Experiment No. 6
Prim's Algorithm
Date of Performance:
Date of Submission:

Experiment No. 6

Title: Prim's Algorithm.

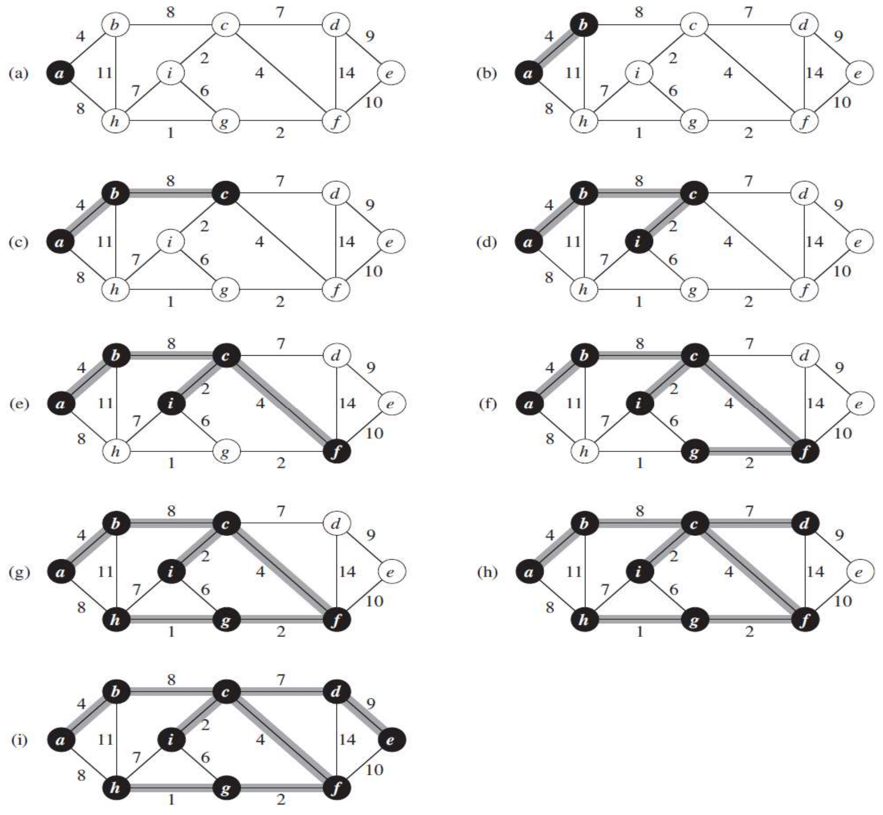
Aim: To study and implement Prim's Minimum Cost Spanning Tree Algorithm.

Objective: To introduce Greedy based algorithms

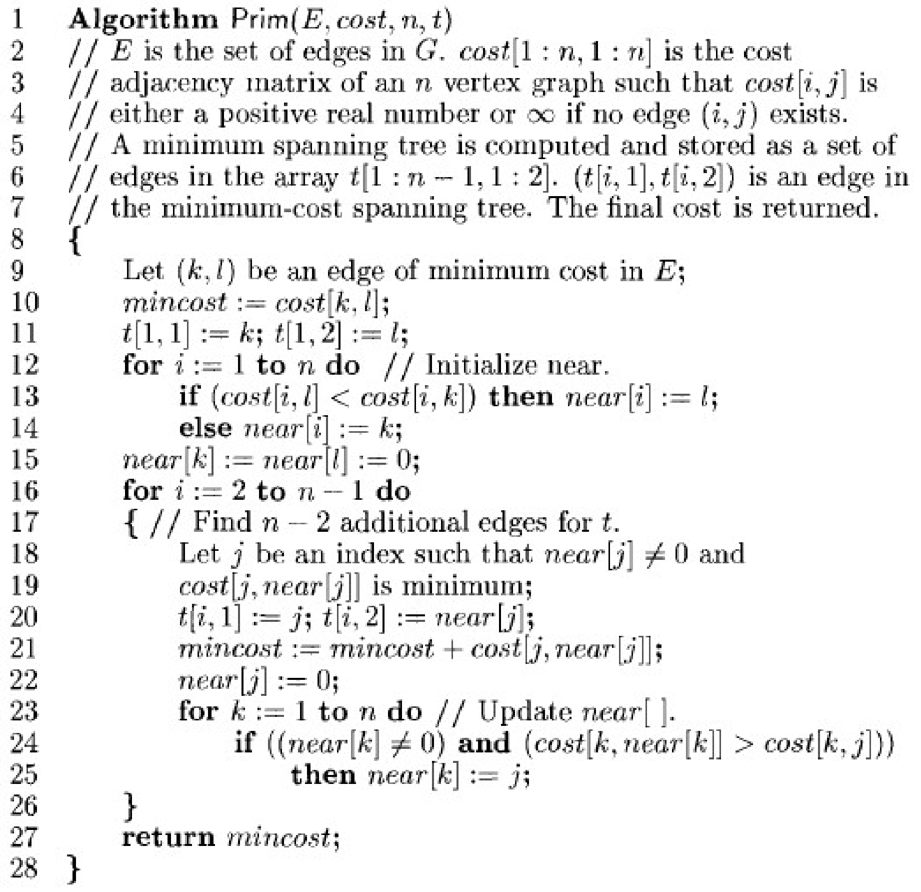
Theory:

Prim's algorithm is a greedy algorithm that finds a minimum spanning tree for a weighted undirected graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. The algorithm operates by building this tree one vertex at a time, from an arbitrary starting vertex, at each step adding the cheapest possible connection from the tree to another vertex.

Example:



Algorithm and Complexity:



```
Algorithm Prim($E, cost, n, t$)
// $E$ is the set of edges in $G$. $cost[1:n, 1:n]$ is the cost
// adjacency matrix of an $n$ vertex graph such that $cost[i,j]$ is
// either a positive real number or $\infty$ if no edge $(i,j)$ exists.
// A minimum spanning tree is computed and stored as a set of
// edges in the array $t[1:n-1, 1:2]$. $(t[i,1], t[i,2])$ is an edge in
// the minimum-cost spanning tree. The final cost is returned.
{
    Let $(k,l)$ be an edge of minimum cost in $E$;
    $mincost := cost[k,l]$;
    $t[1,1] := k$; $t[1,2] := l$;
    for $i := 1$ to $n$ do  // Initialize near.
        if $(cost[i,l] < cost[i,k])$ then $near[i] := l$;
        else $near[i] := k$;
    $near[k] := near[l] := 0$;
    for $i := 2$ to $n-1$ do
    { // Find $n-2$ additional edges for $t$.
        Let $j$ be an index such that $near[j] \neq 0$ and
        $cost[j, near[j]]$ is minimum;
        $t[i,1] := j$; $t[i,2] := near[j]$;
        $mincost := mincost + cost[j, near[j]]$;
        $near[j] := 0$;
        for $k := 1$ to $n$ do // Update $near[\,]$.
            if $((near[k] \neq 0)$ and $(cost[k, near[k]] > cost[k,j]))$
                then $near[k] := j$;
    }
    return $mincost$;
}
```

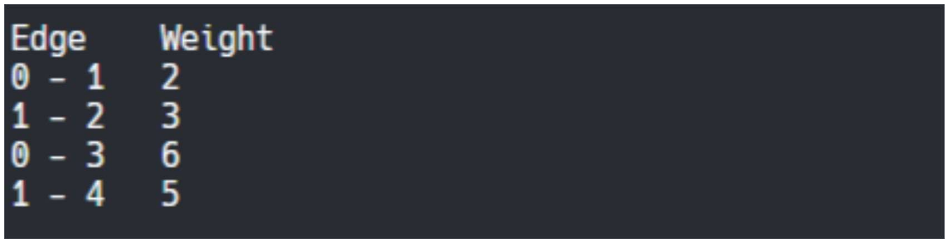
Time Complexity is O(n2), Where, n = number of vertices **Theory:**

Implemenation:

```
// A C program for Prim's Minimum
  // Spanning Tree (MST) algorithm. The program is
  // for adjacency matrix representation of the graph
  #include <limits.h>
  #include <stdbool.h>
  #include <stdio.h>
  // Number of vertices in the graph
  #define V 5
  // A utility function to find the vertex with
  // minimum key value, from the set of vertices
  // not yet included in MST
  int minKey(int key[], bool mstSet[])
```

```
{
  // Initialize min value
  int min = INT_MAX, min_index;

  for (int v = 0; v < V; v++)
    if (mstSet[v] == false && key[v] < min)
      min = key[v], min_index = v;
  return min_index;
}
// A utility function to print the
// constructed MST stored in parent[]
int printMST(int parent[], int graph[V][V])
{
  printf("Edge \tWeight\n");
  for (int i = 1; i < V; i++)
    printf("%d - %d \t%d \n", parent[i], i,
      graph[i][parent[i]]);
}
// Function to construct and print MST for
// a graph represented using adjacency
// matrix representation
void primMST(int graph[V][V])
{
  // Array to store constructed MST
  int parent[V];
  // Key values used to pick minimum weight edge in cut
  int key[V];
  // To represent set of vertices included in MST
  bool mstSet[V];

  // Initialize all keys as INFINITE
  for (int i = 0; i < V; i++)
    key[i] = INT_MAX, mstSet[i] = false;

  // Always include first 1st vertex in MST.
  // Make key 0 so that this vertex is picked as first
  // vertex.
  key[0] = 0;
  // First node is always root of MST
  parent[0] = -1;
  // The MST will have V vertices
  for (int count = 0; count < V - 1; count++) {
    // Pick the minimum key vertex from the
    // set of vertices not yet included in MST
    int u = minKey(key, mstSet);
    // Add the picked vertex to the MST Set
    mstSet[u] = true;
    // Update key value and parent index of
    // the adjacent vertices of the picked vertex.
    // Consider only those vertices which are not
    // yet included in MST
    for (int v = 0; v < V; v++)
      // graph[u][v] is non zero only for adjacent
      // vertices of m mstSet[v] is false for vertices
      // not yet included in MST Update the key only
      // if graph[u][v] is smaller than key[v]
      if (graph[u][v] && mstSet[v] == false
        && graph[u][v] < key[v])
        parent[v] = u, key[v] = graph[u][v];
  }
  // print the constructed MST
  printMST(parent, graph);
}
// Driver's code
int main()
{
  int graph[V][V] = { { 0, 2, 0, 6, 0 },
            { 2, 0, 3, 8, 5 },
            { 0, 3, 0, 0, 7 },
            { 6, 8, 0, 0, 9 },
            { 0, 5, 7, 9, 0 } };

  // Print the solution
  primMST(graph);

  return 0;
}
```

Output:



```
Edge    Weight
0 - 1   2
1 - 2   3
0 - 3   6
1 - 4   5
```

Conclusion: Implementing Prim's algorithm has proven to be effective in generating minimum spanning trees, efficiently connecting all nodes in a graph while minimizing total edge weight. This experiment underscores the algorithm's practical applicability in optimizing network connectivity, demonstrating its importance in various real-world scenarios.