ASSIGNMENT 3

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Download all python codes from

https://github.com/vaibhavchhabra25/EE3900–course/blob/main/Assignment–3/codes

and latex-tikz codes from

https://github.com/vaibhavchhabra25/EE3900–course/blob/main/Assignment–3/main.tex

1 Problem

(Ramsey-4.2-Tangent and Normal-Q.21)
Verify that the perpendicular bisector of the chord joining two points $\mathbf{x_1}$,$\mathbf{x_2}$ on the circle

$$\mathbf{x}^\top\mathbf{x} + 2\begin{pmatrix} g & f \end{pmatrix}\mathbf{x} + c = 0 \tag{1.0.1}$$

passes through the centre.

2 Solution

Since $\mathbf{x_1}$ and $\mathbf{x_2}$ lie on the circle,

$$\mathbf{x_1}^\top\mathbf{x_1} + 2\begin{pmatrix} g & f \end{pmatrix}\mathbf{x_1} + c = 0 \tag{2.0.1}$$

$$\mathbf{x_2}^\top\mathbf{x_2} + 2\begin{pmatrix} g & f \end{pmatrix}\mathbf{x_2} + c = 0 \tag{2.0.2}$$

Subtracting the two, we get,

$$\mathbf{x_2}^\top\mathbf{x_2} - \mathbf{x_1}^\top\mathbf{x_1} = -2\begin{pmatrix} g & f \end{pmatrix}(\mathbf{x_2} - \mathbf{x_1}) \tag{2.0.3}$$

The midpoint of the chord $\mathbf{x_1}\mathbf{x_2}$ is

$$\mathbf{M} = \frac{\mathbf{x_1} + \mathbf{x_2}}{2} \tag{2.0.4}$$

Since perpendicular bisector of $\mathbf{x_1}\mathbf{x_2}$ is perpendicular to $\mathbf{x_1}\mathbf{x_2}$ and passes through $\mathbf{M}$, any general point $\mathbf{P}$ on the perpendicular bisector can be given using the relation

$$(\mathbf{x_2} - \mathbf{x_1})^\top(\mathbf{P} - \mathbf{M}) = 0 \tag{2.0.5}$$

$$\implies (\mathbf{x_2} - \mathbf{x_1})^\top\mathbf{P} - (\mathbf{x_2}^\top - \mathbf{x_1}^\top)\left(\frac{\mathbf{x_1} + \mathbf{x_2}}{2}\right) = 0 \tag{2.0.6}$$

$$\implies (\mathbf{x_2} - \mathbf{x_1})^\top\mathbf{P} = \frac{\mathbf{x_2}^\top\mathbf{x_2} - \mathbf{x_1}^\top\mathbf{x_1} + \mathbf{x_2}^\top\mathbf{x_1} - \mathbf{x_1}^\top\mathbf{x_2}}{2} \tag{2.0.7}$$

Since $\mathbf{x_1}^\top\mathbf{x_2} = \mathbf{x_2}^\top\mathbf{x_1}$, the equation reduces to

$$2(\mathbf{x_2} - \mathbf{x_1})^\top\mathbf{P} = \mathbf{x_2}^\top\mathbf{x_2} - \mathbf{x_1}^\top\mathbf{x_1} \tag{2.0.8}$$

Using (2.0.3), we get

$$2(\mathbf{x_2} - \mathbf{x_1})^\top\mathbf{P} = -2\begin{pmatrix} g & f \end{pmatrix}(\mathbf{x_2} - \mathbf{x_1}) \tag{2.0.9}$$

$$\implies (\mathbf{x_2} - \mathbf{x_1})^\top\mathbf{P} = (\mathbf{x_2} - \mathbf{x_1})^\top\begin{pmatrix} -g \\ -f \end{pmatrix} \tag{2.0.10}$$

Since the centre of the circle is $\mathbf{C} = \begin{pmatrix} -g \\ -f \end{pmatrix}$, we can clearly see that $\mathbf{P} = \mathbf{C}$ satisfies the equation of perpendicular bisector.

Hence, the perpendicular bisector of any chord of a circle passes through the centre of the circle.

For example, consider the circle

$$\mathbf{x}^\top\mathbf{x} + 2\begin{pmatrix} -4 & -5 \end{pmatrix}\mathbf{x} + 5 = 0 \tag{2.0.11}$$

and let $\mathbf{x_1} = \begin{pmatrix} 0 \\ 5 - 2\sqrt{5} \end{pmatrix}$ and $\mathbf{x_2} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$. We can clearly see in the figure that the perpendicular bisector the chord $\mathbf{x_1}\mathbf{x_2}$ passes through the centre of the circle.

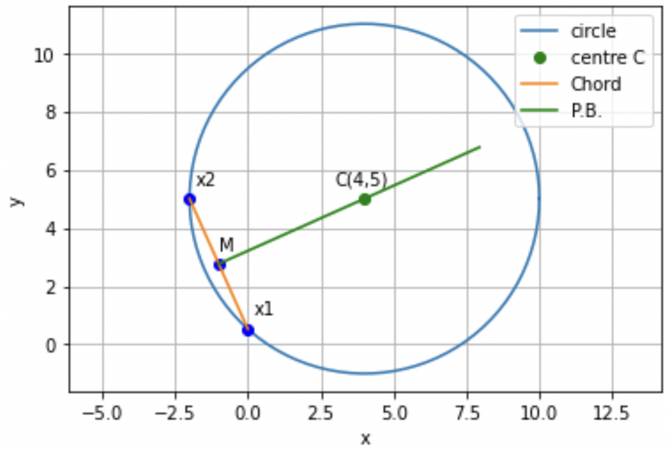

Fig. 0: Example figure