Assignment 2

(1) to compute x^n

```
float compute (float n, int x)
{
                    bool negative = false;
    if ( x = = 0)
        return 1;
    if ( x = = 1)
        return x ;

    if ( x < 0)
    {
        x = - x;
        ~~bool negative~~
        negative = true ;
    }

    float result = n;
  for ( i = 1 ; i < x ; i++ )
  result = result * n
    if ( negative = = true )
      result = 1 / result ;

    return result;
}
```

2

finding smallest integer in a finite sequence of natural numbers

Case 1 ⇒ when the list is sorted

⇒ the smallest integer will be at position Array[0] O(1)

Case 2 ⇒ when the list is not sorted then we only have choice of linear search

```
int x;    // integer we are seaching for

int Array [Size];

for (int i = 0; i <= Size; i++)
{ if (Array[i] == x)
   { cout << "the element is at"
        index << i << endl;
   }
}
```

3

Step 1 //

split the list in 4

x // the element we are trying to find

Size // full size of the list

middle = Size / 2

quarter = middle / 2

list (size) // initial list

l1 (0, quarter)

l2 (quarter + 1, middle)

l3 (middle + 1, middle + quarter)

l4 (middle + quarter + 1, Size)

Step 2 //

```
    if ( x ≤ l1 (quarter) )
        split l1 // recursively split l1
                    into 4
else
  if ( x ≤ l3 (middle + quarter)
     if ( x ≤ l2 ( middle)
         split l2
     split l3
else { ~~split~~
     split l4 }
```

Step 3// before splitting

~~search~~ //check if the element at ~~half~~, 0, middle, quarter
Size == x

⇒ This method will implement binary seach but with splitting it into 4 lists recursivelly

4

a) $f(x) = \cancel{12}\ 17x + 11$ is $O(x^2)$

$17(x) + 11$ is $O(x)$

$O(x)$ is $O(x^2)$

hence $17x + 11$ is $O(x^2)$

b) $f(x) = x^2 + 1000$

$O(x^2)$, $x^2 + 1000 = C(x^2)$

for $k = 10$, $C = 11$

$x^2 + 1000$ is $O(x^2)$

c) $f(x) = x \log x$

$x \log x = C(x)$

$x \log x$ is $O(x)$

$O(x)$ is $O(x^2)$

$x \log x$ is $O(x^2)$

d) $f(x) = x^4/2$

x^4 ↑ is growing faster than x^2

x^4 is not $O(x^2)$

e) $f(x) = 2^x$

2^x ↑ is growing with $O(2^n)$

2^x is not $O(x^2)$

f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

$= x^2 + x$

$= x^2 + x$ is dominated by x^2

hence $x^2 + x$ is $O(x^2)$

$\underline{\underline{5}}$ $f(x)$ is $O(g(x))$ to show

$2^x + 17$ is $O(3^x)$

$f(x)$ is $O(g(x))$ if and only if there exist constant N and C such that

$$|f(x)| \leq C\,|g(x)| \quad \text{for all } x > N$$

f grows faster than g

$2^x + 17 < 2^x + 2^x$ for $x > 5$

$2^x + 17$ is $O(2^x)$

$2^x < 3^x$

2^x is $O(3^x)$

hence

$2^x + 17$ is $O(3^x)$

6 $\left(\frac{x^3+2x}{2x+1}\right)$ is $O(x^2)$

$$\Rightarrow \frac{x^3+2x}{2x+1}$$

$$\Rightarrow \frac{x(x^2+2)}{x(2+\frac{1}{x})}$$

$$= \frac{x^2+2}{2+\frac{1}{x}}$$

$\frac{1}{x}$ is getting smalla and smaller as the function grows

So, $\frac{x^2+2}{2+\frac{1}{x}}$ is dominated by x^2+2

$x^2+2 < x^2+2x^2$

x^2+2 is $O(x^2)$

7

a) $f(x) = 2x^2 + x^3 \log x$

$2x^2 + x^3 \log x$ is $O\,x^4$

because it's donimated by $x^3 \log x$

where $\log < x$

$x^3 \log x < x^3 x$

$= O x^4$

$2x^2 + x^4 \leq 2x^4 + x^4$

So $f(x)\ 2x^4 + x^4$, for $x > 1$

$n = 4$, $C = 3$

$K = 1$

b) $f(x) = 3x^5 + (\log x)^4$

$3x^5 + (\log x)^4 \leq 3x^5 + x^5$

↳ This is because $\cancel{x^5} \log(x)^4 < x^5$

$3x^5 + x^5 = \boxed{4x^5} = O(x^5)$

$n = 5$

$c = 4$

$K = 1$

c $f(x) = \dfrac{(x^4 + x^2 + 1)}{(x^4 + 1)}$

$$\frac{x^4 + x^2 + 1}{x^4 + 1} < \frac{x^4 \cancel{+x^4+x}}{x^4}$$

$$\frac{x^4}{x^4} < x$$

for function $f(x) \leq x$

$n = 1$

$C = 1$

$K = 1$

d $f(x) = \dfrac{(x^3 + 5\log x)}{x^4 + 1}$

$$= \frac{x^3 + 5\log x}{x^4 + 1}$$

$$= \frac{x^3}{(x^4+1)} + \frac{5\log x}{x^4+1}$$

(i) (ii)

$=$ from (i)

$$\frac{x^3}{x^4+1} \leq \frac{x^3}{x^4} = x^{-1}$$

from (ii)

$$\frac{5\log x}{x^4+1} < \frac{5x}{x^4} \quad // 5x < x^4$$

$$\frac{5\log x}{x^4+1} < \frac{x^4}{x^4} \not< 1$$

combining (i) & (ii)

$$< x^{-1}+1$$

$$< x$$

$f(x) \leq x$

$n = 1$

$c = 1$

$k = 1$

8

101^6, $(\log n)^3$, $\sqrt{n}\log n$, $n^{99}+n^{98}$, $(1.5)^n$, $10n^k$, $(n!)^2$

9 $1.1! + 2.2! + \dots n.n! = (n+1)! - 1$

base step $(n=1)$

$1.1! = (1+1)! - 1$

$1 = 2 - 1$

$1 = 1$

LHS = RHS

Inductive step

$1.1! + 2.2! + \dots + k.k! = (k+1)! - 1$

$1.1! + 2.2! + \dots (k+1).(k+1)! = (k+1)! - 1 + (k+1)(k+1)!$

$\Rightarrow (k+1)!(k+2) - 1 = (k+2)! - 1$

So by mathematical induction

$1.1! + 2.2! + \dots + n.n! = (n+1)! - 1$

10 Prove that every positive integer n

$1.2.3 + 2.3.4 + \cdots + n(n+1)(n+2)$
$= n(n+1)(n+2)(n+3)/4$

Base step $\Rightarrow$ $n=1$

$\sum_{i=1}^{1} i(i+1)(i+2) = 1.2.3 = 6$

and

$1(1+1)(1+2)(1+3)/4 = 1.2.3.4/4$
$= 6$

Inductive step

$\cancel{\sum_{i=1}^{k} i(i+1)(i+2) = 1.2.3 + 2.3.4 + \cdots + \frac{k(k+1)(k+2)(k+3)}{4}}$

$\sum_{i=1}^{k} i(i+1)(i+2) =$

$1.2.3 + 2.3.4 + k(k+1)(k+2) + (k+1)(k+2)(k+3)$

$= k(k+1)(k+2)(k+3)/4 + (k+1)(k+2)(k+3)$
$= (k+1)(k+2)(k+3)(k/4 + 1)$
$= (k+1)(k+2)(k+3)(k/4 + 4/4)$
$= (k+1)(k+2)(k+3)(k+4)/4$