 + c_2 e^{1/4 t}\right)$$

$$= \cancel{4} c_2 \frac{1}{\cancel{4}} e^{1/4 t} + c_1 + c_2 e^{1/4 t}$$

$$v = c_1 + 2c_2 e^{1/4 t}$$

$u = 20$, when $t = 0$
$v = 100$, when $t = 0$

$$20 = c_1 + c_2$$
$$-100 = -c_1 + 2c_2$$
$$-80 = -c_2$$

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$c_2 = 80$
$c_1 = -60$

$u = -60 + 80\, e^{1/4 t}$
$v = -60 + 160\, e^{1/4 t}$

Q Solve the simultaneous eq r and θ, given

$$\frac{dr}{dt} - 2r - \theta = 0 \quad , \frac{d\theta}{dt} + r - 4\theta = 0$$

given $\theta(0) = 0$, $r'(0) = 6$

→ $Dr - 2r - \theta = 0$
$(D-2)r - \theta = 0$ — ①
$r + (D-4)\theta = 0$ — ②

from ① $\theta = (D-2)r$
substitute in ②

$r + (D-4)(D-2)r = 0$
$(D^2 - 6D + 9)r = 0$

$m^2 - 6m + 9 = 0$
$(m-3)^2 = 0$
$m = 3, 3$

$r = r_c = (c_1 + c_2 t)e^{3t}$

$\theta = (D-2)r$
$= (D-2)\{(c_1 + c_2 t)e^{3t}\}$

$= \frac{d}{dt}((c_1 + c_2 t)e^{3t}) - 2((c_1 + c_2 t)e^{3t})$

$= 3(c_1 + c_2 t)e^{3t} + e^{3t}(c_2)$

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$$= -2(c_1 + c_2 t)e^{3t}$$

$$\theta = (c_1 + c_2 t)e^{3t} + c_2 e^{3t}$$

$$r = (c_1 + c_2 t)e^{3t}$$
$$\theta = (c_1 + c_2 t)e^{3t} + c_2 e^{3t}$$

$$\theta(0) = 0 \Rightarrow c_1 + c_2 = 0$$
$$r'(0) = 6$$

$$\frac{dr}{dt} = (c_1 + c_2 t)e^{3t} + c_2 e^{3t}$$

$$r'(0) = 3c_1 + c_2$$
$$6 = 3c_1 + c_2$$

$$2c_1 = 6 \qquad \boxed{c_1 = 3,\ c_2 = -3}$$

$$r = (3 - 3t)e^{3t}$$

$$\theta = (3 - 3t)e^{3t} + (-3)e^{3t}$$

$$= \cancel{3e^{3t}} - 3te^{3t} - \cancel{3e^{3t}}$$

$$\theta = -3te^{3t}$$

Q Solve the simultaneous eq^n for x and y

1) $2\frac{dx}{dt} - x + 3y = \sin t$

2) $2\frac{dy}{dt} + 3x - y = \cos t$; $x = \frac{1}{4}$, $y = \frac{-1}{20}$ at $t = 0$

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Homogeneous linear differential eq^n

- Cauchis form
- Legendre's form

→ **Cauchis form of HLDE**

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n y = Q(x)$$

→ **Legendre's form**

$$a_0 (ax+b)^n \frac{d^n y}{dx^n} + a_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} (ax+b) \frac{dy}{dx}$$

$$+ a_n y = Q(x)$$

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1) Cauchis form of HLDE

standard form / General form

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} x \frac{dy}{dx} + a_n y = Q(x)$$

using the substitution

put $x = e^z$

$z = \log x$

$$\frac{dz}{dx} = \frac{1}{x}$$

$$x\frac{dy}{dx} = (1)\frac{dy}{dz} = Dy \qquad D = \frac{d}{dz}$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

Substituting these values in the diff eq^n we get,

LDE with constant coefficient

$$y = \phi(z, c_1 c_2 \cdots)$$
$$y = \phi(\log x, c_1 c_2 \cdots)$$

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Step 1) Identify the type of HLDE OR check whether it is cauchis's type.

Step 2) Used the substitution $x = e^z$, $x\frac{dy}{dx} = Dy$, $x^2\frac{d^2y}{dx^2} = D(D-1)y$ and so on.

Step3) Convert the given diff eq^n $\rightarrow$ LDE with const coefficient $D(D-1)y$

Step4) Find G.S $y = \phi(z, c_1, c_2 \cdots)$

Step 5) Replace $e^z \rightarrow x$ or $z = \log x$
$y = \phi(\log x, c_1, c_2 \cdots)$

Q Solve $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^5$

→ This is cauchy's form of HLDE

put $x = e^z$, $x\frac{dy}{dx} = Dy$, $x^2\frac{d^2y}{dx^2} = D(D-1)y$

$D = \frac{d}{dz}$

$D(D-1)y - 4Dy + 6y = e^{5z}$

$(D^2 - 5D + 6)y = e^{5z}$

$y_c = c_1 e^{2z} + c_2 e^{3z}$

$y_p = \frac{1}{D^2 - 5D + 6} e^{5z}$
$D \to 5$

$= \frac{1}{25 - 25 + 6} e^{5z}$

$y_p = \frac{e^{5z}}{6}$

$y = c_1 e^{2z} + c_2 e^{3z} + \frac{e^{5z}}{6}$

$e^z = x$, $z = \log x$

$y_1 = c_1 x^2 + c_2 x^3 + \frac{x^5}{6}$

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Q Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$

→ $x = e^z$, $x^2\frac{d^2y}{dx^2} = D(D-1)y$, $x\frac{dy}{dx} = Dy$, $D = \frac{d}{dz}$

$D(D-1)y + Dy + y = \sin(\log e^{2z})$

$(D^2 - \cancel{D} + \cancel{D} + 1)y = \sin(\log e^{2z})$

$(D^2+1)y = \sin(\log e^{2z})$

$(D^2+1)y = \sin(2z \log e)$

$(D^2+1)y = \sin(2z(1))$

$m = \pm i$

$y_c = c_1\cos z + c_2 \sin z$

$y_p = \frac{1}{D^2+1}\sin 2z$

$D^2 \to -4$

$= \frac{1}{-4+1}\sin 2z = -\frac{1}{3}\sin 2z$

$y = y_c + y_p$

$= c_1\cos z + c_2\sin z - \frac{1}{3}\sin 2z$

$y = c_1\cos(\log x) + c_2\sin(\log x) - \frac{1}{3}\sin(2\log x)$

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Q Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 2\log x$

→ $x = e^z$, $x^2\frac{d^2y}{dx^2} = D(D-1)y$, $x\frac{dy}{dx} = Dy$, $D = \frac{d}{dz}$

$D(D-1)y - 2Dy - 4y = 2\log e^z$

$D^2 - D - 2D - 4 = 2z\log e$

$(D^2 - 3D - 4)y = 2z$

$m^2 - 3m - 4 = 0$

$m = +4, -1$

$y_c = c_1 e^{4z} + c_2 e^{-z}$

$y_p = \frac{1}{D^2 - 3D - 4}(2z)$

$= \frac{1}{4}\left(\frac{1}{\left(\frac{D^2 - 3D}{4} - 1\right)} 2z\right)$

$= -\frac{1}{4} \frac{1}{\left(1 + \left(\frac{D^2 - 3D}{4}\right)\right)}(2z)$

$= -\frac{1}{4}\left(1 + \frac{D^2 - 3D}{4}\right)^{-1}(2z)$

$= -\frac{1}{4}\left(1 + \frac{D^2 - 3D}{4}\right)(2z)$

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$$= -\frac{1}{4}\left(2z - \frac{8^3}{4^2}\right)$$

$$= -\frac{z}{2} + \frac{3}{8}$$

$$y = c_1 e^{4z} + c_2 e^{-z} - \frac{z}{2} + \frac{3}{8}$$

$$y = c_1 x^4 + c_2 x^{-1} - \frac{\log x}{2} + \frac{3}{8}$$

Symmetric Simultaneous D.E

the diff eq^n of the type $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

where; P, Q, R are some function x, y and z are known as symetric simultaneous linear diff eq^n

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Method 1 :- Method of grouping

Q $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$

→ considering 1st 2 ratios

$$\frac{dx}{y^2} = \frac{dy}{x^2}$$

$$x^2 dx = y^2 dy$$

Integrating both sides

$$\frac{x^3}{3} - \frac{y^3}{3} = c$$

$$x^3 - y^3 = c_1 \quad \text{—} (1)$$ (2 mks)

Consider 2nd and 3rd ratios

$$\frac{dy}{\cancel{x^2}} = \frac{dz}{\cancel{x^2} y^2 z^2}$$

$$y^2 dy = \frac{dz}{z^2}$$

$$\frac{y^3}{3} = -\frac{1}{z} + c_2 \quad \text{—} (2)$$ (2 mks)

Equation (1) and (2) together constitute the solution (1 mks)

Total : 5 mks

Q $$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

considering 1st two ratios,

→ $$\frac{dx}{y^2} = \frac{dy}{-xy} \Rightarrow -x\,dx = y^{\cancel{2}}\frac{dy}{\cancel{y}}$$

$$\Rightarrow -x\,dx = y\,dy$$

$$\Rightarrow -\frac{x^2}{2} = \frac{y^2}{2} + c$$

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$$-\frac{x^2}{2} - \frac{y^2}{2} = c$$

$$-x^2 - y^2 = c$$

$$x^2 + y^2 = c_1 \quad \text{—— (1)}$$

considering 2nd and 3rd ratios,

$$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}(z-2y)}$$

$$-\frac{dy}{y} = \frac{dz}{z-2y}$$

$$z\,dy - 2y\,dy = -y\,dz$$

$$z\,dy + y\,dz - 2y\,dy = 0$$

$$\int d(yz) - \int 2y\,dy = 0$$

$$yz - \frac{\cancel{2}y^2}{\cancel{2}} = c$$

$$yz - y^2 = c_2 \quad \text{—— (2)}$$

Equation (1) and (2) together constitute the solution.

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Q $\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}$

considering 1st two ratios,

$$\frac{dx}{2x} = \frac{dy}{-y} \Rightarrow \frac{1}{2}\log x = -\log y + \log c$$

$$\Rightarrow \frac{1}{2}\log x + \log y = \log c$$

$$\log x + 2\log y = 2\log c$$

$m \log n = \log n^m$

$$\log x + \log y^2 = \log c^2$$

$$\log(xy^2) = \log c^2$$

$\log m + \log n = \log mn$

$$xy^2 = c_1 \quad \text{—— (1)}$$

Considering 2nd and 3rd ratios,

$$\frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}$$

$$-\log y = \frac{\log|4xy^2 - 2z|}{-2} + \log c$$

$$-2\log y = \log|4xy^2 - 2z| + 2\log c$$

$$-\log y^2 + \log|4xy^2 - 2z| = \log c^2$$

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$$\log\left|\frac{4c_1 - 2z}{y^2}\right| = \log c$$

$$\frac{4xy^2 - 2z}{y^2} = c_2 \quad \text{—— (2)}$$

Equation (1) and (2) together constitute the solution.

(1) $$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{xe^{x^2+y^2}}$$

considering 1st two ratios,

$$\frac{dx}{y} = \frac{dy}{-x}$$

$$-x\,dx = y\,dy$$

$$\frac{-x^2}{2} - \frac{y^2}{2} = c$$

$$x^2 + y^2 = c_1 \quad \text{—— (1)}$$

considering 2nd and 3rd ratios.

$$\frac{dy}{-x} = \frac{dz}{xe^{x^2+y^2}} \Rightarrow \frac{dy}{-\cancel{x}} = \frac{dz}{\cancel{x}e^{c_1}}$$

$$-dy = \frac{dz}{e^{c_1}} \Rightarrow -e^{c_1}dy = dz$$

$$-e^{c_1}y = z + c$$

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$$e^{x^2+y^2} + z = c_2 \quad \text{—} (2)$$

Method 2: Method of Multipliers

Q $\dfrac{dx}{y-z} = \dfrac{dy}{z-x} = \dfrac{dz}{x-y}$

$$l(y-z) + m(z-x) + n(x-y) = 0$$

$$l, m, n = 1, 1, 1$$

first set of multipliers are 1, 1, 1

$$dx + dy + dz = 0$$

$$x + y + z = c_1 \quad \text{—} (1)$$

Second set of multipliers are x, y, z

$$x\,dx + y\,dy + z\,dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c$$

$$x^2 + y^2 + z^2 = c_2 \quad \text{—} (2)$$

Equation (1) and (2) together constitute the solution.

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Q $\frac{dx}{3z-4y} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}$

→ first set of multipliers x, y, z

$$\cancel{3xz} - \cancel{4xy} + \cancel{4xy} - \cancel{2xz} + \cancel{2yz} - \cancel{3xz} = 0$$

$$x\,dx + y\,dy + z\,dz = 0$$

$$x^2 + y^2 + z^2 = c_1 \quad \text{—— (1)}$$

Second set of multipliers are $2, 3, 4$

$$2dx + 3dy + 4dz = 0$$

$$2x + 3y + 4z = c_2 \quad \text{—— (2)}$$

Equation (1) and (2) together constitute the solution.

2.) Legendre's Diff eq^n

$$a_0(ax+b)^n \frac{d^n y}{dx^n} + a_1(ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2(ax+b)^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots + a_n y = Q(x)$$

put, $ax+b = e^z$ or $z = \log(ax+b)$

$$(ax+b)\frac{dy}{dx} = aDy$$

A.E ⇒ Auxiliary equation

$$(ax+b)^2 \frac{d^2y}{dx^2} = a^2 D(D-1)y$$

$$(ax+b)^3 \frac{d^3y}{dx^3} = a^3 D(D-1)(D-2)y$$

Q $$(2x+1)^2 \frac{d^2y}{dx^2} - 6(2x+1)\frac{dy}{dx} + 16y = 8(2x+1)^2$$

→ put $2x+1 = e^z$ or $z = \log(2x+1)$

$$(2x+1)\frac{dy}{dx} = 2Dy, \quad D = \frac{d}{dz}$$

$$(2x+1)^2 \frac{d^2y}{dx^2} = 2^2 D(D-1)y$$

Transformed eqⁿ,

$$4D(D-1)y - 6(2Dy) + 16y = 8e^{2z}$$

$$(4D^2 - 4D - 12D + 16)y = 8e^{2z}$$

$$(4D^2 - 16D + 16)y = 8e^{2z}$$

A.E ⇒ $4m^2 - 16m + 16 = 0$

$m = 2, 2$

$$y_c = (c_1 + c_2 z)e^{2z}$$

$$y_p = \frac{1}{4D^2 - 16D + 16}(8e^{2z})$$

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$D \to 2$

$$= \frac{1}{4(2)^2 - 16(2) + 16} \, 8e^{2z}$$

$$= \frac{1}{16 - 32 + 16} \, (8e^{2z})$$

case of failure

$$= \frac{z}{8D - 16} \, 8e^{2z}$$

$D \to 2$

$$= \frac{z}{16 - 16} \, 8e^{2z}$$

case of failure

$$y_p = \frac{z^2}{\cancel{8}} (\cancel{8}e^{2z}) = z^2 e^{2z}$$

$$y = y_c + y_p$$

$$= (c_1 + c_2 z) e^{2z} + z^2 e^{2z}$$

$$= (c_1 + c_2 (\log(2x+1))) (2x+1)^2 + z^2 e^{2z}$$

$$y = [c_1 + c_2 \log(2x+1)](2x+1)^2 + (\log(2x+1))^2 (2x+1)^2$$

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Q $(x+2)^2 \dfrac{d^2y}{dx^2} + 3(x+2)\dfrac{dy}{dx} + y = 4\sin(\log(x+2))$

→ $(x+2) = e^z$ or $z = \log(x+z)$

$$(x+2)\frac{dy}{dx} = Dy$$

$$(x+2)^2 \frac{d^2y}{dx^2} = D(D-1)y$$

Transformed eq^n,

$$D(D-1)y + 3(Dy) + y = 4\sin(z)$$

$$(D^2 - D + 3D + 1)y = 4\sin z$$

$$(D^2 + 2D + 1)y = 4\sin z$$

A.E ⇒ $m^2 + 2m + 1 = 0$

$m = -1, -1$

$$y_c = (c_1 + c_2 z)e^{-z}$$

$$y_p = \frac{1}{D^2 + 2D + 1} 4\sin z$$

$D^2 \to -1$

$$= \frac{1}{\cancel{-1} + 2D \cancel{+1}} 4\sin z = \frac{1}{2D}(4\sin z)$$

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$$= 2\int \sin z \, dz$$

$$y_p = -2\cos z$$

$$y = y_c + y_p$$

$$y = (c_1 + c_2 z)e^{-z} - 2\cos(z)$$

$$y = (c_1 + c_2(\log(x+2)))(x+2)^{-1} - 2\cos(\log(x+2))$$

Q $(2x+3)^2 \dfrac{d^2y}{dx^2} - 2(2x+3)\dfrac{dy}{dx} - 12y = 6x$

$\rightarrow$ $(2x+3) = e^z$ or $z = \log(2x+3)$

$x = \dfrac{e^z - 3}{2}$ $\quad (2x+3)\dfrac{dy}{dx} = 2Dy$

$$(2x+3)^2 \frac{d^2y}{dx^2} = 4D(D-1)y$$

** Always replace both LHS and RHS

Transformed eqn,

$$4D(D-1)y - 2(2Dy) - 12y = 6x$$

$$(4D^2 - 4D - 4D - 12)y = 6x$$

$$(4D^2 - 8D - 12)y = 6x$$

$$(4D^2 - 8D - 12)y = 6\left(\frac{e^z - 3}{2}\right)$$

$$4m^2 - 8m - 12 = 0$$
$$m = 3, -1$$

$$y_c = c_1 e^{-3z} + c_2 e^{-z}$$

$$y_p = \frac{1}{4D^2 - 8D - 12} 6\left(\frac{e^2 - 3}{2}\right)$$

$$= \frac{1}{4D^2 - 8D - 12}(3e^2 - 9)$$

$$= \frac{1}{4D^2 - 8D - 12}(3e^2) - \frac{1}{4D^2 - 8D - 12}(9)\, e^{0x}$$

$D \to 1$

$e^{0x} = 1$

assume there is e^{0x} then $D \to 0$

$$= \frac{1}{4 - 8 - 12}(3e^2) - \frac{1}{0 - 0 - 12}(9)$$

$$y_p = -\frac{3}{16} e^2 + \frac{3}{4}$$

$$y = y_c + y_p$$

$$= c_1 e^{3z} + c_2 e^{-z} - \frac{3}{16} e^2 + \frac{3}{4}$$

$$y = c_1 (2x+3)^3 + c_2 (2x+3)^{-1} - \frac{3}{16}(2x+3) + \frac{3}{4}$$

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Q $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4[\cos(\log(1+x))]$

Q $(4x+1)^2 \frac{d^2y}{dx^2} + 2(4x+1)\frac{dy}{dx} + y = 2x+1$

Q $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2+4x+1$

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