

Q1

① Decision tree using ID3

Entropy of dataset =

$$\text{positives} = 9$$

$$\text{negatives} = \underline{\underline{5}}$$

$$IE(S) = -\frac{9}{14} \log_2 \left(\frac{9}{14} \right) - \frac{5}{14} \log_2 \left(\frac{5}{14} \right)$$
$$= 0.940$$

Attributes

① Outlook

	Positives	Negatives	Entropy
Sunny	2	3	0.971
Rainy	3	2	0.971
Overtcast	0	0	0

AVE \Rightarrow

I(outlook)

$$= \frac{3}{5}$$

$$\frac{3+2}{9+5} \times 0.971$$

+

$$\frac{2+3}{9+5} \times 0.971$$

+

$$= 0.693$$

$$I(\text{outlook}) = 0.693$$

Gain

$$\text{Entropy (S)} - I(\text{outlook})$$

$$= 0.940 - 0.693$$

$$= 0.247 - \textcircled{i}$$

Brain

⑪ I(tempature)

temp.	positive	negative	C
hot	2	2	1
mild	4	2	0.918
cold	3	1	0.811

$$I(\text{temp}) \Rightarrow$$

$$\left(\frac{2+2}{9+5} \times 1 \right) + \left(\frac{4+2}{9+5} \times 0.918 \right) \\ + \left(\frac{3+1}{9+5} \times 0.811 \right)$$

$$I(\text{temp}) \Rightarrow 0.911$$

$$\text{Gain} \Rightarrow$$

$$= 0.940 - 0.911 \\ = 0.029 - \text{(+)}$$

(iii) Humidity

	positive	Negative	E
High	3	4	0.985
Normal	6	1	0.591

$I(\text{humidity})$

$$\left(\frac{3+4}{9+5} \times 0.985 \right) + \left(\frac{6+1}{9+5} \times 0.591 \right)$$

$$= 0.788$$

gain $\Rightarrow E(S) - I(\text{humidity})$

$$= 0.940 - 0.788$$

$$= 0.152$$

— (iii)

⑩ Windy

	positive	negative	
True	3	3	1
False	6	2	0.811

$$\left(\frac{3+3}{9+5} \times 1 \right) + \left(\frac{6+2}{9+5} \times 0.811 \right)$$
$$= 0.892$$

gain
⇒ $E(S) - I(\text{Windy})$

$$\Rightarrow 0.940 - 0.892$$

$$\Rightarrow 0.048$$

— (iv)

from (i), (ii), (iii) & (iv)

Crater

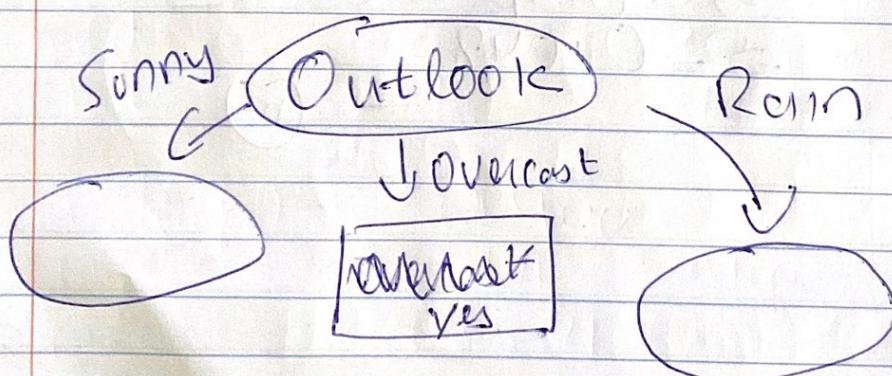
Attribute

gain

outlook	0.247
temp	0.029
humidity	0.152
windy	0.048

From this, we will choose outlook as the base node

that gives us,



Under outlook Attribute

i) For Sunny positive = 2
negative = 3
 $E(S) = \frac{2}{5}$

$$\frac{-2}{2+3} \log_2 \left(\frac{2}{2+3} \right) - \frac{3}{2+3} \log_2 \left(\frac{3}{2+3} \right)$$
$$\Rightarrow 0.971$$

$I(\text{humidity}) \Rightarrow$

	positive	negative	Entropy
high	0	3	0
normal	2	0	0

$$I(\text{humidity}) = 0$$

$$\begin{aligned} \text{gain} &= 0.971 - 0 \\ &= 0.971 \end{aligned}$$

→ i

$I(\text{Wind})$

	Positive	Negative	Entropy
True	1	1	1
False	1	2	0.918

$$I(\text{Wind}) = 0.951$$

$$\begin{aligned} \text{gain} &= 0.971 - 0.951 \\ &= 0.020 \quad \text{(ii)} \end{aligned}$$

$I(\text{Temp})$

	positive	negative	Entropy
cool	1	0	0
mild	0	2	0
hot	1	1	0

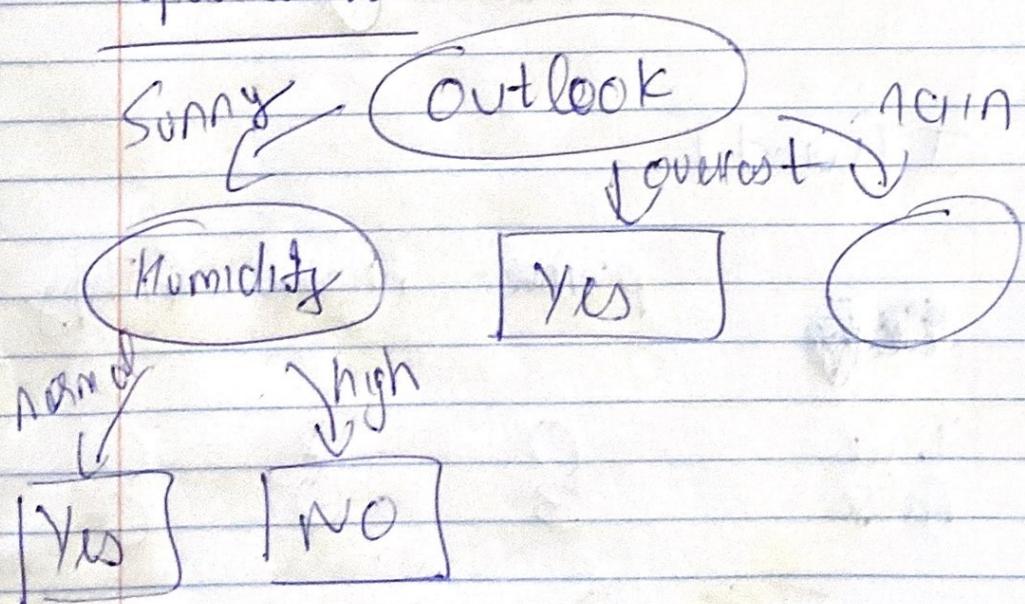
$$\begin{aligned} I(\text{Temp}) &= 0.400 \\ &= 0.971 - 0.400 \\ &= 0.571 \quad \text{(iii)} \end{aligned}$$

from ① & ⑪ & ⑫

Attrib.	Gain
temp	0.571
humidity	0.971 ✓
windy	0.02

humidity is highest so it
is the next node

~~update~~
updated tree



(11)

Fourth Outlook Rainy

 $I(\text{humidity})$

	positive	negative	Entropy
high	1	1	1
normal	2	1	0.918

$$I(\text{humidity}) = 0.951$$

$$\begin{aligned} \text{gain} &= 0.971 - 0.951 \\ &= 0.020 \end{aligned}$$

 $I(\text{wind})$

	positive	negative	Entropy
True	0	2	0
False	3	0	0

$$I(\text{windy}) = 0$$

$$\text{gain} = 0.971$$

(11)

$I(\text{Hum})$

	positive	negative	Entropy
Cool	1	1	1
Mild	1/2	1	0.918

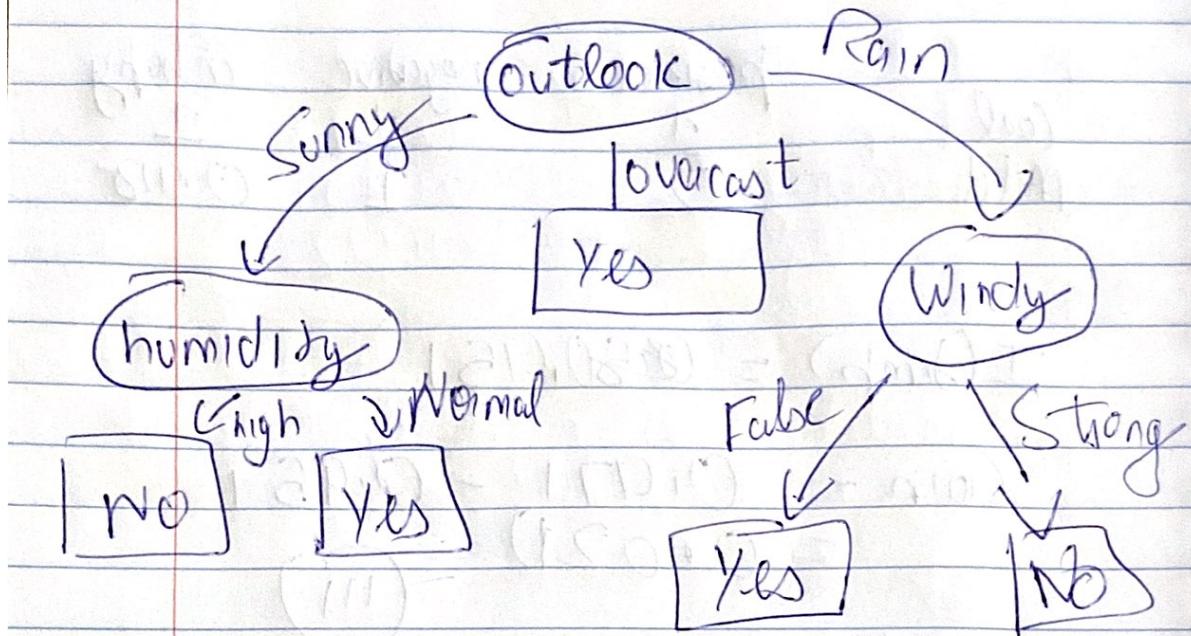
$$I(\text{Hum}) = (0.50, 0.15)$$

$$\begin{aligned} \text{Gain} &= 0.071 - 0.951 \\ &= 0.020 \end{aligned}$$

from (i), (ii) & (iii)

Attribute	Gain
humidity	0.020
wind	0.071
temp	0.020

updated tree →



1.2 PRISM Algorithm

Two classes - "Yes" & "No"

for play = yes		
outlook = Sunny	2/5	①
<u>outlook</u> = Overcast	4/4	longest fraction
outlook = Rainy	3/5	
Temp = hot	2/4	
temp = mild	4/6	
temp = cold	3/4	
humidity = high	3/7	
humidity = normal	6/7	
Windy = true	3/6	②
Windy = false	6/8	longest fraction

from ①

outlook = overcast

For Class = NO	(1)
outlook = Sunny	3/5 ✓ highest fraction
outlook = overcast	0/4
outlook = rainy	2/5
Temp = hot	2/4
Temp = mild	2/6
Temp = cold	1/4
humidity = high	4/7
humidity = normal	1/7
Windy	3/6
Windy	2/8

from (1)

outlook = Sunny

If outlook = overcast then class = Yes
 outlook = Sunny then class = No

2

ID3 uses greedy approach
so it is really fast, It
constructs optimal decision trees
for more than 50% of the
datasets.

It uses Information gain to
construct decision trees with
less cost

Information gain heuristic makes
ID3 fast and cheaper

Z
y
θ

	x_0	x_1	x_2	y (class)
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

Step 1 -

- ① initialising weight &
- ② learning rate
 n ($0 < n < 1$)

Step 2 - repeat 3-5 until convergence

Step 3 - calculating first net output

$$y = \sum_{i=1}^n w_i x_i + b$$

where n is number of inputs

Step 4 apply activation function

Step 5 update weight & bias

Iterations

$$w_t = w_1 + \Delta w_t$$

where

$$\Delta w_t = \lambda (t - o) x_t$$

t = target output

o = output generated

λ = learning rate

$$y' = \begin{cases} \epsilon > 0 & ; y' = 1 \text{ or } y = 0 \end{cases}$$

$$w_0 = -1 \quad w_1 = 0.4$$

$$w_2 = 0.5 \quad \lambda = 0.6$$

x_0	x_1	x_2	y	w_0	w_1	w_2	Σ	y'	$a(y-y')$	
1	1	0	0	0	-1	0.4	0.5	-1	0	6
2	1	0	1	1	-1	0.4	0.5	-0.5	0	0.6
3	1	-1	0	1	-0.8	0.4	0.7	-0.4	0	0.6
4	1	-1	1	1	-0.6	0.6	0.7	-0.7	1	0

for step ① & ② there is no error so we don't need to change weights before we update ③

③

for 2

$$w_0 = -1 + 0.6 = 0.4$$

$$w_1 = 0.4 + 0(0.6) = 0.4$$

$$w_2 = 0.5 + 0.6 = \cancel{0.1}1.1$$

for 3

$$y' = 0, y = 1$$

$$w_0 = -0.8 + 1(0.6) = -0.2$$

$$w_1 = 0.4 + (0.6)1 = 1.0$$

$$w_2 = 0.7 + 0(0.6) = 0.7$$

x_0	x_1	x_2	y	w_0	w_1, w_2	Σ	y'	$a(y-y')$
1	0	0	0	-0.2	1 0.7	-0.2	0	0
1	0	1	1	-0.2	1 0.7	-0.5	0	0
1	1	0	1	-0.2	1 0.7	0.8	1	0
1	1	1	1	0.5	0.4 1.3	2.1	1	0

b $w_0 = -1$ $w_1 = 0$ $w_2 = 0.1$

$$y' \left\{ \begin{array}{l} \Sigma > 0 : y = 1 \\ \Sigma \leq 0 : y = 0 \end{array} \right\}$$

	$\Sigma = 0.2$									
	x_0	x_1	x_2	y	w_0	w_1	w_2	Σ	y'	$a(y-y')$
1	1	0	0	0	-1	0	0.1	-1	0	0
2	1	0	1	0	-1	0	0.1	-0.9	0	0
3	1	1	0	0	-1	0	0.1	-1	0	0
4	1	1	1	1	-1	0	0.1	-0.9	0	0.9

$$w_0 = -1 + 1(0.9) = -0.1$$

$$w_1 = 0 + 0.9 = 0.9$$

$$w_2 = 0.1 + 0.9 = 1.0$$

	x_0	x_1	x_2	y	w_0	w_1	w_2	ϵ	y'	$1(y-y')$
1	1	0	0	0	-0.1	0.9	1	-0.1	0	0
2	1	0	1	0	-0.1	0.9	1	-0.1+1	0	0
3	1	1	0	0	-0.1	0.9	1	-1	0	0
4	1	1	1	1	-0.1	0.9	1	1	1	0