

# QUIZ 2

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Download all latex-tikz codes from

<https://github.com/vaibhavchhabra25/EE3900-course/blob/main/QUIZ-2/main.tex>

## 1 PROBLEM

(3.8(b)) The system function of a casual linear time-invariant system is

$$H(z) = \frac{1 - z^{-1}}{1 + \frac{3}{4}z^{-1}} \quad (1.0.1)$$

The input to this system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + u[-n - 1] \quad (1.0.2)$$

Find the output  $y[n]$ .

## 2 SOLUTION

The  $\mathcal{Z}$  transform of  $x[n]$  is given by

$$X(z) = \mathcal{Z}(x[n]) \quad (2.0.1)$$

$$= \sum_{-\infty}^{\infty} (z^{-1})^n (x[n]) \quad (2.0.2)$$

$$= \sum_{-\infty}^{\infty} (z^{-1})^n \left(\frac{1}{3}\right)^n u[n] + \sum_{-\infty}^{\infty} (z^{-1})^n u[-n - 1] \quad (2.0.3)$$

$$= \sum_0^{\infty} \left(\frac{1}{3}z^{-1}\right)^n + \sum_{-\infty}^{-1} (z^{-1})^n \quad (2.0.4)$$

$$= \sum_0^{\infty} \left(\frac{1}{3}z^{-1}\right)^n + \sum_1^{\infty} z^n \quad (2.0.5)$$

$$= \frac{1}{1 - \frac{1}{3}z^{-1}} - \frac{1}{1 - z^{-1}} \quad (2.0.6)$$

$$= \frac{-\frac{2}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - z^{-1})} \quad (2.0.7)$$

where ROC of  $X(z)$  is  $\frac{1}{3} < |z| < 1$ .

Now, using convolution theorem, since

$$y[n] = h[n] * x[n] \quad (2.0.8)$$

we can write

$$Y(z) = H(z)X(z) \quad (2.0.9)$$

$$= \left(\frac{1 - z^{-1}}{1 + \frac{3}{4}z^{-1}}\right) \frac{-\frac{2}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - z^{-1})} \quad (2.0.10)$$

$$= \frac{-\frac{2}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 + \frac{3}{4}z^{-1})} \quad (2.0.11)$$

$$= \frac{24}{39} \left( \frac{1}{1 + \frac{3}{4}z^{-1}} - \frac{1}{1 - \frac{1}{3}z^{-1}} \right) \quad (2.0.12)$$

$$= \frac{24}{39} \left( \sum_{-\infty}^{\infty} \left(-\frac{3}{4}z^{-1}\right)^n u[n] - \sum_{-\infty}^{\infty} \left(\frac{1}{3}z^{-1}\right)^n u[n] \right) \quad (2.0.13)$$

The output signal  $y[n]$  is the inverse  $\mathcal{Z}$  transform of  $Y(z)$ . So,

$$y[n] = \frac{24}{39} \left( \left(\frac{-3}{4}\right)^n u[n] - \left(\frac{1}{3}\right)^n u[n] \right) \quad (2.0.14)$$

$$y[n] = \frac{24}{39} \left( \left(\frac{-3}{4}\right)^n - \left(\frac{1}{3}\right)^n \right) u[n] \quad (2.0.15)$$