Assignment 2

1

$$2^N = 2^6$$

$\Rightarrow 2 + 4 + 8 + 16 + 32 + 64 \Rightarrow 126$

2 $26^4 - 25^4 \Rightarrow 66351$

3

a) ends with A $\Rightarrow 4^4 \Rightarrow 256$

b) start with T and end with G $\Rightarrow 4^3 = 64$

c) contains only A and T $\Rightarrow 2^5 \Rightarrow 32$

d) do not contain C $\Rightarrow 3^5 = 243$

4

a) divisible by 9

total numbers $\Rightarrow 9 \times 10 \times 10 \times 10$

numbers divisible by 9 $\Rightarrow \dfrac{9 \times 10 \times 10 \times 10}{9}$

$\Rightarrow 1000$

b) total numbers $\Rightarrow$ $9 \times 10 \times 10 \times 10$

even numbers $\Rightarrow$ $\dfrac{9 \times 10 \times 10 \times 10}{2}$

$= 4500$

c) have distinct digits

$\Rightarrow \underset{(\text{first})}{9} \times \underset{(10-1)}{9} \times \underset{(10-2)}{8} \times \underset{(10-3)}{7}$

d) are not divisible by 3

total numbers $\Rightarrow$ $9 \times 10 \times 10 \times 10$

divisible by 3 $\Rightarrow$ $\dfrac{9 \times 10 \times 10 \times 10}{3}$

Not divisible by 3 $\Rightarrow$

$(9 \times 10 \times 10 \times 10) - \left(\dfrac{9 \times 10 \times 10 \times 10}{3}\right)$

$= 6000$

e divisible by 5 or 7

= |divisible by 5| + |divisible by 7| − |divisible by 35|

= 1800 + 1286 − 257

f not divisible by either 5 or 7

= Total − (divisible by 5 or 7)

⇒ 9000 − (180 + 1286 − 257)

g are divisible by 5 but not by 7

~~#~~ |divisible by 5| − |divisible by 35|

= 1800 − 257

h are divisible by ~~B~~ 5 and 7

~~#~~ ~~Total numbers th~~ |divisible by 35|

= 257

5

=> There has to be atleast 5 male or 5 female students

total students = 9
we have 2 pigeon holes

$\boxed{4}$ $\boxed{4}$

if both have equal 4 male and 4 female the 9^{th} students have to be ~~eighter~~ either M or F making either $M = 5$ or $F = 5$

b) Again, we have 2 holes

Case 1 => class have 3 male student

$\underset{M}{\boxed{3}}$ $\underset{F}{\boxed{6}}$

This scenario satisfies there being atleast 3 male students

In every secanio when there are less than 3 male students there will be atleast 7 females

6

Total earnings $= 10^8 - 1$

total people $= 10^8$

So atleast 2 people have same earnings because there ~~is 1~~ ~~more person~~ will be atleast 1 hole having 2 people

7

a) total outcomes $= 2^{8 \text{ coins}}$

H T

$= 2^8 = 256$

b) ~~ext~~ exactly 3 $\Rightarrow$ 8 choose 3

$= \binom{8}{3} = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{3 \times 2}$

$= 4 \times 7 \times 2 = 56$

c. atleast 3 heads

3 or more heads = total − less than 3 heads

$$256 - \left(\binom{8}{0}\binom{8}{1}\binom{8}{2}\right)$$

$$= 256 - (1 + 8 - 28)$$

$$= 219$$

d. Contain same number of heads and tails

total 8
head 4
tail 4

$$= \frac{\text{total}!}{\text{head}!\ \cancel{\text{ta}}(\text{total}-\text{head})!} = \frac{8!}{4!\ 4!} = 70$$

8

a) exactly 3 0's

$\Rightarrow C(10,3) = 120$

b) more 0s than 1s

0's have to be more than 5

$C(10,6) + C(10,7) + C(10,8)$

$+ C(10,9) + C(10,10)$

c) atleast 7 1's

there will be 3 (0)'s, 2(0's), 1(0's)

and 0 (0's)

$C(10,3) + C(10,2) + C(10,1) + C(10,0)$

$= 120 + 45 + 10 + 1 = 176$

d atleast three 1's

total $\Rightarrow 1024 \; (2^{10})$

$-\left(C(10,2) + C(10,1) + C(10,0)\right)$

$= 1024 - (45 + 10 + 1)$

$= 968$

9 we will use

$np = \dfrac{n!}{(n-r)!}$

a) The string ED

$7P_n = \dfrac{7!}{(7-7)!} = \dfrac{7!}{0!} = \dfrac{7!}{1!}$

$= 7!$

b) The string CDE

$$6_P = \frac{6!}{(6-6)!} = 6!$$

c) The string BA and FGH

$$= \frac{5!}{(5-5)!} = 5!$$

d) The string AB, DE and GH

$$= \frac{4!}{(4-4)!} = 4!$$

e)

The string CAB and BED

$$= \frac{3!}{(3-3)!} = 3!$$

f String BCA and ABF

- No permutations

10 expansion $(x+y)^5$

a) $(x+y)^5 = (x+y)^2 (x+y)^3$

$= (x^2+y^2+2xy)(x^3+3x^2y+3xy^2+y^3)$

$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3$

$+ 5xy^4 + y^5$

b) $(x+y)^5 =$

$= \binom{5}{0} x^5 y^0 + \binom{5}{1} x^{5-1} y^1 + \binom{5}{2} x^{5-2} y^2$

$\binom{5}{3} x^{5-3} y^3 + \binom{5}{4} x^{5-4} y^4 + \binom{5}{5} x^0 y^5$

$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

14

a) To choose n red candles and n blue candles
we have 3 case
- picking 2 red candles $-\binom{n}{2}$
- picking 2 blue candles $-\binom{n}{2}$
- picking 1 blue & 1 red $-n^2$

b) $\binom{2n}{2} = \frac{2n(2n-1)}{2}$

$= 2n^2 - n = n^2 + (n^2 - n) = n^2 + 2\binom{n}{2}$

11 $\quad x^5 y^8 \quad \text{in} \quad (x+y)^{13}$

$$= \binom{13}{8} x^5 y^8$$

$$= \frac{13!}{8!\,5!}$$

12 $\quad x^7 \quad \text{in} \quad (1+x)^{11}$

$$\binom{11}{7} x^7$$

$$= \left(\frac{11!}{7!\,4!}\right) x^7$$

13 $\quad x^8 \, y^9 \quad \text{in} \quad (3x+2y)^{17}$

$$= \binom{17}{9} x^8 \, y^9$$

$$= \frac{17!}{9!\,8!} x^8 y^9$$