Assignment 4

1

total Cards = 52
total Cards that are hearts = 13
number of Aces = 4
both Ace and heart = 1

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{17}{52} - \frac{1}{52} = \frac{16}{52}$$

2

$$P(E) = \frac{|E|}{|S|}$$

$|E| = C(51,4)$ $|S| = C(52,5)$

$$= \frac{\frac{51!}{4!(52-4!)}}{\frac{52!}{5!(52-5!)}}$$

$$= \frac{5}{52}$$

3 $P(E) = \frac{|E|}{|S|}$

$|E| = {}_{48}C_4$

$|S| = {}_{52}C_5$

$= \frac{{}_{52}C_5}{{}_{48}C_4}$

$$= \frac{\frac{52!}{5!(52-5)!}}{\frac{48!}{4!(48-4)!}}$$

4

(a) 30

$S = \{1 \ldots\ldots 30\}$

selecting 6 integers from S

$C(30,6)$

$P = \frac{C(6,6)}{C(30,6)} = \frac{1}{C(30,6)}$

(b) $S = \{1 \ldots \ldots 36\}$

total ways to select from S
$\hookrightarrow C(36, 6)$

$P(E)$ $\dfrac{C(6,6)}{C(36,6)} = \dfrac{1}{C(36,6)}$

(c) $S = \{1 \ldots \ldots 42\}$

total ways to select from S
$\hookrightarrow C(42, 6)$

$P(E) = \dfrac{C(6,6)}{C(42,6)} = \dfrac{1}{C(42,6)}$

(d) $S = \{1 \ldots \ldots 48\}$

total ways to select from S
$\hookrightarrow C(48, 6)$

$P(E) = \dfrac{C(6,6)}{C(48,6)} = \dfrac{1}{C(48,6)}$

5

(a) 40

$|S| = C(40,6)$

$|E| = C(34,6)$

$P(E) = \frac{|E|}{|S|} = \frac{C(34,6)}{C(40,6)} = 0.35$

(b) ~~8~~ 48

$|S| = C(48,6)$

$|E| = C(42,6)$

$P(E) = \frac{|E|}{|S|} = \frac{C(42,6)}{C(48,6)} = 0.43$

(c) $|S| = C(56, 6)$

$|E| = \cancel{C}C(50, 6)$

$$P(E) = \frac{|E|}{|S|} = \frac{C(50,6)}{C(56,6)} = 0.49$$

(d) $S = C(64, 6)$

$E = C(58, 6)$

$$P(E) = \frac{|E|}{|S|} = \frac{C(58,6)}{C(64,6)} = 0.54$$

6

number of ways to select 7 ~~8~~ numbers out of 80 +ve integers

$= C(80,7)$

number of ways for it to be in ~~8~~ 11 numbers selected

$C(11,7)$

$$P(E) = \frac{|E|}{|S|} = \frac{C(11,7)}{C(80,7)}$$

7 $P(E) = \cancel{|S|\,|E|}\ \frac{|E|}{|S|}$

$|S| = C(40, 6)$

$|E| = C(6, 5) \times C(40-6, 1)$

$$P(E) = \frac{C(6,5) \times C(34, 1)}{C(40, 6)}$$

8 Three winners can ocupphy prizen in 3! ways

· Ways to choose a winner $C(100, 3)$

$$P(E) = \frac{|E|}{|S|} = \frac{3!}{C(100,3)} = \frac{1}{26950}$$

9 $P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$

$5P \qquad 2p$

$\Rightarrow 7p = 1$

$p = \frac{1}{7}$

(p) of $1, 2, 4, 5, 6 = \frac{1}{7} \qquad p(3) = \frac{2}{7}$

B

10

(a) 1 precedes 3

$E = \{123, 213, 132\}$

$|E| = 3$

$|S| = 3! = 6$

$P(E) = \frac{3}{6} = \frac{1}{2}$

(b) $|S| = 3! = 6$

$E|\cancel{\theta} = \{(3\cdot12,\ 23\cancel{3}1,\ 32,1)\}$

$= 3$

$P(E) = \frac{|E|}{|S|} = \frac{3}{6} = \frac{1}{2}$

c $|S| = 3! = 6$

$|E| = \{312, 321\}$

$= 2$

$P(E) = \frac{2}{6} = \frac{1}{3}$

11 $P(E|F) = \frac{P(E \cap F)}{P(F)}$

$P(E \cap F) = \frac{1}{32}$

$P(F) = \frac{1}{2}$

$= \frac{\frac{1}{32}}{\frac{1}{2}} = \frac{1}{16}$

12

$E = \{100, 010, 001, 111\}$

$|E| = 4$

$E \cap F = \{100, 010, 001, 111\}$

$\cap$

$\{100, 110, 101, 111\}$

$E \cap F = 2$

$|S| = \{000, 010, 001 \ldots\ldots 111\}$

$= 8$

$p(E) = \frac{|E|}{|S|}$

$p(E \cap F) = p(E) \times p(F)$

$p(E) = \frac{|E|}{|S|} = \frac{4}{8} = \frac{1}{2}$

$p(F) = \frac{|F|}{|S|} = \frac{4}{8} = \frac{1}{2}$

$$p(E \cap F) = \frac{|E \cap F|}{|S|}$$

$$= \frac{2}{8} = \frac{1}{4}$$

$$p(E \cap F) = \frac{1}{4}$$

$$= p(E) \times p(F)$$

E and F are independent.

13 probability of selecting for orang box 1 = $3/14$

$$\Downarrow$$

$$\left(\frac{1}{2}\right)\left(\frac{3}{7}\right)$$

probability of selecting orange box 2 = $\frac{5}{22}$

$$\Downarrow$$

$$\left(\frac{1}{2}\right)\left(\frac{5}{11}\right)$$

prob of second orange = $\cancel{\left(\frac{5}{22}\right)}$ / $\dfrac{\frac{5}{22}}{\frac{3}{14} + \frac{5}{22}}$

14 $p(E) = 5\%$

$P(F) = 95\%$

$p(G/F) = 98\%$

$P(E_2/F) = 12\%$

$p(G/E_2) =$

$=$

$$\frac{98\% \times 5\%}{98\% \times 5\% + 12\% \times 95\%} = 0.3006$$