# ASSIGNMENT 5

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Download all python codes from

> https://github.com/vaibhavchhabra25/EE3900–course/blob/main/Assignment–5/codes

and latex-tikz codes from

> https://github.com/vaibhavchhabra25/EE3900–course/blob/main/Assignment–5/main.tex

## 1 Problem

(Quadratic Forms-Q-2.25)
Find the equation of the parabola which is symmetric about the y-axis, and passes through the point $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

## 2 Solution

Since the parabola is symmetric about y-axis, it is the axis of the parabola. Let the vertex of the parabola be $\mathbf{v} = \begin{pmatrix} 0 \\ k \end{pmatrix}$ and the focus be $\mathbf{f} = \begin{pmatrix} 0 \\ k+a \end{pmatrix}$.
Then the point of intersection of the directrix and y-axis will be $\begin{pmatrix} 0 \\ k-a \end{pmatrix}$.
Since, directrix of the parabola is perpendicular to the axis, the equation of directrix will be

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = k - a \tag{2.0.1}$$

Let $\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $c = k - a$.
Let $\mathbf{x}$ be any general point on the parabola.
By the definition of a parabola, the distance between $\mathbf{x}$ and the focus is equal to the perpendicular distance between $\mathbf{x}$ and the directrix.
So we can write

$$\|\mathbf{x} - \mathbf{f}\| = \frac{\left|\mathbf{n}^\top \mathbf{x} - c\right|}{\|\mathbf{n}\|} \tag{2.0.2}$$

$$\implies \|\mathbf{x} - \mathbf{f}\|^2 \|\mathbf{n}\|^2 = \left|\mathbf{n}^\top \mathbf{x} - c\right|^2 \tag{2.0.3}$$

$$\implies (\mathbf{x} - \mathbf{f})^\top (\mathbf{x} - \mathbf{f}) \|\mathbf{n}\|^2 = (\mathbf{n}^\top \mathbf{x})^2 - 2c\mathbf{n}^\top \mathbf{x} + c^2 \tag{2.0.4}$$

$$\|\mathbf{n}\|^2 \mathbf{x}^\top \mathbf{x} - 2\|\mathbf{n}\|^2 \mathbf{f}^\top \mathbf{x} + \|\mathbf{n}\|^2 \|\mathbf{f}\|^2 = \mathbf{x}^\top \mathbf{n}\mathbf{n}^\top \mathbf{x} - 2c\mathbf{n}^\top \mathbf{x} + c^2 \tag{2.0.5}$$

$$\implies \mathbf{x}^\top (\|\mathbf{n}\|^2 \mathbf{I} - \mathbf{n}\mathbf{n}^\top)\mathbf{x} + 2(c\mathbf{n} - \|\mathbf{n}\|^2 \mathbf{f})^\top \mathbf{x} + \|\mathbf{n}\|^2 \|\mathbf{f}\|^2 - c^2 = 0 \tag{2.0.6}$$

Putting values of $\mathbf{n}$, $\mathbf{f}$ and $c$, we get

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} 0 & -2a \end{pmatrix} \mathbf{x} + 4ak = 0 \tag{2.0.7}$$

Since $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ lies on the parabola,

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 0 & -4a \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix} + 4ak = 0 \tag{2.0.8}$$

$$\implies 4 + 12a + 4ak = 0 \tag{2.0.9}$$

$$\implies ak = -1 - 3a \tag{2.0.10}$$

Using (2.0.7) and (2.0.10), the required equation of parabola is

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 0 & 4a \end{pmatrix} \mathbf{x} - 12a - 4 = 0 \tag{2.0.11}$$