Assignment 2

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Download all python codes from

https://github.com/vaibhavchhabra25/AI1103-course/blob/main/Assignment-2/Codes/simulation_code.py

and latex codes from

https://github.com/vaibhavchhabra25/AI1103-course/blob/main/Assignment-2/main.tex

1 Problem

(GATE EC-problem 21) Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively.
Then, for all values of x

1) $F(x) - G(x) \leq 0$
2) $F(x) - G(x) \geq 0$
3) $(F(x) - G(x))x \leq 0$
4) $(F(x) - G(x))x \geq 0$

2 Solution

If X is a random variable, the cumulative distribution functions of U and 2V can be written in terms of X as

$$F(x) = \Pr(X \leq x) \tag{2.0.1}$$

$$G(x) = \Pr(2X \leq x) \tag{2.0.2}$$

Or,

$$G(x) = \Pr(X \leq x/2) \tag{2.0.3}$$

Using 2.0.1 in 2.0.3, we can see that

$$G(x) = F(x/2) \tag{2.0.4}$$

So,

$$F(x) - G(x) = F(x) - F(x/2) \tag{2.0.5}$$

As F is Cumulative Distribution Function, it is non-decreasing.
That means for $x \geq y$, $F(x) \geq F(y)$.

Using this, we can form the following table:

Case	$F(x) - F(x/2)$	$(F(x) - F(x/2))x$
$x \geq 0$	≥ 0	≥ 0
$x \leq 0$	≤ 0	≥ 0

TABLE 4

From the table we can see that for any value of x,

$$(F(x) - F(x/2))x \geq 0 \tag{2.0.6}$$

Or, using 2.0.4,

$$(F(x) - G(x))x \geq x \tag{2.0.7}$$

So, option 4 is correct.