GATE ASSIGNMENT 1

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Download all latex-tikz codes from

https://github.com/vaibhavchhabra25/EE3900–course/blob/main/GATE_Assignment–1/main.tex

1 Problem

(EC 2017-Q.33) Consider an LTI system with magnitude response

$$|H(f)| = \begin{cases} 1 - \dfrac{|f|}{20}, & |f| \le 20 \\ 0, & |f| > 20 \end{cases}$$

and phase response

$$\arg H(f) = -2f$$

If the input to the system is

$$x(t) = 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 16\sin\left(40\pi t + \frac{\pi}{8}\right) + 24\cos\left(80\pi t + \frac{\pi}{16}\right)$$

then what is the average power of the output signal $y(t)$.

2 Solution

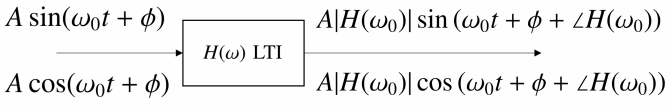

Fig. 0: Output of LTI

1) For input signal $8\cos\left(20\pi t + \frac{\pi}{4}\right)$,

$$f_1 = 20\pi/2\pi = 10\text{Hz} \tag{2.0.1}$$

Since $|f_1| \le 20$,

$$|H(f_1)| = 1 - \frac{10}{20} = \frac{1}{2} \tag{2.0.2}$$

Also,

$$\arg H(f_1) = -2f_1 = -20 \tag{2.0.3}$$

So, the output signal $y_1(t)$ is

$$y_1(t) = \left(8 \times \frac{1}{2}\right)\cos\left(20\pi t + \frac{\pi}{4} - 20\right) \tag{2.0.4}$$

$$\implies y_1(t) = 4\cos\left(20\pi t + \frac{\pi}{4} - 20\right) \tag{2.0.5}$$

2) For input signal $16\sin\left(40\pi t + \frac{\pi}{8}\right)$,

$$f_2 = 40\pi/2\pi = 20\text{Hz} \tag{2.0.6}$$

Since $|f_2| \le 20$,

$$|H(f_2)| = 1 - \frac{20}{20} = 0 \tag{2.0.7}$$

So, the output signal $y_2(t) = 0$.

3) For input signal $24\cos\left(80\pi t + \frac{\pi}{16}\right)$,

$$f_3 = 80\pi/2\pi = 40\text{Hz} \tag{2.0.8}$$

Since $|f_3| > 20$,

$$|H(f_3)| = 0 \tag{2.0.9}$$

So, the output signal $y_3(t) = 0$.

So, the total output signal is

$$y(t) = y_1(t) + y_2(t) + y_3(t) \tag{2.0.10}$$

$$\implies y(t) = 4\cos\left(20\pi t + \frac{\pi}{4} - 20\right) \tag{2.0.11}$$

Average power of this output signal

$$P_{y(t)} = \frac{4^2}{2} = 8\text{W} \tag{2.0.12}$$