Assignment 5

Question 4 (Perceptron learning algorithm)

In this question, there is no threshold so we consider the threshold as 0 (default). First, we declare that

$$\vec{x} \cdot \vec{w} > 0,\ output\ 1;$$
$$\vec{x} \cdot \vec{w} \leq 0,\ output\ 0.$$

(it is also good, if you consider $\geq$ 0 is positive and <0 is negative).

Then, we have two choices: 1. ignore the threshold and simply perform the algorithm; 2. adjust the threshold and use augmented inputs and biased weight.

See the sample solution to question 4.a:

1. don't consider the threshold:

$$\begin{aligned}
&Take\ initial\ weights\ as\ \overrightarrow{w_0} = (0,0,0)\\
&\overrightarrow{x_1} \cdot \overrightarrow{w_0} = 0,\ output\ 0\ is\ correct;\\
&\overrightarrow{x_2} \cdot \overrightarrow{w_0} = 0,\ output\ 0\ is\ incorrect,\ \overrightarrow{w_1} = \overrightarrow{w_0} + \overrightarrow{x_2} = (1,0,1);\\
&\overrightarrow{x_3} \cdot \overrightarrow{w_1} = 1,\ output\ 1\ is\ correct;\\
&\overrightarrow{x_4} \cdot \overrightarrow{w_1} = 2,\ output\ 1\ is\ correct;\\
&\overrightarrow{x_1} \cdot \overrightarrow{w_1} = 1,\ output\ 1\ is\ incorrect;\ \overrightarrow{w_2} = \overrightarrow{w_1} - \overrightarrow{x_1} = (0,0,1);\\
&\overrightarrow{x_2} \cdot \overrightarrow{w_2} = 1,\ output\ 1\ is\ correct;\\
&\overrightarrow{x_3} \cdot \overrightarrow{w_2} = 0,\ output\ 0\ is\ incorrect;\ \overrightarrow{w_3} = \overrightarrow{w_2} + \overrightarrow{x_3} = (1,1,1);\\
&\overrightarrow{x_4} \cdot \overrightarrow{w_3} = 3,\ output\ 1\ is\ correct;\\
&\overrightarrow{x_1} \cdot \overrightarrow{w_3} = 1,\ output\ 1\ is\ incorrect;\ \overrightarrow{w_4} = \overrightarrow{w_3} - \overrightarrow{x_1} = (0,1,1);\\
&\overrightarrow{x_2} \cdot \overrightarrow{w_4} = 1,\ output\ 1\ is\ correct;\\
&\overrightarrow{x_3} \cdot \overrightarrow{w_4} = 1,\ output\ 1\ is\ correct;\\
&\overrightarrow{x_4} \cdot \overrightarrow{w_4} = 1,\ output\ 1\ is\ correct;\\
&\overrightarrow{x_1} \cdot \overrightarrow{w_4} = 0,\ output\ 0\ is\ correct.
\end{aligned}$$

Thus, the final weight vector is (0,1,1).

See another solution to question 4.b:

2. adjust the threshold and using augmented inputs:

$Since\ the\ threshold\ is\ 0,\ w_{n+1}=0,\ the\ initial\ weights\ is\ \overrightarrow{w_0}=(0,0,0,0)$
$The\ augmented\ inputs\ are$
$\overrightarrow{x_1}=(1,0,0,1)$
$\overrightarrow{x_2}=(1,0,1,1)$
$\overrightarrow{x_3}=(1,1,0,1)$
$\overrightarrow{x_4}=(1,1,1,1).$

$\overrightarrow{x_1}\cdot\overrightarrow{w_0}=0,\ output\ 0\ is\ correct;$
$\overrightarrow{x_2}\cdot\overrightarrow{w_0}=0,\ output\ 0\ is\ correct;$
$\overrightarrow{x_3}\cdot\overrightarrow{w_0}=0,\ output\ 0\ is\ correct;$
$\overrightarrow{x_4}\cdot\overrightarrow{w_0}=0,\ output\ 0\ is\ incorrect,\ \overrightarrow{w_1}=\overrightarrow{w_0}+\overrightarrow{x_4}=(1,1,1,1);$
$\overrightarrow{x_1}\cdot\overrightarrow{w_1}=2,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_2}=\overrightarrow{w_1}-\overrightarrow{x_1}=(0,1,1,0);$
$\overrightarrow{x_2}\cdot\overrightarrow{w_2}=1,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_3}=\overrightarrow{w_2}-\overrightarrow{x_2}=(-1,1,0,-1);$
$\overrightarrow{x_3}\cdot\overrightarrow{w_3}=-1,\ output\ 0\ is\ correct;$
$\overrightarrow{x_4}\cdot\overrightarrow{w_3}=-1,\ output\ 0\ is\ incorrect,\ \overrightarrow{w_4}=\overrightarrow{w_3}+\overrightarrow{x_4}=(0,2,1,0);$
$\overrightarrow{x_1}\cdot\overrightarrow{w_4}=0,\ output\ 0\ is\ correct;$
$\overrightarrow{x_2}\cdot\overrightarrow{w_4}=1,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_5}=\overrightarrow{w_4}-\overrightarrow{x_2}=(-1,2,0,-1);$
$\overrightarrow{x_3}\cdot\overrightarrow{w_5}=0,\ output\ 0\ is\ correct;$
$\overrightarrow{x_4}\cdot\overrightarrow{w_5}=0,\ output\ 0\ is\ incorrect,\ \overrightarrow{w_6}=\overrightarrow{w_5}+\overrightarrow{x_4}=(0,3,1,0);$
$\overrightarrow{x_1}\cdot\overrightarrow{w_6}=0,\ output\ 0\ is\ correct;$
$\overrightarrow{x_2}\cdot\overrightarrow{w_6}=1,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_7}=\overrightarrow{w_6}-\overrightarrow{x_2}=(-1,3,0,-1);$
$\overrightarrow{x_3}\cdot\overrightarrow{w_7}=1,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_8}=\overrightarrow{w_7}-\overrightarrow{x_3}=(-2,2,0,-2);$
$\overrightarrow{x_4}\cdot\overrightarrow{w_8}=0,\ output\ 0\ is\ incoreect,\ \overrightarrow{w_9}=\overrightarrow{w_8}+\overrightarrow{x_4}=(-1,3,1,-1);$
$\overrightarrow{x_1}\cdot\overrightarrow{w_9}=-1,\ output\ 0\ is\ correct;$
$\overrightarrow{x_2}\cdot\overrightarrow{w_{10}}=-1,\ output\ 0\ is\ correct;$
$\overrightarrow{x_3}\cdot\overrightarrow{w_{10}}=1,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_{11}}=\overrightarrow{w_{10}}-\overrightarrow{x_3}=(-2,2,1,-2);$
$\overrightarrow{x_4}\cdot\overrightarrow{w_{11}}=-1,\ output\ 0\ is\ incorrect,\ \overrightarrow{w_{12}}=\overrightarrow{w_{11}}+\overrightarrow{x_4}=(-1,3,2,-1);$
$\overrightarrow{x_1}\cdot\overrightarrow{w_{12}}=-2,\ output\ 0\ is\ correct;$
$\overrightarrow{x_2}\cdot\overrightarrow{w_{12}}=0,\ output\ 0\ is\ correct;$
$\overrightarrow{x_3}\cdot\overrightarrow{w_{12}}=1,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_{13}}=\overrightarrow{w_{12}}-\overrightarrow{x_3}=(-2,2,2,-2);$
$\overrightarrow{x_4}\cdot\overrightarrow{w_{13}}=0,\ output\ 0\ is\ incorrect,\ \overrightarrow{w_{14}}=\overrightarrow{w_{13}}+\overrightarrow{x_4}=(-1,3,3,-1);$
$\overrightarrow{x_1}\cdot\overrightarrow{w_{14}}=-2,\ output\ 0\ is\ correct;$
$\overrightarrow{x_2}\cdot\overrightarrow{w_{14}}=1,\ output\ 1\ is\ incorrect,\ \overrightarrow{w_{15}}=\overrightarrow{w_{14}}-\overrightarrow{x_2}=(-2,3,2,-2);$
$\overrightarrow{x_3}\cdot\overrightarrow{w_{15}}=-1,\ output\ 0\ is\ correct;$
$\overrightarrow{x_4}\cdot\overrightarrow{w_{15}}=1,\ output\ 1\ is\ correct;$
$\overrightarrow{x_1}\cdot\overrightarrow{w_{15}}=-4,\ output\ 0\ is\ correct;$
$\overrightarrow{x_2}\cdot\overrightarrow{w_{15}}=-2,\ output\ 0\ is\ correct.$

Thus, the final weight vector is (-2,3,2,-2).