P=NP?

Will P v NP be solved in the next 50 years?

Extended Project Dissertation Level 3

Vaibhav Krishnakumar

Ibstock Place School

Candidate Number – 6257

Centre Number – 11032

Teacher Assessor – Mr. Tomlinson

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* The Gantt chart is spread over 2 pages. The first part of the chart is on page 7 and the second part of the chart on page 6. The reversal of page ordering is intentional; it makes it easier to read across the rows and interpret the graph.

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Project Proposal form

Learner Name	**Vaibhav Krishnakumar**	**Learner number**	**6257**
Centre Name	**Ibstock Place School**	**Centre Number**	**11032**
Teacher Assessor	**Mr Tomlinson**	**Date**	**11/01/2013**
Unit	**Dissertation**		

Proposed project title **Will P v NP be solved in the next 50 years?**

Section One: Title, objective, responsibilities

Title or working title of project (in the form of a question, commission or design brief)

Will P v NP be solved in the next 50 years?

Research an important mathematical question which has profound implications for the field of Computer Science.

Project objectives (e.g., what is the question you want to answer? What do you want to learn how to do? What do you want to find out?):

P v NP is a famous unsolved problem in the relatively short history of computer science. While most scientists believe that P $\neq$ NP, there is no proof for this as of yet. The efforts of mathematicians round the world in the best part of 50 years have failed, and it is worth pursuing whether or not this problem will be solved in the near future. I intend to read about the problem and look at some of the attempts to solve it, forming my own conclusion on whether it will be solved in the near future.

If it is a group project, what will your responsibilities be? **N/A**

Section Two: Reasons for choosing this project

Reasons for choosing the project (e.g., links to other subjects you are studying, personal interest, future plans, knowledge/skills you want to improve, why the topic is important):

I will be studying Maths and Computer Science at University next year and the project will provide a good introduction to the topic. My subjects at A-Level link in well (Maths and Further Maths) but at the same time I will be able to extend myself beyond the topics that are part of the syllabus, especially in the field of Computing which is a subject I have never studied in school. I will also be able to improve my ability to write long discussions, as I am not doing any essay subjects at A-Level. The topic itself is quite important to society as it dictates the future of computers and crucially, their efficiency. With the ever-increasing importance of computers, a solution to the P v NP would make an unprecedented leap forward into computing technology.

Section Three: Activities and timescales	
Activities to be carried out during the project (e.g., research, development and analysis of ideas, writing, data collection, numerical analysis, rehearsal techniques, production meetings, administration, evaluation, preparing for the presentation, etc.)	How long this will take:
Choosing a Project Question / Gantt Chart / Project Proposal Form	2 weeks
Research and Data Collection – Library (IPS and Roehampton University) / Internet / Lectures / Journals	4 weeks
Introduction and Literature Review	12 weeks
Summary of arguments and Discussion	17 weeks
Conclusion and Abstract	4 weeks
Project redraft based on feedback	13 weeks
Evaluation, Review and Appendices	3 weeks
Preparation of PowerPoint and Notes for Presentation	4 weeks
Please refer to Gantt Chart for more details about the task breakdown and timescales.	

Milestone one: Interim Deadline 1 – Project Proposal form, Introduction, Gantt Chart, Literature Review, Activity Log

Target date (set by tutor-assessor): 25/03/2013

Milestone two: Interim Deadline 2 – Project Proposal form, Introduction, Gantt Chart, Literature Review, Discussion, Activity Log, Bibliography

Target date (set by tutor-assessor): 23/09/2013

Milestone three: Interim Deadline 3 – Draft of entire project (5500-6000 words)

Target date (set by tutor-assessor): 07/01/2014

Section Four: Resources

What resources will you need for your research, write up and presentation (e.g., libraries, books, journals, equipment, rehearsal space, technology and equipment, venue, physical resources, finance):

I used JSTOR to access journals on my topic area and NEXIS (a Roehampton University Library search system) for any news articles published worldwide on the P v NP problem. My school library also had access to the New Scientist for some of my articles. I could access most of the articles in the 'Communications of the ACM' magazine online so I didn't require hard copies for them. Unfortunately, most of the textbooks published about the topic are too technical for my level of research. There was minimal use of IT – MS Word to type up the dissertation and other relevant documents, MS Excel for the Gantt Chart and MS PowerPoint for the oral presentation at the end of the project. I did not use any specialist software while doing the project.

Despite being a specialist topic, there are many resources available on the internet; lack of research material should not be a problem while answering my project question.

What topics will your areas of research will cover?

Mathematics, Computer Science, Computer Security, Computability of problems, Algorithm efficiency

Comments and agreement from tutor-assessor

Is the learner taking this project as part of the Diploma? Yes / **No**

If yes, which Diploma are they taking? _____

Comments (optional):

Is project derived from work which has been/will be submitted for another qualification? Yes / **No**

Which qualification (title and unit)? _____

Comments (optional):

I confirm that the project is not work which has been or will be submitted for another qualification and is appropriate.

Agreed: **Vaibhav Krishnakumar**

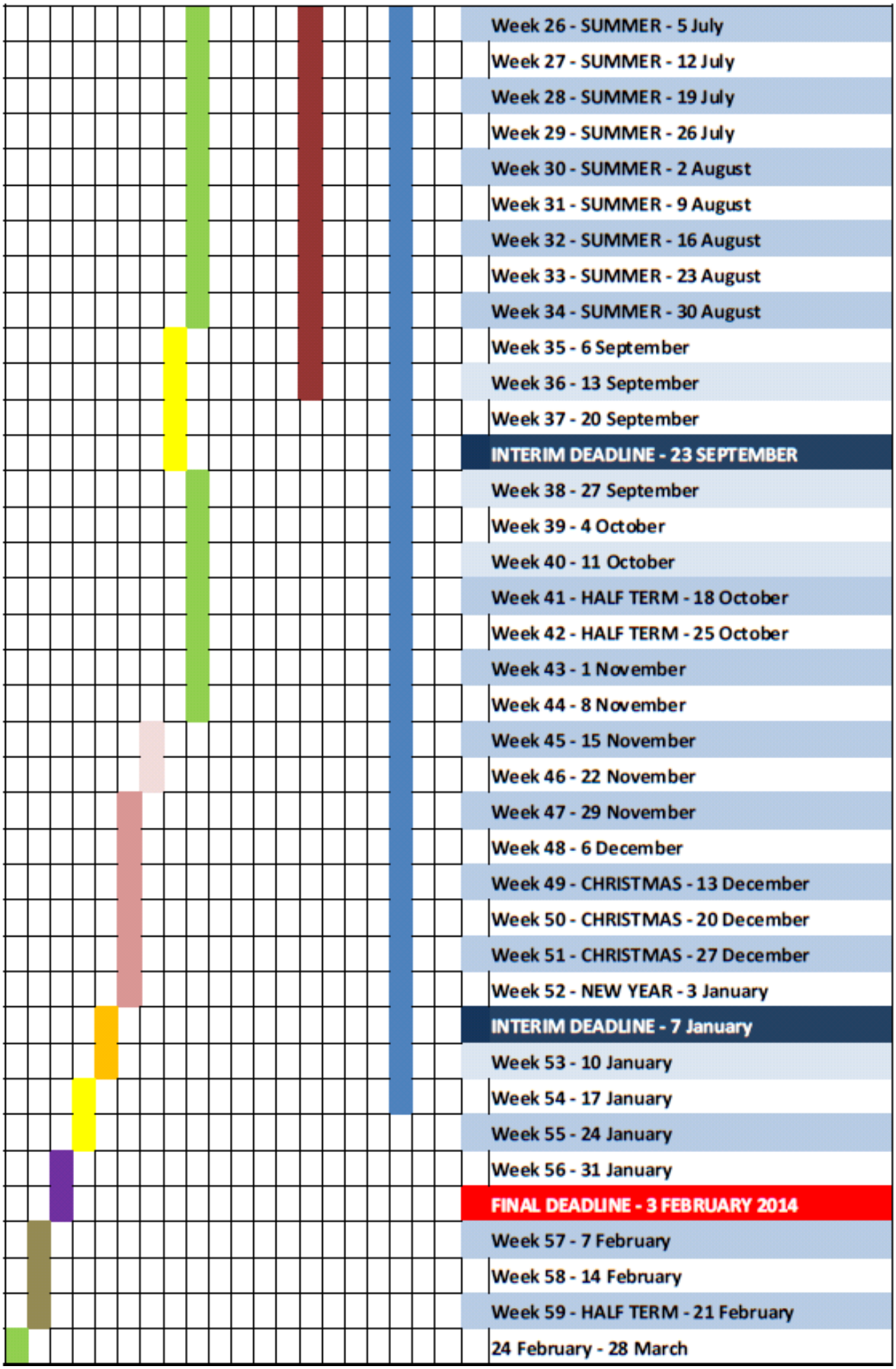
11/01/2013

Comments and agreement from project proposal checker

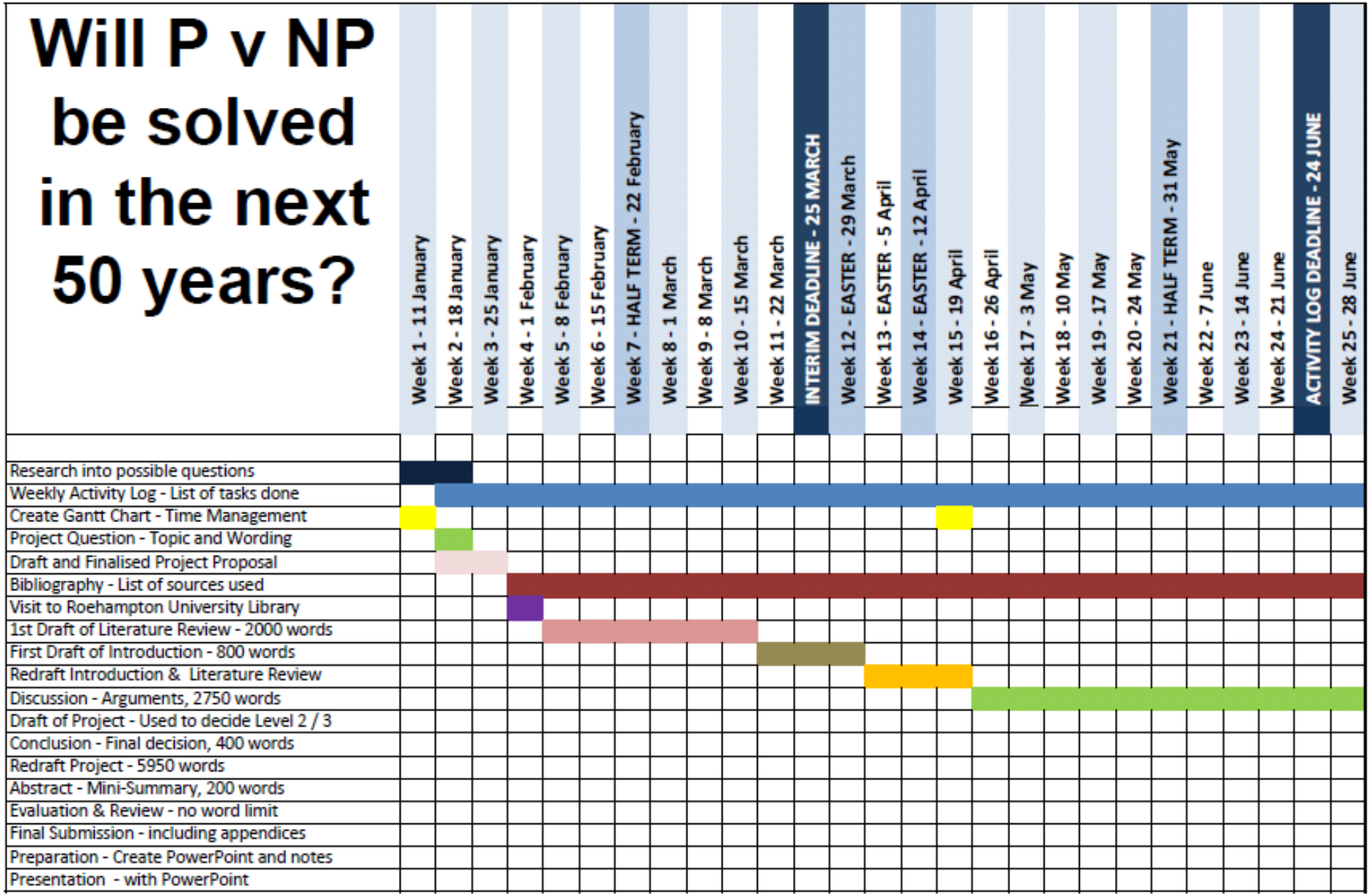
Comments (optional):

I confirm that the project is appropriate.

Agreed: **J Tomlinson**


Week 26 - SUMMER - 5 July
Week 27 - SUMMER - 12 July
Week 28 - SUMMER - 19 July
Week 29 - SUMMER - 26 July
Week 30 - SUMMER - 2 August
Week 31 - SUMMER - 9 August
Week 32 - SUMMER - 16 August
Week 33 - SUMMER - 23 August
Week 34 - SUMMER - 30 August
Week 35 - 6 September
Week 36 - 13 September
Week 37 - 20 September
INTERIM DEADLINE - 23 SEPTEMBER
Week 38 - 27 September
Week 39 - 4 October
Week 40 - 11 October
Week 41 - HALF TERM - 18 October
Week 42 - HALF TERM - 25 October
Week 43 - 1 November
Week 44 - 8 November
Week 45 - 15 November
Week 46 - 22 November
Week 47 - 29 November
Week 48 - 6 December
Week 49 - CHRISTMAS - 13 December
Week 50 - CHRISTMAS - 20 December
Week 51 - CHRISTMAS - 27 December
Week 52 - NEW YEAR - 3 January
INTERIM DEADLINE - 7 January
Week 53 - 10 January
Week 54 - 17 January
Week 55 - 24 January
Week 56 - 31 January
FINAL DEADLINE - 3 FEBRUARY 2014
Week 57 - 7 February
Week 58 - 14 February
Week 59 - HALF TERM - 21 February
24 February - 28 March

Will P v NP be solved in the next 50 years?



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Project Activity Log

Learner Name	**Vaibhav Krishnakumar**	**Learner number**	**6257**
Centre Name	**Ibstock Place School**	**Centre Number**	**11032**
Unit Name	**Dissertation**	**Unit number**	
Teacher Assessor	**Mr Tomlinson**		
Proposed project title	**Will P v NP be solved in the next 50 years?**		

This form should be used to record the process of your project and be submitted as evidence with the final piece of work.
You may want to discuss:

- what you have done (e.g., from one week to the next)
- if you are working in a group, what discussions you have had
- any changes that you have or will need to make to your plans
- what resources you have found or hope to find
- what problems you are encountering and how you are solving them
- what you are going to do next

Date	Comments
14th December 2012	**Describe and give reasons for your activity:** I created a Gantt chart which gave me a clear list of the tasks I had to do and the timescales involved. This will help with the time management of my project once I get further into it.
	What problems, if any, did you encounter? Explain how you overcame these. It was hard to do the Gantt Chart without a template as I had to use the stacked bar function on Microsoft Excel. I later found a template on the internet which helped make it a lot easier.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
11th January 2013	**Describe and give reasons for your activity:** I did the project proposal form over the Christmas holidays, and I had to confirm my project title. I started research into the topic area. This was scary at first as it showed that the project had really started.
	What problems, if any, did you encounter? Explain how you overcame these. My original title was "Is P=NP?" which, when I started researching I found to be too hard to answer with an argument. I changed the project question but I still don't think the question is perfect as it doesn't have ethical considerations. I might refine the question further in the future to help with this.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? Yes, I have changed my project title to "Will P = NP ever be solved?" This will hopefully make it easier to argue in an essay, as I found the first title too hard to argue effectively.

Date	Comments
18th January 2013	**Describe and give reasons for your activity:** I continued with my research and found some good articles on the internet supporting both arguments. I refined myself to only one side of the argument to make the project more logical. One particularly informative source was a research study by a Canadian scientist published in JSTOR.
	What problems, if any, did you encounter? Explain how you overcame these. None
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
25th January 2013	**Describe and give reasons for your activity:** I visited the Roehampton University Library for 2 hours to use their online access journals for more information. EBSCO and NEXIS were both very useful resources, with EBSCO giving access to journals, magazines other research material. NEXIS allowed me to see P v NP articles in newspapers worldwide, and I have saved the many resources on PDF files to review later.
	What problems, if any, did you encounter? Explain how you overcame these. Sometimes, alternate search terms had to be used to find relevant articles. I first searched for P v NP, then P = NP as different sources came up. Taking a lead from one of the articles, I also searched for "Limits of Computing" and "Turing Machines" which are both linked to the topic and are alternate ways of defining the problem.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
Date	**Comments**
15th February 2013	**Describe and give reasons for your activity:** I summed up the research for one side of the argument, and have now found 5 to 6 solid reasons. This allows me to clearly focus on a counter argument for each reason in turn, making my research more structured.
	What problems, if any, did you encounter? Explain how you overcame these. The wording of the question involves predicting the long term future which wouldn't make a good argument. I have changed the title of my project slightly to reflect this opinion.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? Yes. I have changed the wording of my question, so it now reads – "Will P v NP be solved in the next 50 years?" This would allow me to form a better argument as it puts a limiting time frame which is easier to predict than the far future.
1st March 2013	**Describe and give reasons for your activity:** I finished the research and summary of the counter arguments for my project question. Now it is much clearer to set up my project and start writing as I have a rough idea as to the main points I need to make on each side. This will make it easier when I start writing my discussion.
	What problems, if any, did you encounter? Explain how you overcame these. Learning the new referencing system was quite hard at first, as once I summarised an article I usually forgot to add it to the bibliography immediately. This causes problems later as I have to find the article on the internet again to get the author and webpage. But I got used to referencing the sources as I started using more of them.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No

Date	Comments
8th March 2013	**Describe and give reasons for your activity:** I made good progress on my literature review, and my total word count is now 1000 words which almost half way through. I used most of my sources and summarised them, which was useful as it gave me a better indication of how much information I had collected, and how much more I need. Discussing the reliability of the sources was easier for me as I had done similar things for History GCSE.
	What problems, if any, did you encounter? Explain how you overcame these. Rather than being a problem, creating the bibliography was annoying, as I had to note down immediately any information I had used or I would forget to reference it. I started getting used to do this as I used more sources.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No. I am still on target with my Gantt chart.
15th March 2013	**Describe and give reasons for your activity:** Today was another productive lesson, and I am now almost 2/3 of my way through the literature review. The most interesting source I found today was a film about my project question. It is set in an alternate universe, with a thrilling plot. It is an interesting perspective on the topic, and I might watch the film to gain a different insight into the question.
	What problems, if any, did you encounter? Explain how you overcame these. There are no significant problems, although I will have to step up my work rate slightly to achieve the deadline next week. I am slightly behind on my Gantt chart as I am meant to have started my introduction by now. But I can catch up on the work and be back on schedule after the deadline.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
Date	**Comments**
22nd March 2013	**Describe and give reasons for your activity:** I finished my literature review and continued with my introduction. I will need to do some work over the weekend but I have almost finished the first 2000 or so words of my project. The deadline is next week, on Monday the 25th of March. I will be able to submit the work by the interim deadline.
	What problems, if any, did you encounter? Explain how you overcame these. None
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
19th April 2013	**Describe and give reasons for your activity:** Due to revised deadlines, I had to re draw my Gantt chart and re-organise my project. I can now go a bit more slowly in terms of writing my discussion, as I have longer to do it.
	What problems, if any, did you encounter? Explain how you overcame these. The re-planning of my project took time, so I have to get back to research and do extra work to ensure that I'm up to date.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? Yes, the deadlines have been changed and I have drawn up a revised Gantt Chart to show this.

Date	Comments
26th April 2013	**Describe and give reasons for your activity:** I received the feedback for my first submission, and continued working on research and getting my literature review finished. The feedback was very useful as it gave me an early indication of what I need to change to do better in my project. Correcting these errors now is easier than making changes later in the project.
	What problems, if any, did you encounter? Explain how you overcame these. I was meant to start my discussion today, but due to revised deadlines, I had to make changes to my plans. The Gantt chart reflects these changes. I will start my discussion next week and continue over the coming months.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
3rd May 2013	**Describe and give reasons for your activity:** I started the discussion today, and this has helped me consolidate my ideas and use my sources to build the argument. I have decided to split my discussion in three sub parts, which makes it easier for me to write and also helps someone reading it understand better. This is similar to what others have done in the exemplar dissertations.
	What problems, if any, did you encounter? Explain how you overcame these. I didn't know how to start my discussion as it was a new part of the project. It was a "writer's block" moment. However I read a template discussion and saw a few examples before starting my own, which helped the flow of the project more.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
Date	**Comments**
10th May 2013	**Describe and give reasons for your activity:** I continued writing my discussion, and it is surprising that after only using 3 of my sources, I have already covered close to 1000 words of content. The 2750 word limit seems less intimidating now, and I think it is getting easier to write and get into the flow of the argument.
	What problems, if any, did you encounter? Explain how you overcame these. I am still trying to find a moral aspect to my argument, although the movie source which I found which discussed the moral implications of a solution. It is a film which I would preferably like to watch before writing about it in my literature review.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
17th May 2013	**Describe and give reasons for your activity:** I continued work on the discussion of my project, and started the second part of my discussion which looks at the moral arguments to do with P v NP. Being rare for a mathematical problem to have a moral aspect, I only have one source which is the film mentioned earlier.
	What problems, if any, did you encounter? Explain how you overcame these. I cannot watch the movie in school as the server blocks access to video streaming. Also, it has only released in the US so getting a copy can be difficult. Luckily, I have found a good review of the film which describes the plot in detail; hopefully this means I do not have the film myself.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No

Date	Comments
24th May 2013	**Describe and give reasons for your activity:** I have taken a break from my discussion and worked on improving my Literature Review and Introduction today. This has affected my time management but I have left enough time in my Gantt Chart to make adjustments. I used the feedback from the original deadline submission to improve the literature review, but reading back at it I made further improvements and added new sources which I found.
	What problems, if any, did you encounter? Explain how you overcame these. To improve my literature review, I had to stop working on my discussion for the lesson today, which puts me behind schedule according to the Gantt Chart. That is not a problem as I can do some work over half term to catch up.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
7th June 2013	**Describe and give reasons for your activity:** I finished editing the literature review and introduction, and have put both those documents to one side. I started focusing on my discussion to add to the content of it, and I'm on track to finish off one set of arguments for my project question before the summer holidays.
	What problems, if any, did you encounter? Explain how you overcame these. I didn't have any problems this week; I'm back on schedule on my Gantt Chart as well so everything is going according to plan.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No
Date	**Comments**
14th June 2013	**Describe and give reasons for your activity:** With the deadline approaching next week, I read through my activity log and edited it to make it flow better and easier to read. This did not take long, and I continued writing my discussion too.
	What problems, if any, did you encounter? Explain how you overcame these. My discussion is 1500 words long and I need to have written 2000 words by the deadline in September. I may need to do some work over the summer to bring it up to the required level.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? Yes, this is the last lesson for the academic year so I had to make some changes to my Gantt Chart, but I am on track to meet the deadline on the 23rd of September.
4th September 2013	**Describe and give reasons for your activity:** I finished my discussion over the summer holidays, which eases the pressure so that I am no longer hurried by the deadline. Today, I started editing my introduction; I made some changes to the wording and added new information to make the project flow better.
	What problems, if any, did you encounter? Explain how you overcame these. None
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? Although I am now ahead of schedule, I am not editing my Gantt chart now as I might have to make further changes later and it would be easier to make them all at once.

Date	Comments
11th September 2013	**Describe and give reasons for your activity:** I read through my entire project and did some final editing to make sure there are no grammatical errors and that the project flows well. I created the bibliography which was relatively easy as I had all the links and authors stored in a document. I have referenced all my sources. I started work on the evaluation and abstract. I should finish these next week, allowing me to hand in a draft of the entire project by the deadline.
	What problems, if any, did you encounter? Explain how you overcame these. Reading through the entire project made me realise how some of the arguments didn't link up perfectly or where there were flaws in my reasoning. It took time to fix these gaps and hopefully I have a more convincing argument now.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? None
18th September 2013	**Describe and give reasons for your activity:** I finished writing my evaluation and abstract and read through the activity log from start to finish making sure there are no gaps in my explanation of actions. I brought together all my work on one document so that it is easier to submit next Monday. I have created a 'master copy' of the entire extended project which is ready to be submitted to my tutor on the 23rd.
	What problems, if any, did you encounter? Explain how you overcame these. While working on the project, I had saved bits of my document across folders and files on my computer (school and home). It took a while to find all the required documents and create the master copy. Next time, it would be easier to have all work relevant to extended project on one folder.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? None
Date	**Comments**
25th September 2013	**Describe and give reasons for your activity:** Over the weekend, I read through the entire project to make a final check before printing it out. I submitted a copy to my assessor on Monday for the deadline and am awaiting feedback. Once I receive it, I can just continue making the final edits and have a final submission ready by mid-October.
	What problems, if any, did you encounter? Explain how you overcame these. I had intended to print the Gantt Chart separately and hand the two documents in, but I found a better solution. I inserted the Gantt chart directly into the word document, which took some time and a few attempts before I got it perfectly. Now all the relevant components of the project are saved in one big document.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? Yes, surprisingly, I have finished my project ahead of schedule. This interim deadline was meant to be about 60% of the project but I have submitted a draft of my whole project. I am keeping with the original Gantt chart as I might have further changes to make.
2nd October 2013	**Describe and give reasons for your activity:** I received some feedback on my project and made the required changes and edits. I restructured my arguments in the discussion to help it flow better, and merged my conclusion with the discussion and made it slightly shorter. I also reordered the documents based on examples of dissertations online.
	What problems, if any, did you encounter? Explain how you overcame these. My conclusion and evaluation were written as two separate parts of the project, essentially describing the same thing. Editing this, I merged my conclusion and discussion and finished editing the evaluation to make it easier to read.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No, I have finished most of my project and am waiting for the rest of my feedback to finish the edits and changes. I am well ahead of the schedule.

Date	Comments
9th October 2013	**Describe and give reasons for your activity:** I received detailed feedback on my project which was useful as I know what changes I have to make next. Some changes are minor, and I did them during the lesson. I will finish the other, bigger changes next week as they will take longer. I should have a revised draft ready to hand in in 2 weeks.
	What problems, if any, did you encounter? Explain how you overcame these. There were no major problems although some of the changes were hard to make as I had to edit multiple documents (each individual one & the master copy of all documents together). This took longer than expected.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No, I am still ahead of schedule and have until January to finish my draft and make any changes.
16th October 2013	**Describe and give reasons for your activity:** I finished off the major changes I had to make to my introduction and literature review. This took a while as I had to re-write some of the paragraphs and reorder them to make it flow better with the new changes. I made fewer changes to my discussion as my tutor is largely happy with it.
	What problems, if any, did you encounter? Explain how you overcame these. My introduction needed more work as I originally wrote it when I didn't have a full idea of the project. It was much easier to edit it now as I know what I have done and could make the changes.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No, I am finishing of the final edits to my draft, I have half term to make any final minor adjustments.
Date	**Comments**
13th November 2013	**Describe and give reasons for your activity:** I worked through the rest of my feedback and have now finished editing all the documents. I can now start working on my project presentation as I have been told I will be entered for a Level 3 project. My dissertation is done and I can focus on the presentation next.
	What problems, if any, did you encounter? Explain how you overcame these. There were no real problems as I have finished the dissertation aspect of the project.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No, I am still working ahead of schedule and have until January to submit the final draft.
4th December 2013	**Describe and give reasons for your activity:** I worked on creating a PowerPoint and the notes for my presentation. Most of the work is done; the little that is remaining I can finish over the Christmas holidays. So the entire project, with the dissertation and the presentation, will be finished in time for the final deadline on the 7th of January.
	What problems, if any, did you encounter? Explain how you overcame these. I finished the dissertation ahead of schedule but as I missed a few lessons due to other commitments, I have to do some work over the holidays to finish the presentation. But due to the extra buffer time, I am still on target to meet the final deadline so there is no time pressure.
	Have you changed or adapted your original plans? (Yes/No) If Yes, why and in what way? No, I am back on schedule and the whole project will be finished by the first week of January.

ABSTRACT

P v NP is a major unsolved problem in the fields of Mathematics and Computer Science. Put simply, it asks whether any problem whose solution can be verified easily can also be solved easily. While intuitively the answer seems no, there is no rigorous mathematical proof that sets this out. A solution to the problem has far reaching consequences in many different fields of science, from cryptography to algorithm research and logistics. In my extended project, I look at whether a proof that P = NP or P ≠ NP will be found in the next 50 years. I consider different arguments such as faster computers, increased awareness of the problem, moral repercussions of a solution and the importance of the problem with regard to making technological advances in the future. Having looked at a variety of sources and considered both sides of the discussion, my conclusion is that P v NP is unlikely to be solved in the next 50 years. However, I believe that the information gathered and the research done in the journey en route to a proof will be much more valuable to science than simply a proof for the P v NP problem.

WORD COUNT - 198

INTRODUCTION

As computers get more advanced, components have to be made even smaller, and problems have to be solved even faster. The reduction in the size of microchips is already limited. Moore's Law, conceived by Gordon Moore (co-founder of Intel), tells us that every 18 months, the number of transistors on a processor doubles (Intel, 2005). At this rate, by 2040, components will reach a minimum size. This limit will not be due to human capability but the laws of physics themselves; making a transistor smaller, although possible, would make it redundant as the information transfer would no longer be reliable. So, within the next 30 years, sizing components will be at a stalemate. However, the other advancement - reduction in problem solving time will still be possible (Linden, 2013). The key to unlock the secret is solving the P v NP problem. A solution to this can tell us whether humans are efficient problem-solvers, or if there are faster algorithms to solve problems which we have not found yet. Thus, it is a key question, with huge implications for the future of computing. My project question for the Extended Project dissertation is "Will P v NP be solved in the next 50 years?" In the introduction, I will explain what the P v NP problem is and give an example to make it easier to understand. Although a detailed knowledge of the mathematics behind the problem is not required, a basic explanation would be useful to follow the arguments outlined.

The P v NP problem was conceived by Stephen Cook in 1971. In his formal description of the problem, Cook says that the P v NP problem asks "whether every language accepted by some nondeterministic algorithm in polynomial time is also accepted by some (deterministic) algorithm in polynomial time."(Cook and Levin, 1971) Put simply, the P v NP problem asks whether every problem whose solution can be verified easily can also be solved easily. Intuitively, the answer seems obvious; most scientists believe that $P \neq NP$ but there is no formal proof that outlines this. The best effort of scientists in the last 40 years has not yielded a solution, and it is widely regarded as one of the most important unsolved problems in the field of Computer Science.

The best explanation of the P v NP problem is through examples, and there are many of them from different sources. The factorisation of large numbers is one such problem. To split 39 into its prime factors would take little time and effort (3 x 13). A bigger number, 786 for example, would take longer but a computer can still do it in fractions of seconds (2 x 3 x 131). However, a number with over a million digits which needs to be factorised would take significantly longer. If given a possible set of values, it would be relatively easy to check if the solution works – a computer could multiply the numbers together and produce an answer which can then be compared with the original number. However, for the computer to come up with the two numbers by itself would be near impossible. A brute force search for the answer has an expected running time longer than the age of the universe! The P v NP problem asks whether there is an easier way of solving this problem, i.e. whether there exists an algorithm with mathematicians and programmers have not been ingenious enough to come up with yet. This particular example, prime factorisation, has huge implications for internet security. Most encryptions on the internet, such as credit card details for purchases, are secured using the RSA encryption method, which is based on the inability to factorise large prime numbers quickly. So if an easy way is found to factorise these numbers, internet security is compromised at the highest level.

My project links well with my A Level subjects – Maths and Physics – and also prepares me for my university course in computer science. The P v NP problem is very complicated and I will only be looking at one aspect of it. However, it would still be useful to learn more about a topic that is pivotal for the future of computers. Moreover, due to my choice of A-levels, I do not write many essays; conducting my own research and writing an extended piece of dissertation will be a useful skill in itself.

In my literature review, I will introduce the various sources that have contributed information to my research question. One of my most important sources was the ACM magazine, which publishes articles related to computing on a monthly basis; articles by Fortnow, Lindley and Rosenberger proved very useful in backing up my arguments. In my discussion, I will consider the impact of technological advances, improved awareness and the morality of the problem, as well as the implications of a solution being found by 2063. The morality aspect is unique as it is rare for a mathematical problem to have ethical considerations; an interesting source which highlights this is a movie about the P v NP problem. To keep the dissertation unbiased, I have not yet formed an opinion on my project question. This will allow me to give a fair decision at the end of the project as opposed to simply finding evidence to back my arguments.

WORD COUNT - 888

LITERATURE REVIEW

In the introduction, I described the P v NP problem in a simple manner which is easier to understand than the description of Cook and Levin. I will now be looking at some of the sources that I have used for my research into this topic area. The P v NP problem is widely recognised as an open question by computer scientists around the world. As a result, there is no lack of evidence available on the internet and in journals. However, due to the technical nature of the problem, most of the books on the topic are textbooks which are too complicated for my level of research. The majority of my sources are either journals or articles on the internet; while this doesn't necessarily make them less reliable, it would be good to have multiple sources backing the same point of view. I will be analysing most of my sources in the literature review, although some of the smaller or less informative ones will be mentioned in my discussion later.

Most sources agree that the P v NP problem is still an open question, i.e. an answer exists. One of my most useful articles for this topic is by Lance Fortnow which appears in the magazine Communications of the ACM (Fortnow, 2009). He opens by saying: when I was asked "to write this piece for Communications, my first reaction was the article could be written in two words: Still open." The article talks about the optimism for a quick solution to the problem due to advances in technology and computing when it was first conceived. But that optimism has now faded. The article informally describes the P v NP problem and also looks at attempts in recent history to solve it, one way or another. This article contains a lot of information, but Fortnow states that is not too technical; it is written for a layman's audience. This is important in improving the awareness of the problem, so that more people understand what P v NP actually is. The article seems unbiased and openly admits the mistakes made by the early computer scientists who tried to solve the problem. It appears in the ACM, a magazine whose reliability I will be looking at later as most of my sources are from this publication.

Fortnow's opinion is backed by the results from the poll conducted by William Gasarch, a professor of computer science at University of Maryland. He asked a group of researchers their opinion on the P v NP problem, in 2002 and again in 2012 (Rosenberger, 2012). In the 2002 poll, 70% of the respondents had a definitive opinion on the problem, as opposed to 30% who either didn't have an opinion or claimed that it didn't have a solution. However, these statistics have to be used with caution as they are the opinions of research scientists, not facts. The reputation of the scientists gives some credibility to their statements, but even amongst professionals, there are differences in opinion. As a source of information, this article is quite useful in gauging the opinion of the 'public'. Rosenberger's article too appears in the ACM, so I will consider its reliability later in the review.

An article by 'the Explainer', a group of people from the MIT News Office, gives reasons why this problem is important for computing. (Targeted News Service, 2009) The fundamental question in computer science is "how long will it take a computer to perform a certain action?" The P v NP problem helps answer this question. The article also mentions other fields in computer science which are affected by the problem, such as artificial intelligence, quantum computing and cryptography (particularly internet security). This

article is also quite informative about the wording of the P v NP question and explores the significance of the letters P and N in this problem. It was published for newspaper release to a wide audience, and is likely to be factually correct. Moreover, MIT is a world renowned pioneer in computer science and this report was written by experts in the field. This is a reliable source which could prove useful later in my discussion as it addresses the implications of a solution to P v NP being found.

The P v NP problem received big publicity due to the article by Fortnow, mentioned earlier. The New York Times covered this news and published an article which mentions some of the consequences of a successful solution. (Markoff, 2009) Amongst other benefits, the efficient design of transistor chips and a solution to the travelling salesman problem depend on the solution to the problem. The article mentions that a solution that P is equal to NP would lead "to a burst of new economic and technological productivity." This article was written for the public audience and is not technical or mathematical. It is useful as it provides some good examples of problems in the P and NP categories and looks at the future of progress in computer science more generally.

An American newspaper, the Pittsburgh Post-Gazette gives information on a scientist who has contributed a lot to the P v NP problem. (Templeton, 2007) Dr Steven Rudich is a computer science professor at the Carnegie Mellon University. He is one of the scientists of the opinion that no one will ever solve the problem. His work in the field of computer science is well recognised and he has advanced our understanding of the P v NP problem considerably, despite not having directly solved it. His main contribution (for which he shared the Gödel Prize in 2007) was to prove that "natural proofs", common in computational complexity, would not work in trying to solve the P v NP problem. This was a widely followed approach before Rudich's proof was published, and gives a strong argument against my project question, i.e. P v NP will not be solved in the next 50 years. However, Rudich is one of the few scientists who have this opinion. While his proof gives a strong reason to back him, the majority of professionals believe otherwise. This contradiction is useful for me to compare the two sides of the argument. The article is quite reliable, and was published in a recognised newspaper.

During my visit to the Roehampton library, I looked through the majority of articles on P v NP directly, but one of the new search terms that came up was "computability" which was mentioned in quite a few of the articles. Computability is linked to P v NP as it shows the most efficient way of computing a problem. This new search term gave me some more relevant results, and one of them was about the limits of computers. This article, (Lindley, 2008) talks about the uses of algorithms in the real world, such as economics, game theory and the creation of Sudoku puzzles. It gives more examples of NP problems and has references from many other eminent scientists, mostly based in Princeton University, USA. It is quite reliable as it has been peer-reviewed and been published in a reputable magazine. The article provides good arguments about the practical implications of a solution to the real-world.

Communications from the ACM has proved a very good source of information, as the majority of my articles come from this magazine. This magazine is published monthly by Association for Computing Machinery (ACM), a company dedicated to advances in

computing. The magazine is sent out to all members of the ACM, which includes over 100,000 people from many leading IT companies. This magazine has been published for the last 50 years and is well-regarded by computer science professors. It is quite a reliable source of information. It is worth noting that the magazine is published for a fee, so there is a monetary interest for the company; however, this is unlikely to affect the credibility of the information published.

Apart from the magazine, online journals also provided useful information. One of the journals I found on JSTOR was originally published by the Canadian Journal of Philosophy, and part of the journal has an interesting view on the P v NP problem (Thagard, 1993). The author gives reasons why so many scientists believe that P ≠ NP when they should, in theory, approach the problem with an open-mind. It is useful to gauge the opinions of professionals who know the P v NP problem very well and have often dedicated their entire careers to solving it. Being a formal publication, it is thoroughly researched, referenced and validated, giving reliable information about the thought processes of these scientists.

One of the problems I identified with my question early on was the lack of a moral theme to the topic. Morality is quite important in an argument and at first glance, P v NP didn't seem to have any moral issues. However, I was surprised to come across an article in the New Scientist about a mathematical film which discusses the P v NP problem (Aron, 2012). The film "Travelling Salesman" is set in a fictional world where P = NP; it discusses the moral consequences of this alternate universe. The thriller film has been highly commended by critics, and even compared to James Bond movies! A film with a mathematical undertone has rarely caused such a stir. The key reliability aspect of the film is of course that it has been made to entertain and make money. Some of the consequences are exaggerated and unrealistic. But even keeping this in mind, the film might be a useful source to evaluate the morality of the P v NP problem and question whether we should be looking for a solution in the first place.

A controversial source which I used was a paper by Dominic Cummings titled 'Some Thoughts on Education and Political Priorities' (Cummings, 2013). Cummings was an adviser to Michael Gove, the secretary of state for education, and his word carries some weight in the world of politics. He believes that politicians should focus more on science and that changes should be made to the school syllabus to make it more accessible to students. He uses various examples to illustrate this, one of them being the P v NP problem. He argues that further research into this topic is essential to make mankind more efficient as a race. The paper gives some of the consequences of solving the P v NP problem in more practical terms and discusses the efficiency of algorithms more generally. It was considered controversial as he published the paper a few months before resigning from office, amidst suggestions that it was written as an attack on MPs and their policies. Setting aside the political intentions that Cummings had while writing the paper, the scientific information it provides is useful for my project.

Finally, I came across an article mysteriously titled 'Machines of the Infinite', published in the Scientific American (Pavlus, 2012). Although at first glance the article seemed unrelated to my project, further reading revealed some relevant information. The article talks about the P v NP problem from a historical perspective. Gödel and von Neumann, both famous

mathematicians and computer scientists exchanged letters while discussing the P v NP problem and the potential of a computer equipped with the power of solving any problem in the shortest of times (a Turing machine). The article describes the P v NP question as an organising principle of computer science and discusses the implications of a computer ("infinite machine") with such powerful problem-solving capabilities. It also talks about an optimism for a quick solution in the 1970s (and one student's foolish bet for an ounce of gold that P v NP would be solved by 2000) and how P v NP links to the modern age with Google's search algorithms for example. The article is very informative as it has the content of letters exchanged between two of the most eminent people in the field of complexity theory. The article is quite reliable and has been published in a reputable magazine. Despite being interesting and providing a lot of information, it is of a highly technical nature, making it less useful than some of my other sources.

I have summarised a majority of the sources that have been useful for my argument and will be using them extensively in my discussion. The sources, on the whole, are reliable and provide useful information. The ACM, New Scientist and Scientific American are reliable magazines published widely and recognised by eminent scientists as credible sources of information. The main arguments of technological advances, improved awareness and morality are all mentioned as well as the implications of a solution. With such a wide range of information now at my disposal, I am suitably prepared to start my discussion. By adding my own arguments to the content of these sources, I will now go onto discuss my opinion on the project question.

WORD COUNT - 2154

DISCUSSION

The question of P v NP has been 'open' for the last 30 years and many eminent computer scientists have done research on the topic. While there is no rigorous proof, many scientists have an opinion on the question. Their views might shape those of their students at universities around the world, leading to a widely accepted consensus, but no mathematical proof. As of now, the majority of the scientific community believes that P ≠ NP (Rosenberger, 2012). In the poll conducted by William Gasarch in 2012, 81% of the computer scientists who answered believed that the two set of problems (P and NP) were not equivalent. In the same poll, only about half the people (53%) believed that a solution would be found by the end of the century. Although this paints a bleak picture, what is worse is that optimism is waning. Gasarch had conducted a similar poll in 2002; in that poll, only 61% believed that P ≠ NP and 62% of the computer scientists believed that there would be a solution to the problem by 2100. This drop in confidence in a period of only 10 years highlights the difficulty of the problem. This stark view of reality is echoed by Professor Gasarch's own views (he didn't participate in the survey). He believes that P ≠ NP and a proof would take 200 to 400 years.

Looking at the opinion of the scientists, the future looks gloomy. However, as Plato's famous quote says, "necessity is the mother of invention". The need for a faster, more efficient way of solving problems may become too important to ignore and an increased effort into the P v NP problem could lead a solution sooner than expected. The importance of the problem cannot be underestimated. As Lance Fortnow wrote in the ACM Magazine (Fortnow, 2009), if P = NP was indeed proven, it would "make the whole Internet look like a footnote in history." If after 2040, we need to make processors more powerful, a solution to P v NP will be highly useful and there is a good chance that a solution will be found to help advances in technology. For professional mathematicians, the fame of solving this problem will be enough to motivate them to find a solution. However, in addition to the scientific urgency, the Clay Mathematics Institute, Massachusetts has provided a monetary incentive too. At the turn of the century, in May 2000, the Clay Institute came up with a list of seven unsolved problems from the 1900s, and gave a formal announcement of a $1 million prize for a correct solution to each of the problems (Clay, Institute, 2013). In the 13 years since, only 1 of the problems has been solved (the Poincare Conjecture, solved by Grigoriy Perelman). Apart from giving an indication of the difficulty of the problems, this also means that there are 6 other problems worth $1 million each to solve – the P v NP being one of them. Could amateur mathematicians solve the problem solely motivated by the money? (This is not to say that *all* amateur mathematicians attempt mathematical problems simply for the money. Indeed, Perelman - an amateur - refused the cash prize, which was clearly not his motive.)

The difficulty of the P v NP problem cannot be underestimated, but the question remains: Can an amateur solve it? The prize money is aimed at younger, less known mathematicians and is intended to further their interest in maths. In his blog, Professor Lipton of Georgia Tech shares his view on the millennium problems (Lipton, 2013). One of the first requirements for a mathematical problem to be solved by an amateur is that it should be approachable. As Lipton mentions, only 2 of the Millennium problems can be understood with relative ease by a layman audience, P v NP being one of them. The other millennium problems, such as the Hodge conjecture for example (Which cohomology classes in $H^{k,k}(X)$ come from complex sub- varieties Z?), require a technical background to simply understand the problem statement. As Lipton goes on to mention, one of the biggest problems for an amateur is a lack of formal training and this is essential for a mathematical proof to be accepted in professional circles. This does not mean that the amateur cannot prove P v NP;

it is a case of being clear enough to convince a professional mathematician that the proof is valid. Without formal training in writing proofs (which very few amateurs receive), their work is often ignored even if they have valuable contributions to make. Perhaps we cannot expect something like the story of amateur mathematician George Dantzig (who reputedly solved two unsolved problems in statistics on the assumption that they were set as homework tasks), but as Lipton concludes, there are definitely many ways that amateurs can advance our understanding.

It is not often that a mathematical problem has a moral aspect to it but such is the power of the P v NP problem. One of the most interesting sources I came across was a movie titled 'The Travelling Salesman' which released in 2013. It is set in a fictitious world where P = NP. Briefly, the film is about 4 top mathematicians who have proven that P = NP. The US Government, for obvious reasons, is offering to pay them off and retain the solution to use it for military purposes. However, the mathematicians question the morality of their action and one of them refuses to hand over his part of the solution. The thriller received good reviews although mathematicians criticised it for over-dramatizing the effects of a solution. In his article published in the New Scientist (Aron, 2012), Aron looks at the moral repercussions of P = NP as shown by the movie. As with any scientific breakthrough, there is the good and the bad. Advantages of living in such a world include the eradication of AIDS and cancer through high speed gene sequencing or improving logistics at Heathrow's Air Traffic Control tower making the future safer than ever. However, the P v NP problem also has the capability to crack through cryptography at the highest level, with internet security compromised or financial systems vulnerable to attack. The movie brought great publicity for the P v NP, especially among the amateur circles. With increased awareness of the problem and its implications, more people are likely to try to solve the problem. This could increase our knowledge and understanding of P v NP, even if it doesn't lead to a proof.

Considering the negatives of P = NP, should we be looking for a solution at all? The popular opinion is that it is unlikely P = NP, but that shouldn't be the view of a researcher trying to solve the problem. A person trying to solve the problem should be unbiased, and try and explore various alternatives as opposed to simply trying to prove his or her opinion. But as Fortnow mentions in his article (Fortnow, 2009), computer scientists in the 1970s were convinced that P ≠ NP and many researchers wanted the glory of proving it. In the rush for the solution, the problem was not approached in a systematic manner. With the advance of computers, the problem gained importance, but there was little attempt to consider the case of P = NP. This would have been detrimental, especially in the early stages of the problem. This initial rush can have long term consequences and compromise a solution in the next few years. A tiny detail missed in those years could have provided a key to the solution. Lipton too agrees with this (Lipton, 2013); in his blog, he wishes that one could forget the difficulty of the P v NP problem and start afresh, arguing that this could potentially have a significant psychological advantage.

Advances in technology have occurred thick and fast in the last 30 years, and will doubtless continue for many years to come. However, does a faster computer necessarily mean that a solution will be found? Not according to Andrew Wiles, the Cambridge mathematician most famous for proving Fermat's Last Theorem. His belief is that a computer is useful for ruling out some cases in maths problems, but not for proofs. In fact, all his working to prove Fermat's theorem involved no computers or internet, and his "most useful tools were pencil and paper." Similarly, in P v NP, a computer can only do an exhaustive search for a solution to the problem. Returning to the prime factorisation problem mentioned in the introduction, the computer will try all possible factors until it gets to the correct solution. Such a search for the answer is highly ineffective and could potential take billions of years. So while technological advances will result in a faster computer, this does not necessarily mean that a

solution to P v NP will be found. Fermat's Last Theorem took 300 years to be solved, and P v NP might well take the same time, if not more.

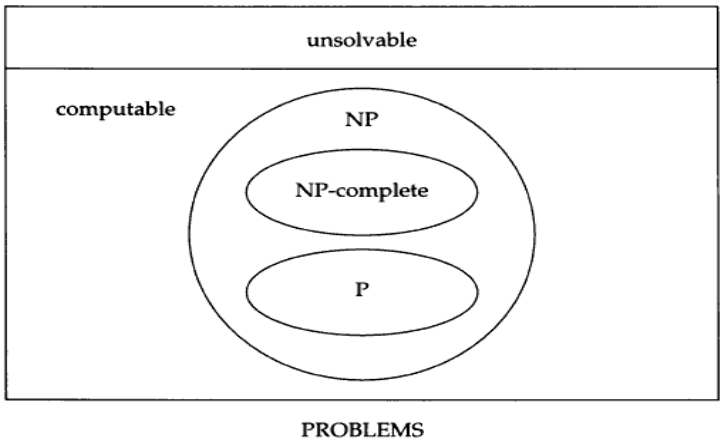

Figure 1. Composition of the class of problems, on the assumption that P≠NP.

The diagram above, taken from a journal published on JSTOR (Thagard, 1993), shows how mathematical problems are classified, assuming that P ≠ NP (Figure 1). Among the problems that have a solution (computable problems), some have solutions that can easily be verified (NP). Within this set, there are some problems which can also be easily solved (P). To prove that P ≠ NP, it is only required to show that there is at least one problem which can be solved easily, but whose solution cannot be verified in a shorter time period (i.e. more easily). This is called proof by contradiction, most famously used to prove the irrationality of $\sqrt{2}$. Such a proof would require much less time and effort, but although many scientists have tried to find such a mathematical problem, there has been no success. Referring back to the diagram, there is a class of problems called NP-complete; this is a class of problems of equal difficulty; a computer would take the same amount of time to solve all the problems in this category and the solutions would be similar, if not the same. Although solving an NP-complete problem does not prove P v NP directly, it would be a huge step forward in the right direction. One of the most famous NP-complete problems, which also lends its name to the P v NP film mentioned earlier, is the Travelling Salesman problem. NP-complete problems are notorious for being deceptively hard, and this problem is no exception. Put simply, the Travelling Salesman problem (a.k.a. the Chinese Postman problem) asks: Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city? (Pierce, 2013) Since its conception in 1930, numerous scientists have attempted to solve this general problem, in vain. If P = NP, this problem could be solved more easily.

It is amazing that there is so much potential locked in one equation and the key is waiting to be found. Considering the two cases, if it is proven that P = NP this would mean that there is an easy way to solve some of the hardest problems in mathematics and computer science which humans (programmers) have not been clever enough to come up with. The biggest impact would occur in the field of cryptography, where the commonly used RSA cryptography would crumble, as large prime numbers could, in theory, be factorised easily (Sudan, 2010). This leads on to internet security, and the failure of credit card payments, financial systems being vulnerable etc. However, the advantages would be superior logistical systems – a better transport industry, optimisation of airports through safer air traffic control, better organisation of communication grids and internet networks. P = NP would also lead to faster ways to solve algorithms with more variables, so weather

predictions could be made more accurate or tsunamis and earthquakes could be predicted more easily. The list is endless. On the other hand, if P ≠ NP, there wouldn't be much practical change. The consequences would be far less exciting and the benefits to the working of the world wouldn't be extreme. (Pavlus, 2010) However, in addition to making the person with the proof $1 million richer, the benefits to theoretical mathematics and computer science would be huge. New fields of research will be opened and a proof would also have a profound impact on existing fields such as AI, game theory and algorithm research. If P ≠ NP, NP-complete problems such as the Travelling Salesman problem could take many more years of research before a satisfactory solution is found. The question to answer is if it is worth the compromise, whether the advantages of having a proof either way outweigh the disadvantages of not having a proof.

In my opinion, we should continue to persevere and try to find a proof for the P v NP problem. Although the problem is based in maths and computing, it has far reaching consequences in other fields of study. Even if it takes more than 50 years to reach a conclusion, the research done and the information gathered during the process will be very useful in advancing these fields of study. (Cummings, 2013) Many computer scientists have spent their whole careers on the P v NP problems, and their work cannot go for a waste. However, deciding that we should continue to search for a proof is the easy bit. When Gasarch was asked what he thought about the future of the P v NP problem, he gave this worrying answer: "My impression is that we are really at a standstill. There has been no progress and there is no real plan of attack." (Rosenberger, 2012) The question "What next?" remains unanswered, making it highly doubtful that a solution will be found by 2064.

While the future doesn't look optimistic, there is one reason which is going in favour of a relatively quicker solution for P v NP than other problems. In 2010, an Indian-born HP Labs employee based in the USA sent an e-mail to top computer science professors (Katz, 2010). Attached was a paper, titled simply "P ≠ NP". Unfortunately, within 1 month, there was enough evidence to suggest that the proof was fundamentally flawed. Last heard of in August 2010, Deolalikar was returned his draft and he says he is still working on correcting the mistakes highlighted. The validation of a proof for any mathematical problem, let alone one of the Millennium problems, will take a long time. We have to wait and see if Deolalikar manages to correct the flaws in his argument and return the proof for peer reviewing. However, based on reports on the Internet and other rumours, it is unlikely that Deolalikar will manage to fix the proof.

When Deolalikar's attempt at the proof became public, New Scientist, a reputable magazine, responded by conducting a study to see how long, on average, mathematical problems took to be solved. The record holder is the Honeycomb conjecture, which was conceived in ~AD 350 and took more than 1500 years to solve. Some of the longer lasting problems include the Poincare Conjecture - a Millennium problem - and Fermat's Last Theorem. While announcing the results, New Scientist produced the graph below to show their findings (Figure 2 – Martel, 2011). According to the graph, 53 years after a problem is conceived, there is a 50% chance that it is solved. The P v NP problem, being only about 45 years old, still has a long way to go to get a higher probability of a correct solution. Based on these findings, the likelihood of Deolalikar's proof being right is low.

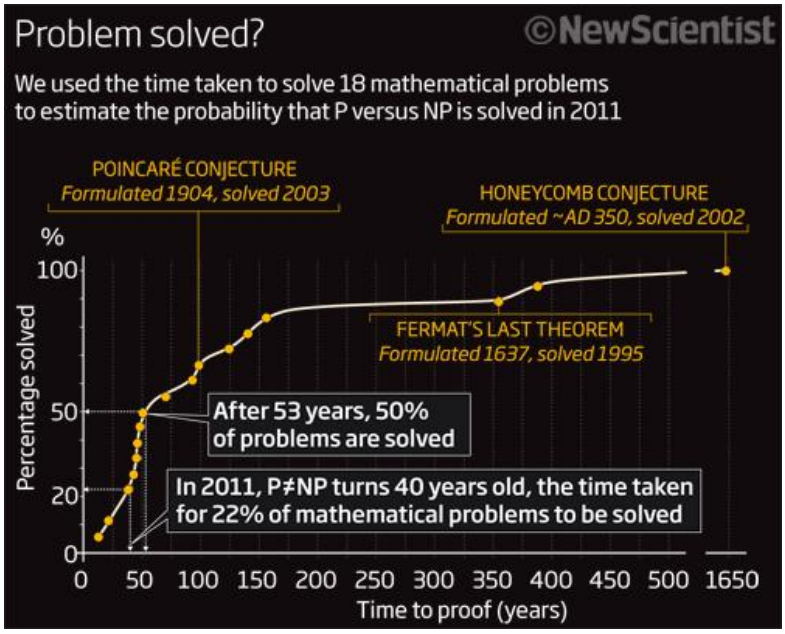

Figure 2 - New Scientist / ASP

Despite no success in the last 40 years, P v NP remains an interesting question. It is one of the few millennium problems which is well-known outside of mathematical circles; events such as Deolalikar's attempted proof, coverage by magazines such as the New Scientist and even a film made about help more of the public know about its existence and importance. As argued earlier, this is good news; if more people know about the problem, more people will try to solve it. The importance of P v NP cannot be underestimated as we have just entered the digital age. With computers more important than ever before, and growing more important by the year, P v NP has huge implications for the future.

Having considered both sides of the argument, and taking into account that P v NP is one of the 6 hardest problems in the world I have formed the opinion that although P v NP will be solved, it is highly likely that a proof will take more than 50 years. Although this result is disappointing to find out, the evidence for this side of the argument both outnumber and outweigh the evidence for P v NP being solved by 2060.

The importance and difficulty of the P v NP problem cannot be underestimated. Many mathematicians have spent their entire lives focusing on this one problem, often helping advance our understanding but not quite getting to a final solution. It is unlikely that P v NP will be solved within the lifetime of one of these research scientists. This is backed up by New Scientist's graph of mathematical problems. The only other millennium problem on the graph took just under 100 years to be solved, and P v NP might have to wait longer.

Often, mathematical problems lie dormant for many years and then are suddenly solved by a flash of inspiration. Examples of problems solved in this manner include Poincare's Conjecture, Fermat's Last theorem, the Honeycomb Conjecture and others. While P v NP could have a similar pattern, I believe we do not have a sufficient understanding of complexity theory at the moment to solve the problem. Fermat claimed he had a proof for his theorem in the 17th century, but Wiles' proof involved topics 'invented' much later. A proof for P v NP published in the next century might contain new, unknown maths that we have not 'invented' yet.

So, linking back to my introduction, does a failure to find a solution to P v NP by 2040 stop technological advances? I don't think so. As history has taught us, when humans are confronted with a problem, they find a way of solving it. In fact, research has already started into something that can make computers faster without making transistors smaller; scientists are investigating the idea of quantum computers and their commercial viability. The prime factorisation problem, mentioned in the introduction, is an NP problem for digital computers. However, it is a P problem for quantum computers, i.e. it can be solved much more quickly. (Linden, 2013) So while we may not have a proof for the P v NP problem, advances in quantum computing will ensure that computers will get faster.

The final question that remains unanswered is of course, when? If not in the next 50 years, when will P v NP be solved? In my opinion, Gasarch's views are a bit extreme; 200-400 years seems a long time, even for a problem as difficult as this. However, I still believe it will take us until the next century to be at a sufficiently advanced stage in mathematical research to solve P v NP. But does it really matter? I believe that getting a final solution, a proof that P = NP or otherwise is not as important as the data gathered in the process. While not getting a proof would count as a 'failure' in the public eye, I think mathematicians would still gain a huge amount from simply *trying* to solve the problem; the amount of information generated would sustain research in numerous fields of science for years to come. And with further research, progress would continue irrespective of P = NP or P ≠ NP.

WORD COUNT - 3340

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EVALUATION

At the start of my project, my aim was to learn more about the P v NP problem as it is something I will be studying in detail in my first year of university. Reflecting back, I have definitely achieved this objective. However, I have got so much more than simply learning about a topic of maths. This is the first time I have done a piece of research based work, and the dissertation is the longest piece of writing I have ever done. Despite being apprehensive at the start of the project (the idea of writing 6500 words was daunting), I feel much more confident about writing extended essays now.

I have learnt a lot throughout the project. I built on my history source work skills in my literature review, as I read through and analysed a huge variety of resources. The skill of filtering out irrelevant content and focusing on what is important for my project was very useful; this will help me process large quantities of data much more easily in the future. The type of sources I had proved to be a limitation; I had no original research and depended almost excessively on online sources. The nature of my topic is such that I couldn't rely on books as the topics are too technical for my level of understanding. One of my most interesting sources for the project was 'The Travelling Salesman' movie. I was very surprised to find an action thriller film with the mathematical problem I was studying at the core of it! I look forward to watching the film in cinemas once it releases to the public, although the article in New Scientist did reveal most of the plot.

One thing that I didn't find particularly useful was my Gantt chart. Although it gave a good indication of the deadlines at the start of the project, once I had started writing my discussion I found it hard to stick to the plan I had created. I often jumped around tasks and had to make adjustments to the chart throughout my project. While I did meet all my deadlines relatively comfortably, I have to admit that the Gantt chart didn't play a big part in helping with that. If I were to do a similar project in the future, I would perhaps create a more realistic Gantt chart with enough "buffer time" to jump to different tasks and incorporate mistakes or major changes.

Overall, the extended project has been a very rewarding experience. The process of researching a topic of my own choice allowed me to do something which I enjoyed while sufficiently extending me beyond the school syllabus. I learnt a lot about the P v NP problem and this area of maths and computer science; it was also a useful experience to undertake a research project and write an extended piece of writing. I will continue to read around this topic further and look forward to studying the P v NP problem in much more depth at university.

WORD COUNT - 506

PRESENTATION