Assignment 3
CS 310: Discrete Computational Structures
University of Regina
Department of Computer Science
Fall 2018

Due date: October 26, 2018 at 1:30 pm

1. (1 point) How many bit strings are there of length six or less, not counting the empty string?

2. (1 point) How many strings are there of four lowercase letters that have the letter x in them?

3. (12 points) How many 5-element DNA sequences

   (a) end with A?

   (b) start with T and end with G?

   (c) contain only A and T?

   (d) do not contain C?

4. (24 points) How many positive integers between 1000 and 9999 inclusive

   (a) are divisible by 9?

   (b) are even?

   (c) have distinct digits?

   (d) are not divisible by 3?

   (e) are divisible by 5 or 7?

   (f) are not divisible by either 5 or 7?

   (g) are divisible by 5 but not by 7?

   (h) are divisible by 5 and 7?

5. (8 points) Suppose that there are nine students in a discrete mathematics class at a small college.

   (a) Show that the class must have at least five male students or at least five female students.

(b) Show that the class must have at least three male students or at least seven female students.

6. (1 point) Show that if there are 100,000,000 wage earners in the United States who earn less than 1,000,000 dollars (but at least a penny), then there are two who earned exactly the same amount of money, to the penny, last year.

7. (12 points) A coin is flipped eight times where each flip comes out either heads or tails. How many possible outcomes

   (a) are there in total?

   (b) contain exactly three heads?

   (c) contain at least three heads?

   (d) contain the same number of heads and tails?

8. (12 points) How many bit strings of length 10 have

   (a) exactly three 0s?

   (b) more 0s than 1s?

   (c) at least seven 1s?

   (d) at least three 1s?

9. (18 points) How many permutations of letters $ABCDEFGH$ contain

   (a) the string ED?

   (b) the string CDE?

   (c) the strings BA and FGH?

   (d) the strings AB, DE and GH?

   (e) the strings CAB and BED?

   (f) the strings BCA and ABF?

10. (4 points) Find the expansion of $(x + y)^5$

    (a) using combinatorial reasoning.

    (b) using the binomial theorem.

11. (1 point) Find the coefficient of x^5y^8 in $(x + y)^{13}$.

12. (1 point) What is the coefficient of x^7 in $(1 + x)^{11}$?

13. (1 point) What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$?

14. (4 points) Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$

    (a) using combinatorial argument.

    (b) by algebraic manipulation.