Assignment 2

Vaibhav Chhabra

AI20BTECH11022

Download all python codes from

and latex codes from

1 Problem

(GATE EC-problem 21) Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively.
Then, for all values of x

1) $F(x) - G(x) \leq 0$
2) $F(x) - G(x) \geq 0$
3) $(F(x) - G(x))x \leq 0$
4) $(F(x) - G(x))x \geq 0$

2 Solution

If X is a random variable, the cumulative distribution functions of U and 2V can be written in terms of X as

$$F(x) = \Pr(X \leq x) \qquad (2.0.1)$$

$$G(x) = \Pr(2X \leq x) \qquad (2.0.2)$$

Or,

$$G(x) = \Pr(X \leq x/2) \qquad (2.0.3)$$

Using 2.0.1 in 2.0.3, we can see that

$$G(x) = F(x/2) \qquad (2.0.4)$$

(i) For $x \geq 0$

$$x/2 \leq x \qquad (2.0.5)$$

As, F is a cumulative distribution function, it is non-decreasing. Thus,

$$F(x/2) \leq F(x) \qquad (2.0.6)$$

Using 2.0.3

$$G(x) \leq F(x) \qquad (2.0.7)$$

Or,

$$F(x) - G(x) \geq 0 \qquad (2.0.8)$$

(ii) For $x \leq 0$

$$x \leq x/2 \qquad (2.0.9)$$

As, F is non-decreasing,

$$F(x) \leq F(x/2) \qquad (2.0.10)$$

Using 2.0.3

$$F(x) \leq G(x) \qquad (2.0.11)$$

Or,

$$F(x) - G(x) \leq 0 \qquad (2.0.12)$$

From the two cases we get that x and $F(x) - G(x)$ have same sign. Thus, we can combine the cases as

$$(F(x) - G(x))x \geq 0 \qquad (2.0.13)$$

which is true for all values of x.
So, option 4 is correct.