

Assignment 3

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Download all latex codes from

<https://github.com/vaibhavchhabra25/AI1103-course/blob/main/Assignment-3/main.tex>

So,

$$E(XY) = 19/2 \quad (2.0.10)$$

Let Z be a random variable defined as

$$Z = X - Y \quad (2.0.11)$$

Then using (2.0.1),

$$E(Z) = E(X - Y) = E(X) - E(Y) = 0 \quad (2.0.12)$$

Now, using (2.0.12)

$$Var(Z) = E(Z^2) - (E(Z))^2 = E(Z^2) \quad (2.0.13)$$

$$Var(Z) = E((X - Y)^2) \quad (2.0.14)$$

$$Var(Z) = E(X^2) + E(Y^2) - 2E(XY) \quad (2.0.15)$$

Using (2.0.6), (2.0.7) and (2.0.10),

$$Var(Z) = 10 + 10 - 2 \times 19/2 \quad (2.0.16)$$

$$Var(Z) = 1 \quad (2.0.17)$$

Theorem 2.1. (Chebychev's Inequality) Let T be an arbitrary random variable, with finite mean $E(T)$, then for all $a > 0$,

$$\Pr(|T - E(T)| \geq a) \leq \frac{Var(T)}{a^2} \quad (2.0.18)$$

Proof. Let T be a random variable with probability distribution function $f(T)$ and $a > 0$ be any real number. Then,

$$\Pr(|T - E(T)| \geq a) = \int_{-\infty}^{-E(T)-a} f(T)dT + \int_{E(T)+a}^{\infty} f(T)dT \quad (2.0.19)$$

$$\Pr(|T - E(T)| \geq a) = \int_{|T-E(T)| \geq a} f(T)dT \quad (2.0.20)$$

1 PROBLEM

(GATE 2001 (MA), Q. 2.24) Let (X, Y) be a two-dimensional random variable such that $E(X) = E(Y) = 1/2$, $Var(X) = Var(Y) = 1$ and $Cov(X, Y) = 1/2$. Then, $P(|X - Y| > 6)$ is

- | | |
|------------------|---------------------|
| 1) less than 1/6 | 3) equal to 1/3 |
| 2) equal to 1/2 | 4) greater than 1/2 |

2 SOLUTION

Given,

$$E(X) = E(Y) = 3 \quad (2.0.1)$$

$$Var(X) = Var(Y) = 1 \quad (2.0.2)$$

$$Cov(X, Y) = 1/2 \quad (2.0.3)$$

Now,

$$Var(X) = E(X^2) - (E(X))^2 \quad (2.0.4)$$

Substituting given values, we get,

$$1 = E(X^2) - 3^2 \quad (2.0.5)$$

So,

$$E(X^2) = 10 \quad (2.0.6)$$

Similarly for Y ,

$$E(Y^2) = 10 \quad (2.0.7)$$

Also,

$$Cov(X, Y) = E(XY) - E(X)E(Y) \quad (2.0.8)$$

Substituting given values, we get,

$$1/2 = E(XY) - (3)(3) \quad (2.0.9)$$

Now,

$$\begin{aligned}
 \text{Var}(T) &= \int_{-\infty}^{\infty} (T - E(T))^2 f(T) dT \\
 &\geq \int_{|T-E(T)| \geq a} (T - E(T))^2 f(T) dT \\
 &\geq a^2 \int_{|T-E(T)| \geq a} f(T) dT \quad (2.0.21)
 \end{aligned}$$

So, we finally get,

$$\text{Var}(T) \geq a^2 \int_{|T-E(T)| \geq a} f(T) dT \quad (2.0.22)$$

Using (2.0.20),

$$\text{Var}(T) \geq a^2 \Pr(|T - E(T)| \geq a) \quad (2.0.23)$$

Or,

$$\Pr(|T - E(T)| \geq a) \leq \frac{\text{Var}(T)}{a^2} \quad (2.0.24)$$

□

Applying Chebychev's Inequality for Z with $a = 6$, we get,

$$\Pr(|Z - E(Z)| \geq 6) \leq \frac{\text{Var}(Z)}{6^2} \quad (2.0.25)$$

Using (2.0.12) and (2.0.17),

$$\Pr(|Z - 0| \geq 6) \leq \frac{1}{36} \quad (2.0.26)$$

As $Z = X - Y$,

$$\Pr(|X - Y| \geq 6) \leq \frac{1}{36} \quad (2.0.27)$$

So, option 1 is correct.