Assignment 2

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Download all python codes from

https://github.com/vaibhavchhabra25/AI1103-course/blob/main/Assignment-2/Codes/simulation_code.py

and latex codes from

https://github.com/vaibhavchhabra25/AI1103-course/blob/main/Assignment-2/main.tex

1 Problem

(GATE EC-problem 21) Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively.
Then, for all values of x

1) $F(x) - G(x) \leq 0$
2) $F(x) - G(x) \geq 0$
3) $(F(x) - G(x))x \leq 0$
4) $(F(x) - G(x))x \geq 0$

2 Solution

If X is a random variable, the cumulative distribution functions of U and 2V can be written in terms of X as

$$F(x) = \Pr(X \leq x) \tag{2.0.1}$$

$$G(x) = \Pr(2X \leq x) \tag{2.0.2}$$

Or,

$$G(x) = \Pr(X \leq x/2) \tag{2.0.3}$$

Using 2.0.1 in 2.0.3, we can see that

$$G(x) = F(x/2) \tag{2.0.4}$$

Now, lets examine the options

1)

$$F(x) - G(x) \leq 0 \tag{2.0.5}$$

For $x \geq 0$

$$x/2 \leq x \tag{2.0.6}$$

As, F is a cumulative distribution function, it is non-decreasing. Thus,

$$F(x/2) \leq F(x) \tag{2.0.7}$$

Using 2.0.4,

$$G(x) \leq F(x) \tag{2.0.8}$$

Or,

$$F(x) - G(x) \geq 0 \tag{2.0.9}$$

This contradicts option-1, so its wrong.

2)

$$F(x) - G(x) \geq 0 \tag{2.0.10}$$

For $x \leq 0$

$$x \leq x/2 \tag{2.0.11}$$

As, F is a cumulative distribution function, it is non-decreasing. Thus,

$$F(x) \leq F(x/2) \tag{2.0.12}$$

Using 2.0.4,

$$F(x) \leq G(x) \tag{2.0.13}$$

Or,

$$F(x) - G(x) \leq 0 \tag{2.0.14}$$

This contradicts option-2, so its wrong.

3)

$$(F(x) - G(x))x \leq 0 \tag{2.0.15}$$

For $x \geq 0$, we saw that $F(x) - G(x) \geq 0$ As both x and $F(x) - G(x)$ are positive simultaneously, their product will also be positive. Thus,

$$(F(x) - G(x))x \geq 0 \tag{2.0.16}$$

This contradicts option-3, so its wrong.

4)

$$(F(x) - G(x))x \geq 0 \tag{2.0.17}$$

We saw that for $x \geq 0$, $F(x) - G(x) \geq 0$ and for $x \leq 0$, $F(x) - G(x) \leq 0$. In either case, x and $F(x) - G(x)$ have same sign. So, their product will be positive,

$$(F(x) - G(x))x \geq 0 \tag{2.0.18}$$

This proves that option-4 is true for all x.