

DB-SCAN

Density-Based Clustering



Clustering based on density (local cluster criterion), such as density-connected points

Each cluster has a considerable higher density of points than outside of the cluster

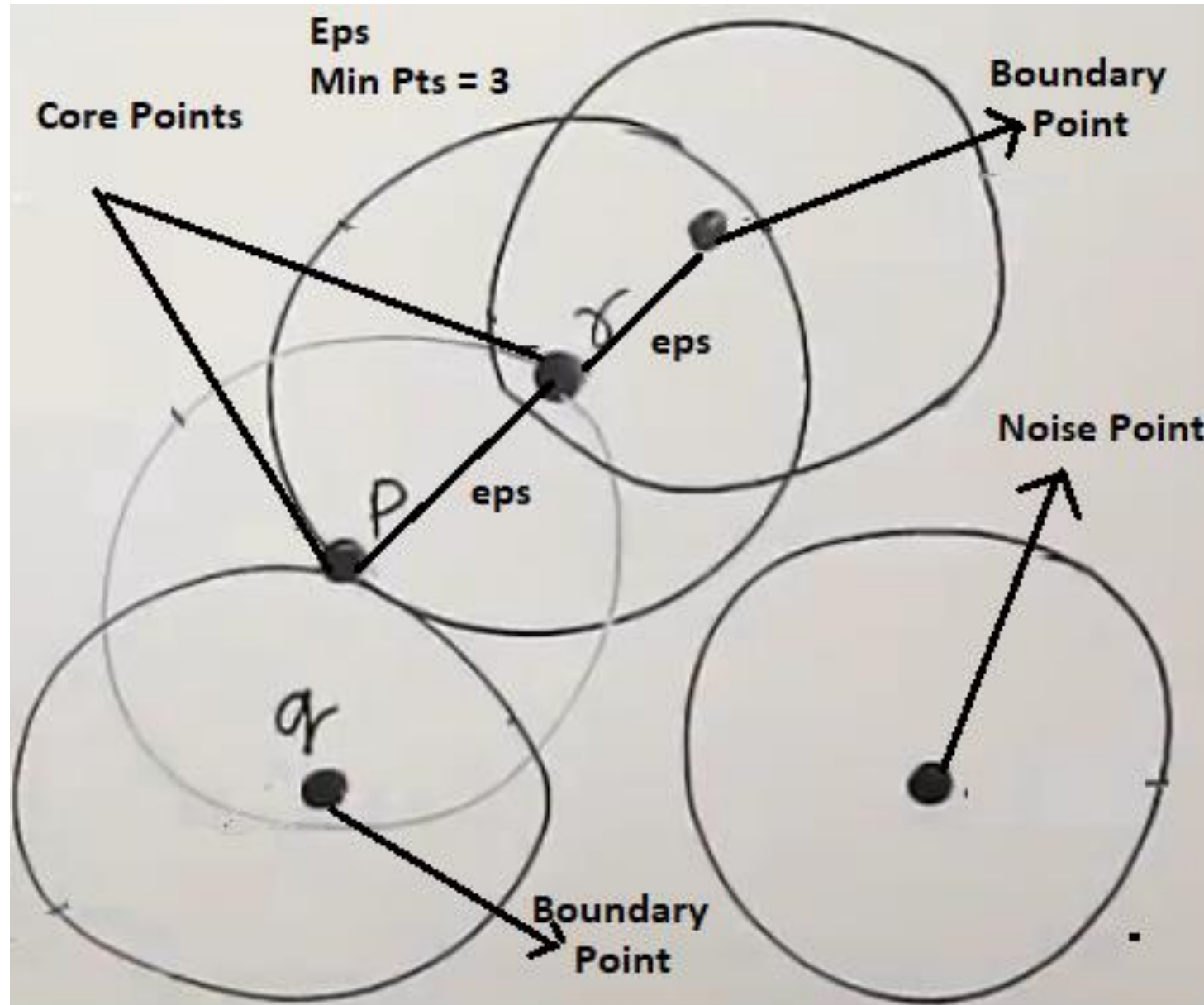
DBSCAN

DBSCAN is a density-based algorithm.

- Density = number of points within a specified radius r (Eps)
- A point is a **core point** if it has more than a specified number of points (MinPts) within Eps

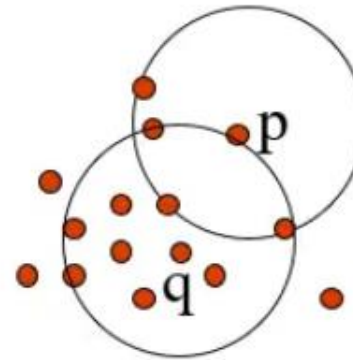
These are points that are at the interior of a cluster

- A **border point** has fewer than MinPts within Eps, but is in the neighborhood of a core point
- A **noise point** is any point that is not a core point or a border point.



Density-Based Clustering: Basic Concepts

- Two parameters:
 - Eps: Maximum radius of the neighbourhood
 - MinPts: Minimum number of points in an Eps-neighbourhood of that point
- NEps(p): $\{q \text{ belongs to } D \mid \text{dist}(p,q) \leq \text{Eps}\}$
- Directly density-reachable: A point p is directly density-reachable from a point q w.r.t. Eps, MinPts if
 - p belongs to NEps(q)
 - core point condition:
 - $|\text{NEps}(q)| \geq \text{MinPts}$



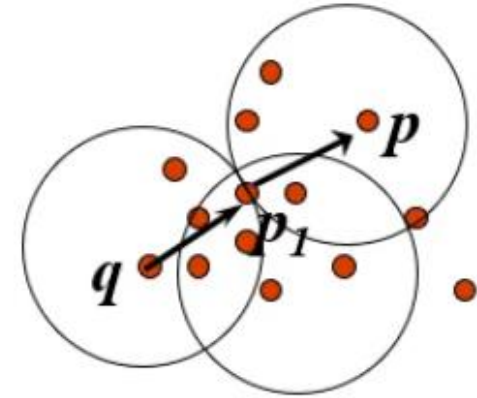
MinPts = 5

Eps = 1

Density-Reachable and Density-Connected

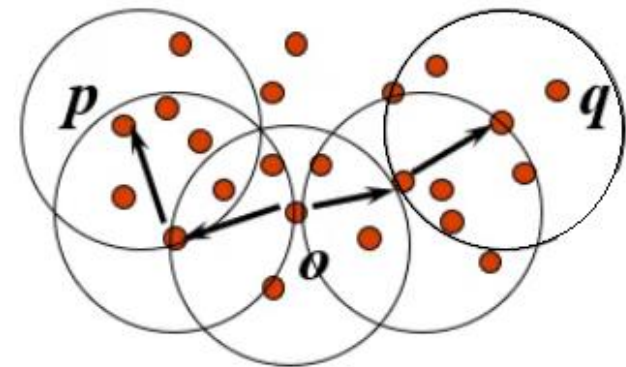
- Density-reachable:

- A point p is density-reachable from a point q w.r.t. Eps , $MinPts$ if there is a chain of points p_1, \dots, p_n , $p_1 = q$, $p_n = p$ such that p_{i+1} is directly density-reachable from p_i



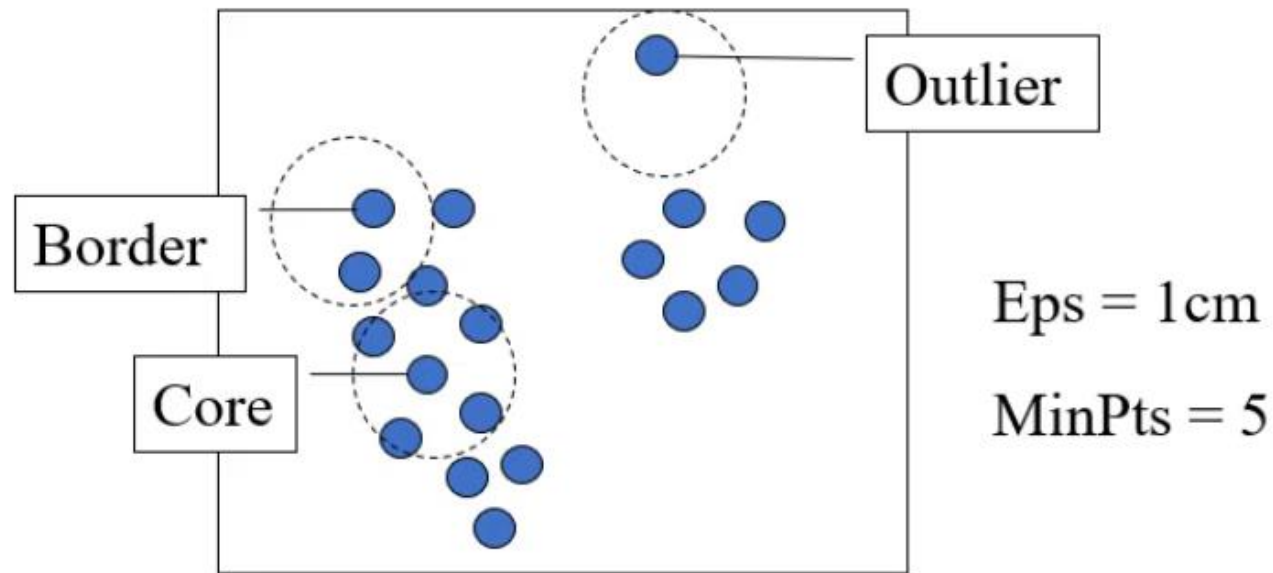
- Density-connected

- A point p is density-connected to a point q w.r.t. Eps , $MinPts$ if there is a point o such that both, p and q are density-reachable from o w.r.t. Eps and $MinPts$



DBSCAN: Density-Based Spatial Clustering of Applications with Noise

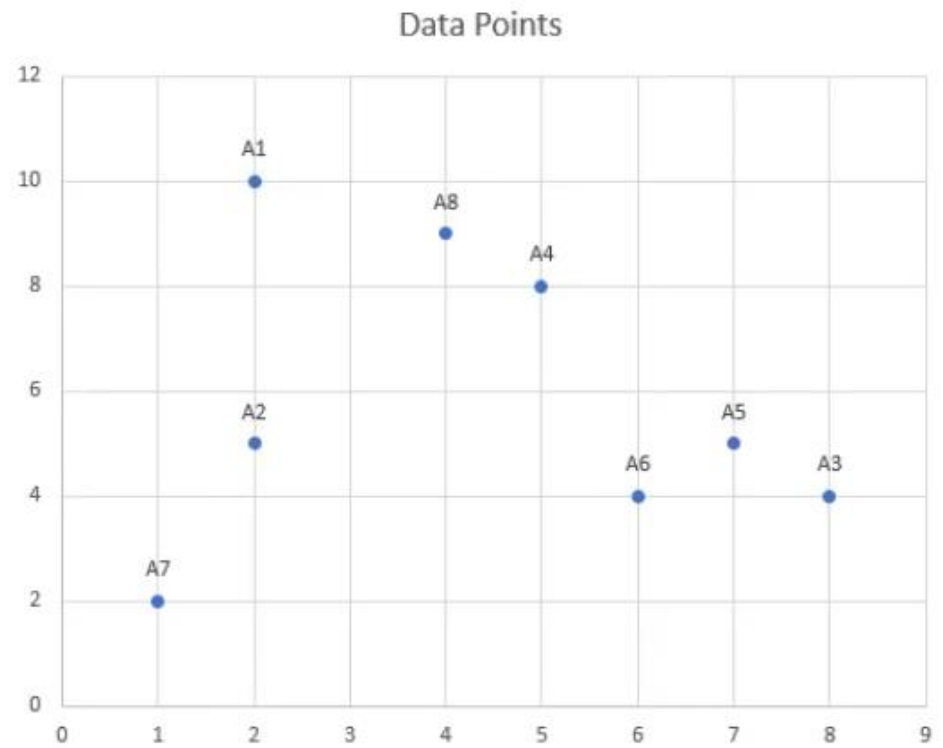
- Relies on a density-based notion of cluster: A cluster is defined as a maximal set of density-connected points
- Discovers clusters of arbitrary shape in spatial databases with noise



Min-pts = 3



	x	y
A1	2	10
A2	2	5
A3	8	4
A4	5	8
A5	7	5
A6	6	4
A7	1	2
A8	4	9



Euclidean Distance	A1	A2	A3	A4	A5	A6	A7	A8	
A1		0							
A2		5	0						
A3		8.49	6.08	0					
A4		3.61	4.24	5	0				
A5		7.07	5	1.41	3.61	0			
A6		7.21	4.12	2	4.12	1.41	0		
A7		8.06	3.16	7.28	7.21	6.71	5.39	0	
A8		2.24	4.47	6.4	1.41	5	5.39	7.62	0

	O	O	C	C	C	C	O	C
Euclidean Distance	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	5	8.49	3.61	7.07	7.21	8.06	2.24
A2	5	0	6.08	4.24	5	4.12	3.16	4.47
A3	8.49	6.08	0	5	1.41	2	7.28	6.4
A4	3.61	4.24	5	0	3.61	4.12	7.21	1.41
A5	7.07	5	1.41	3.61	0	1.41	6.71	5
A6	7.21	4.12	2	4.12	1.41	0	5.39	5.39
A7	8.06	3.16	7.28	7.21	6.71	5.39	0	7.62
A8	2.24	4.47	6.4	1.41	5	5.39	7.62	0
	A1	A2	A3,A5,A6	A4,A8	A3,A5,A6	A3,A5,A6	A7	A4,A8

$\epsilon = 2$ and min pt = 2

A1, A2 and A7 are outliers

There are 2 clusters formed:

Cluster1: A3,A5,A6

Cluster2: A4,A8

	B/O	B/O	B/O	C	C	C
Euclidean Distance	A	B	C	D	E	F
A	0	0.7	5.7	3.6	4.2	3.2
B	0.7	0	4.9	2.9	3.5	2.5
C	5.7	4.9	0	2.9	1.4	2.5
D	3.6	2.9	2.9	0	1	0.5
E	4.2	3.5	1.4	1	0	1.1
F	3.2	2.5	2.5	0.5	1.1	0
	A,B	A,B	C,E	D,E,F	C,D,E,F	D,E,F

$\epsilon = 2$ and min pt = 3

Visiting the Border/Outlier Points:

C: Neighborhood points = E which is a core point

Thus C is a Border point

A: Neighborhood points = B which is a B/O

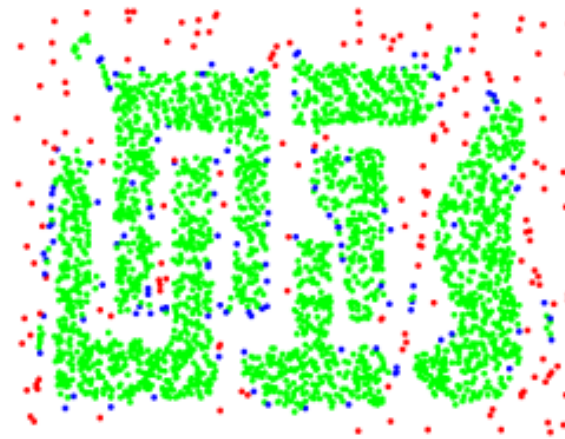
B: Neighborhood points = A which is a B/O

Thus A and B are outlier

DBSCAN: Large Eps



Original Points



Point types: **core**,
border and **noise**

DBSCAN: Optimal Eps



Original Points

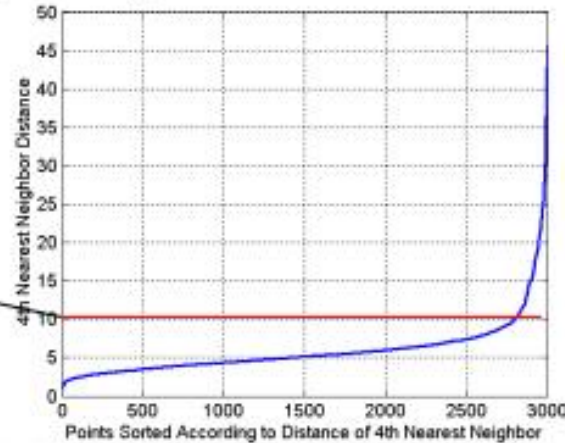


Clusters

Determining Eps and MinPts

- Idea is that for points in a cluster, their k^{th} nearest neighbors are at roughly the same distance
- Noise points have the k^{th} nearest neighbor at farther distance
- So, plot sorted distance of every point to its k^{th} nearest neighbor (e.g., $k=4$)

Thus, $\text{eps}=10$



DB Scan algorithm

1. $S = \{x_1, x_2, \dots, x_n\} \in R^m$
2. Chose value for $r > 0$ and $\epsilon > 0$ where r = no of points and ϵ = distance of two points
3. $A_i = \{x \in S : d(x_i, x) < \epsilon\}$; $i = 1, 2, 3, \dots, n$
4. if $|A_i| \geq r$, calculate all those A_i
5. Take union of A_i & A_j if $A_i \cap A_j \neq \emptyset$
6. Repeat 5 till no union take place.

DBSCAN: Complexity

Time Complexity: $O(n^2)$ —for each point it has to be determined if it is a core point, can be reduced to $O(n \cdot \log(n))$ in lower dimensional spaces by using efficient data structures (n is the number of objects to be clustered);

Space Complexity: $O(n)$.

Questions?