GATE ASSIGNMENT 2

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Download all latex-tikz codes from

https://github.com/vaibhavchhabra25/EE3900-course/blob/main/GATE_Assignment-2/main.tex

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1 Problem

(EC 2006-Q.54) The unit-step response of a system starting from rest is given by

$$c(t) = 1 - e^{-2t} \quad \text{for } t \geq 0 \tag{1.0.1}$$

The transfer function of the system is:

1) $\dfrac{1}{1+2s}$
2) $\dfrac{2}{2+s}$
3) $\dfrac{1}{2+s}$
4) $\dfrac{2s}{1+2s}$

2 Solution

The input function $r(t)$, to the system is a unit-step function. So,

$$r(t) = u(t) \tag{2.0.1}$$

The unit-step response for the system is

$$c(t) = 1 - e^{-2t} \quad \text{for } t \geq 0 \tag{2.0.2}$$

Definition 1 (Laplace transform)**.** *The Laplace transform of a function f(t) is defined as*

$$\mathcal{L}\{f(t)\}(s) = \int_0^\infty f(t)e^{-st}dt \tag{2.0.3}$$

Definition 2 (Transfer function)**.** *The transfer function of a system is defined as the ratio of Laplace transform of the output $c(t)$, to the Laplace transform of the input $r(t)$, under zero initial conditions.*

$$T(s) = \frac{\mathcal{L}\{c(t)\}(s)}{\mathcal{L}\{r(t)\}(s)} \tag{2.0.4}$$

The system is given to be starting from rest, so we have zero initial conditions.
Now,

$$\begin{aligned}
\mathcal{L}\{c(t)\}(s) &= \mathcal{L}\{1 - e^{-2t}\}(s) && (2.0.5)\\
&= \int_0^\infty (1 - e^{-2t})e^{-st}dt && (2.0.6)\\
&= \int_0^\infty e^{-st}dt - \int_0^\infty e^{-(2+s)t}dt && (2.0.7)\\
&= \frac{1}{s} - \frac{1}{2+s} && (2.0.8)\\
&= \frac{2}{s(2+s)} && (2.0.9)
\end{aligned}$$

And

$$\begin{aligned}
\mathcal{L}\{r(t)\}(s) &= \mathcal{L}\{u(t)\}(s) && (2.0.10)\\
&= \mathcal{L}\{1\}(s) && (2.0.11)\\
&= \int_0^\infty e^{-st}dt && (2.0.12)\\
&= \frac{1}{s} && (2.0.13)
\end{aligned}$$

By Definition 2,

$$\begin{aligned}
T(s) &= \frac{\mathcal{L}\{c(t)\}(s)}{\mathcal{L}\{r(t)\}(s)} && (2.0.14)\\
&= \frac{\frac{2}{s(2+s)}}{\frac{1}{s}} && (2.0.15)\\
&= \frac{2}{2+s} && (2.0.16)
\end{aligned}$$

So, option 2 is correct.

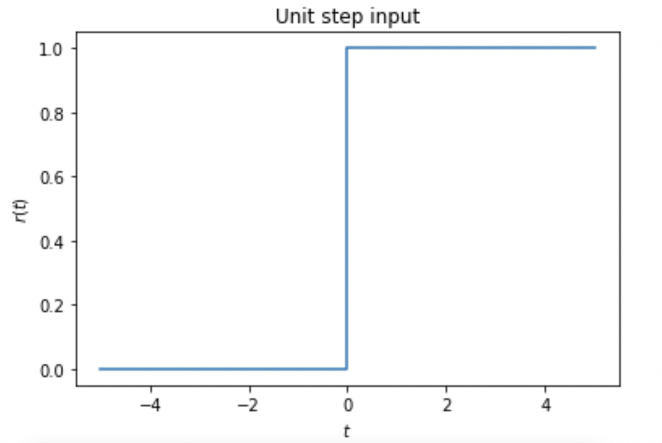

Fig. 4: Input Function Plot

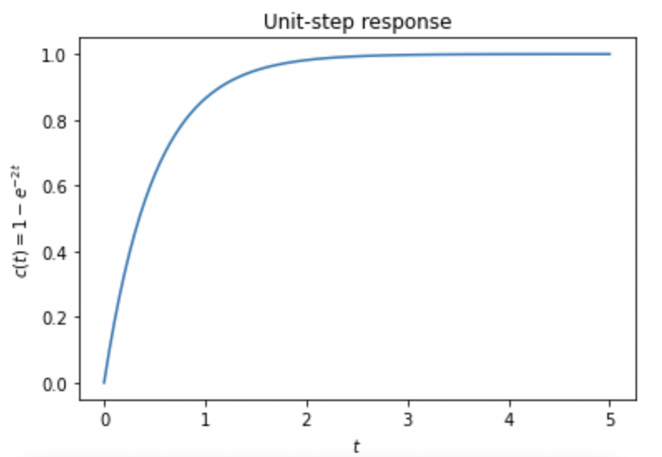

Fig. 4: Output Function Plot

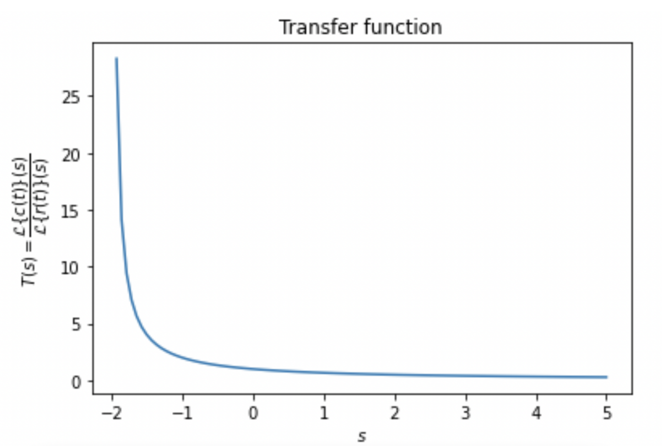

Fig. 4: Transfer Function Plot