Assignment 1

1 List the first 10 terms of each Sequence

a) 10, 7, 4, 1, -2, -5, -8, -11, -14, -17

b) ~~For n = 0 → 1~~
~~n = 1 → 1 + 2~~

$\sum_{k=1}^{n} k$

n	
0	1
1	1+2 = 3
2	3+3 = 6
3	6+4 = 10
4	10+5 = 15
5	15+6 = 21
6	21+7 = 28
7	28+8 = 36
8	36+9 = 45
9	45+10 = 55

c

n^{th} term is $3^n - 2^n$

from $n\,[1 \Rightarrow 10]$

$n = 1 \Rightarrow 3^1 - 2^1 = 3 - 2 = 1$

$n = 2 \Rightarrow 3^2 - 2^2 = 9 - 4 = 5$

$n = 3 \Rightarrow 3^3 - 2^3 = 27 - 8 = 19$

$n = 4 \Rightarrow 3^4 - 2^4 = 81 - 16 = 65$

$n = 5 \Rightarrow 3^5 - 2^5 = 243 - 32 = 211$

$n = 6 \Rightarrow 3^6 - 2^6 = 729 - 64 = 665$

$n = 7 \Rightarrow 3^7 - 2^7 = 2187 - 128 = 2059$

$n = 8 \Rightarrow 3^8 - 2^8 = 6561 - 256 = 6305$

$n = 9 \Rightarrow 3^9 - 2^9 = 19{,}683 - 512 = 19{,}171$

$n = 10 \Rightarrow 3^{10} - 2^{10} = 59{,}049 - 1024 = 58{,}025$

= 1, 5, 19, 16, 211, 665, 2059, 6305,

119171, 58025

d) n^{th} term is $\lfloor \sqrt{n} \rfloor$

n	
1	$\lfloor \sqrt{1} \rfloor = 1$
2	$\lfloor \sqrt{2} \rfloor = \lfloor 1.141 \rfloor = 1$
3	$\lfloor \sqrt{3} \rfloor = 1$
4	$\lfloor \sqrt{4} \rfloor = 2$
5	$\lfloor \sqrt{5} \rfloor = 2$
6	$\lfloor \sqrt{6} \rfloor = 2$
7	$\lfloor \sqrt{7} \rfloor = 2$
8	$\lfloor \sqrt{8} \rfloor = 2$
9	$\lfloor \sqrt{9} \rfloor = 3$
10	$\lfloor \sqrt{10} \rfloor = 3$

$\Rightarrow$ 1, 1, 1, 2, 2, 2, 2, 2, 3, 3

e) First 2 terms 1 & 5

n	
1	1
2	5
3	6
4	11
5	17
6	28
7	45
8	73
9	118
10	191

(g)

n	
1	1
2	$1+1 = 2$
3	$2 \times 1 = 2$
4	$2+2 = 4$
5	$4 \times 2 = 8$
6	$8+3 = 11$
7	$11 \times 3 = 33$
8	$33+4 = 37$
9	$37 \times 4 = 148$
10	$148+5 = 153$

h)

n	
1	$1! = 1 \leq 1 = 1$
2	$2! = 2$
3	$2! \leq 3 = 2$
4	$2! \leq 4 = 2$
5	$2! \leq \cancel{4}5 = 2$
6	$3! \leq 6 = 3$
7	$3! \leq 7 = 3$
8	$3! \leq 8 = 3$
9	$3! \leq 9 = 3$
10	$3! \leq 10 = 3$

1

2 Fust 6 terms of the sequence

a) $a_n = -2a_{n-1}$; $a_0 = -1$

$a_1 = 2$
$a_2 = -4$
$a_3 = 8$
$a_4 = -16$
$a_5 = 32$
$a_6 = -64$

b) $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = -1$

$a_2 = -3$
$a_3 = -2$
$a_4 = 1$
$a_5 = 3$
$a_6 = 2$
$a_7 = -1$

c) $a_n = 3a^2_{n-1}$, $a_0 = 1$

$a_1 = 3$
$a_2 = 27$
$a_3 = 2187$
$a_4 = 14348907$
$a_5 = 617673396283947$
$a_6 = 114456127343087498854969642 7$

d) $a_n = n a_{n-1} + a_{n-2}^2$, $a_0 = -1$
$a_1 = 0$

$a_2 = 1$
$a_3 = 3$
$a_4 = 13$
$a_5 = 74$
$a_6 = 613$
$a_7 = 9767$

e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, $a_0 = 1$
$a_2 = 2$

$a_3 = 2$
$a_4 = 1$
$a_5 = 1$
$a_6 = 2$
$a_7 = 2$
$a_8 = 1$

3

a) $a_n = 0$

for $a_n = -3a_{n-1} + 4a_{n-2}$

$= -3(0) + 4(0) = 0 = a_n$

// Solution of recurrence

b) $a_n = 1$

$a_n = -3a_{n-1} + 4a_{n-2}$

$= -3(1) + 4(1) = -3 + 4 = 1 = a_n$

// Solution of recurrence relations

c) $a_n = (-4)^n$

$a_n = -3a_{n-1} + 4a_{n-2}$

$= -3(-4)^{n-1} + 4(-4)^{n-2}$

$= (-4)^{n-2}(-3(-4) + 4)$

$= (-4)^{n-2}(12+4) = (-4)^{n-2}(16)$

$= (-4)^{n-2}(-4)^2 = a_n$

// Solution of recurrence relations

d) $a_n = 2(-4)^n + 3$

$= -3a_{n-1} + 4a_{n-2}$

$= -3(2(-4)^{n-1}+3) + 4(2(-4)^{n-2}+3)$

$= (-4)^{n-2}(-3(-8)+8) + (-3)(3) + 4(3)$

$= (-4)^{n-2}(32) + 12 - 9$

$= (-4)^{n-2}(-4)^2(2) + 3$

$= 2(-4)^n + 3$

$= a_n$

// thus it's a recurrance relation

4

a) $3, 6, 11, 18, 27 \ldots 102 \ldots$

$$a_1 + 2 = 3$$
$$a_2 + 2 = 6$$
$$a_3 + 2 = 11$$

$$a_1 = 3 - 2 = 1 \quad (1)^2$$
$$a_2 = 6 - 2 = 4 \quad (2)^2$$
$$a_3 = 11 - 2 = 9 \quad (3)^2$$

pattern $= n^2 + 2$
next three

$= 123, 146, 171$

b) $7, 11, 15, 19, 23, 27 \ldots 43 \ldots$

$$a_1 \pm b_1 = 7$$
$$a_2 \pm b_2 = 11$$
$$a_3 \pm b_3 = 15$$

$$4 + 3 = 7$$
$$4(2) + 3 = 11$$
$$4(3) + 3 = 15$$

pattern $= 4(n) + 3$
next three $= 47, 51, 55$

c

1
10
11
100
101
110
111
1000
1001
1010
1011

binary numbers incrementing

pattern ⇒ binary number incrementing
next three ⇒ 1100
1101
1110

d 1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5 . . . 5 . . .

? ~~B~~ couldn't crack this one

∴ maybe some marks for honesty

e 0, 2, 8, 26, 80, 242, 728

$$\begin{aligned} 2 &= 3-1 = (3)^1-1 \\ 8 &= 9-1 = (3)^2-1 \\ 26 &= 27-1 = (3)^3-1 \end{aligned}$$

pattern $\Rightarrow 3^n - 1$

next three terms $\Rightarrow$ 59048, 177146
531440

f

1	$\rightarrow$	1×1	pattern:
3	$\rightarrow$	1×3	multiplying
15	$\rightarrow$	3×5	last
105	$\rightarrow$	15×7	number
945	$\rightarrow$	105×9	with
⋮			odd sequential
⋮			numbers
⋮			
34459425			

Next three $\Rightarrow$ 654729075
13749310575
316234143225

g 1, 0 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1

~~1, 0, 1~~
pattern : number of digits 1 → 1
2 → 0
3 → 1
4 → 0

There are 1 one the 2 zero's the 3 one's the 4 zero's

h 2, 4, 16, 256, 65536

2 → starting point
4 → square of last term
16 → square of last term

5

a) $$\sum_{i=1}^{3}\sum_{j=1}^{2}(i-j)$$

$\Rightarrow \cancel{((3-2)+(2-1))}$

$\Rightarrow \cancel{((1)+(1))=2}$

$$=\sum_{i=1}^{3}(i-1)+(i-2)$$

$$=\sum_{i=1}^{3}(2i-3)$$

$$=\sum_{i=1}^{3}((2(1)-3)+(2(2)-3)+(2(3)-3)$$

$$=-1+1+3=3 \text{ Ans}$$

b) $$\sum_{i=0}^{3}\sum_{j=0}^{2}(3i+2j)$$

$$\sum_{i=0}^{3}((3i+2(2))+(3i+2(1))+(3i+2(0))$$

$$\sum_{i=0}^{3}=3i+4+3i+2+3i+0$$

$$=9i+6$$

$$\sum_{i=0}^{3} (9i+6)$$

$$= (9(3)+6) +$$
$$(9(2)+6) +$$
$$(9(1)+6) +$$
$$(9(0)+6)$$

$$= 54 + 24$$

$$= 78$$

c. $$\sum_{i=1}^{3} \sum_{j=0}^{2} j$$

$$= 3(2+1+0)$$

$$= 3(3) = 9$$

d. $$\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$$

$$= \sum_{i=0}^{2} i^2(3)^3 + i^2(\cancel{3}2)^3 + i^2(1)^3 + i^2(0)^3$$

$$= \sum_{i=0}^{2} 27i^2 + 8i^2 + 1i^2 + 0$$

$$= \sum_{i=0}^{2} 36i^2$$

$$= \sum_{i=0}^{2} 36i^2$$

$$= 36(2)^2 + 36(1)^2 + 36(0)^2$$

$$= 36(4) + 36$$

$$= 144 + 36$$

$$= 180$$

6

a) The integers that are greater than 10

$\rightarrow$ Countably infinite

from 10 $\Rightarrow$ it's approaching infinity

$f : \mathbb{Z}_{>10} \rightarrow N$ ~~#tag~~

One to one correspondence $\rightarrow$ $z \rightarrow z - 11$

b) Coutably infinite

$f: (2K+1 \mid K \in \mathbb{Z}_{<0} \} \rightarrow N)$

one-to-on $\rightarrow$ $2K+1 \rightarrow -K-1$

c) Finite , $2(10,000,00 - 1) + 1$

d) Uncountable, $|0 \rightarrow 2|$ containd
$|0 \rightarrow 1|$

e) Countable

$f: (2, 3) \times \mathbb{Z}_{>0} \rightarrow N$

one-to-one correspondance:

$$(n, 2) \rightarrow \begin{cases} 2(2-1) & \text{if } n = 2 \\ 2(2-1)+1 & n = 3 \end{cases}$$

f Coutable, infinte

$f: \{ 10k \mid k \in \mathbb{Z} \} \rightarrow N$

One to one $\rightarrow$ $10k \rightarrow \begin{cases} 2k & \text{for +ve} \\ -2k-1 & \text{for -ve} \end{cases}$

7

a) Integer not divisible by 3

: Countable

- we can map a one-to one with natural numbers not including the multiples of 3 and alternating b/w both +ve and -ve elements

0	→	1
1	→	-1
2	→	2
3	→	-2
4	→	4
5	→	-4
6	→	5
7	→	-5
8	→	~~6~~ 7
9	→	-~~6~~ 7

h) Integers divisible by 5 but not by 7

: Countable

- we can map a one-to-one with natural numbers not including the multiple's of 7 which in this case would be multiples of 35

$0 \rightarrow 5$
$1 \rightarrow -5$
$2 \rightarrow 10$
$3 \rightarrow -10$
$4 \rightarrow 15$
$5 \rightarrow -15$
$6 \rightarrow 20$
$7 \rightarrow -20$
$8 \rightarrow +25$
$9 \rightarrow -25$
$10 \rightarrow 30$
$11 \rightarrow -30$
$12 \rightarrow 40$
$13 \rightarrow -40$

c Real numbers with decimal representation of all 1's

Countable :

~~[illegible]~~

$N = \{ K, I \}$ where K belongs to the set of rational numbers

d Not Countable