

## Assignment One (CS 435/890BN, Spring/Summer 2019)

(Weight: 3%, Due: July 18, 5:30pm)

1. (20 points)

A generalization of the Caesar cipher, known as the affine Caesar cipher, has the following form: For each plaintext letter  $p$ , substitute the ciphertext letter  $C$ :

$$C = E([a, b], p) = (ap + b) \bmod 26$$

A basic requirement of any encryption algorithm is that it be one-to-one. That is, if  $p \neq q$ , then  $E(k, p) \neq E(k, q)$ . Otherwise, decryption is impossible, because more than one plaintext character maps into the same ciphertext character. The affine Caesar cipher is not one-to-one for all values of  $a$ . For example, for  $a = 2$  and  $b = 3$ , then  $E([a, b], 0) = E([a, b], 13) = 3$ .

- Are there any limitations on the value of  $b$ ? Explain why or why not.
- Determine which values of  $a$  are not allowed.
- Provide a general statement of which values of  $a$  are and are not allowed. Justify your statement.

2. (20 points) Key:  $K = \begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}$ , Inverse of the Key:  $K^{-1} = \begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix}$

- Encrypt the message "MEET ME AT THE USUAL PLACE" using the Hill cipher (plaintext and ciphertext as column vectors, which the column vector is placed after the key matrix) with the key  $K$ . Show your calculation steps and the result.
- Show the calculations for the corresponding decryption of the ciphertext to recover the original plaintext.

3. (20 points) Describe the  $n$  times  $m$  columns transposition cipher. Indicate the number of searches required by brute-force attack and method to break the key or reduce the search time by cryptanalysis attack. ( $m$  columns transposition cipher, and apply  $n$  times)

4. (30 points)

Note: this problem refer to details of DES that are described in Appendix S of the textbook

This problem provides a numerical example of encryption using a one-round version of DES. We start with the same bit pattern for the key  $K$  and the plaintext, namely:

**Hexadecimal notation:** 0 1 2 3 4 5 6 7 8 9 A B C D E F

**Binary notation:** 0000 0001 0010 0011 0100 0101 0110 0111

1000 1001 1010 1011 1100 1101 1110 1111

- Derive  $K_1$ , the first-round subkey.
- Derive  $L_0, R_0$ .
- Expand  $R_0$  to get  $E[R_0]$ , where  $E[\cdot]$  is the expansion function of Table S.1.
- Calculate  $A = E[R_0] \oplus K_1$ .
- Group the 48-bit result of (d) into sets of 6 bits and evaluate the corresponding S-box substitutions.
- Concatenate the results of (e) to get a 32-bit result,  $B$ .
- Apply the permutation to get  $P(B)$ .
- Calculate  $R_1 = P(B) \oplus L_0$ .
- Write down the ciphertext.

**(a) Initial Permutation (IP)**

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

**(b) Inverse Initial Permutation (IP<sup>-1</sup>)**

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

**(c) Expansion Permutation (E)**

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

**(d) Permutation Function (P)**

16	7	20	21	29	12	28	17
1	15	23	26	5	18	31	10
2	8	24	14	32	27	3	9
19	13	30	6	22	11	4	25

Table S.1 Permutation Tables of DES

5. (10 points) Describe the major differences between stream ciphers and block ciphers.