

Assignment 3

①

17 Convert following EBNF to BNF

$$S \rightarrow A \{ b A \}^*$$

$$A \rightarrow a | b | A$$

Solution \Rightarrow

Converting

$$\langle S \rangle \Rightarrow \langle S \rangle b \langle A \rangle | \langle A \rangle$$

$$\langle A \rangle \Rightarrow a \langle A \rangle | ab \langle A \rangle$$

23 Finding weakest precondition

a) $a = 2^*(b-1) - 1 \{ a > 0 \}^?$

$$\Rightarrow \{ 2^*(b-1) - 1 > 0 \}^?$$

$$\Rightarrow \{ 2^* b - 3 > 0 \}^?$$

$$\Rightarrow \{ b > 3/2 \}^?$$

b) $b = (c+10)/3 \{ b > 6 \}^?$

$$\Rightarrow \{ (c+10)/3 > 6 \}^?$$

$$\Rightarrow \{ c+10 > 18 \}^?$$

$$\Rightarrow \{ c > 8 \}^?$$

$$\underline{\underline{c}} \quad a = 2^* b - 1 \quad \{ a > 1 \}$$

$$\Rightarrow \{ 2^* a + 2^* b - 1 > 1 \}$$

$$\Rightarrow \{ 2^* a + 2^* b > 2 \}$$

$$\Rightarrow \{ 2^* b > 2 - a \}$$

$$\Rightarrow \{ b > (2 - a)/2 \}$$

$$\Rightarrow \{ b > 1 - (a/2) \}$$

$$d) \quad x = 2^* y + x - 1 \quad \{ x > 11 \}$$

$$\Rightarrow \{ 2^* y + x - 1 > 11 \}$$

$$\Rightarrow \{ 2^* y + x > 12 \}$$

$$\Rightarrow \{ 2^* y > 12 - x \}$$

$$\Rightarrow \{ y > 6 - (x/2) \}$$

24

$$\textcircled{i} \begin{cases} a = 2 * b + 1; \\ b = a - 3 \\ b < 0 \end{cases}$$

$$\Rightarrow \begin{cases} a = 2 * b + 1; & \textcircled{i} \\ b = a - 3 & \textcircled{ii} \\ b < 0 \end{cases}$$

Applying Axiom to \textcircled{i} & \textcircled{ii}

$$\Rightarrow \begin{cases} a - 3 < 0 \\ a < 3 \end{cases} \quad \textcircled{iii}$$

$$\Rightarrow \begin{cases} 2 * b + 1 < 3 \\ 2 * b < 2 \\ b < 1 \end{cases}$$

$b < 1$ is weakest for Dom \textcircled{i} & \textcircled{ii}

$$\textcircled{ii} \begin{cases} a = 3 * (2 * b + a); \\ b = 2 * a - 1 \\ b > 5 \end{cases}$$

$$\Rightarrow \text{Applying Axiom to both statements}$$

$$\begin{cases} 2 * a - 1 > 5 \\ 2 * a > 6 \\ a > 3 \end{cases}$$

So ~~$\{a > 3\}$~~ $\{a > 3\}$ is weakest
for statement 2 and becomes
post condition for statement 1.

$$\{3 * (2 * b + a) > 3\}$$

$$\{2 * b + a > 1\}$$

$$\{a > 1 - 2 * b\}$$

hence $\{a > 1 - 2 * b\}$ is weakest

25

a) if $(a == b)$

$$b = 2 * a + b$$

else

$$b = 2 * a;$$

$$\{b > 1\}$$

post condition = $b > 1$ So

$$\Rightarrow 2^*a + 1 > 1 \rightarrow 2^*a > 0 \rightarrow a > 0$$

$$\Rightarrow 2^*a > 1 \rightarrow a > 1/2$$

$$\Rightarrow a > 1/2 \rightarrow a > 0 \text{ ~~was~~$$

^{as}
but $a > 0 \nrightarrow a > 1/2$

$$\{ a > \frac{1}{2} \}$$

b) if $(x < y)$

$$x = x + 1$$

else

$$x = 3^*x$$

$$\{ x < 0 \}$$

post condition $\Rightarrow x < 0$
so

$$\Rightarrow x + 1 < 0 \rightarrow x < -1$$

$$\Rightarrow 3^*x < 0 \rightarrow \cancel{x < 0} \rightarrow x < 0$$

$$\Rightarrow x < -1 \rightarrow x < 0$$

$$x < 0 \nrightarrow x < -1$$

$$\Rightarrow \{ x < -1 \}$$

if $(x > y)$

$$y = 2 * x + 1$$

else

$$y = 3 * x - 1;$$

$$\{ y > 3 \}$$

post condition $\rightarrow y > 3$

$$= 2 * x + 1 > 3 \Rightarrow 2 * x > 2 \Rightarrow x > 1$$

$$\Rightarrow 3 * x - 1 > 3 \Rightarrow 3 * x > 4 \Rightarrow x > \frac{4}{3}$$

$$= x > \frac{4}{3} \Rightarrow x > 1$$

$$x > 1 \nrightarrow x > \frac{4}{3}$$

the weakest precondition $\{ x > \frac{4}{3} \}$

2 Problems, 1, 2, 4, 5, 6, 8

1

(a) $A \Rightarrow aB | b | cBB$

(b) $B \Rightarrow aB | bA | aBb$

(c) $C \Rightarrow aaA | b | caB$

\Rightarrow Grammar

(a) $AaB | b | cBB$

\rightarrow RHS for Non terminal A ~~is~~ FIRSTs value

~~First~~ $(aB) = \{a\}$

$FIRST(AaB) = \{a\}$

$FIRST(b) = \{b\}$

$FIRST(cBB) = \{c\}$

all of these are pairwise disjoint to each other

b) $BaB \mid bA \mid aBb$

→ RNS of non-terminal B First Value

$$\text{FIRST}(aB) = \{a\}$$

$$\text{FIRST}(bA) = \{b\}$$

$$\text{FIRST}(aBb) = \{a\}$$

The ~~first~~ $\text{FIRST}(ab)$ and $\text{FIRST}(aBb)$
have common element "a"

c) $CaA \mid b \mid caB$

$$\text{FIRST}(CaA) = \{a\}$$

$$\text{FIRST}(b) = \{b\}$$

$$\text{FIRST}(caB) = \{c\}$$

All the ~~parts~~ ~~over~~ ~~a~~ sets are pairwise
disjoint to each other

2

$$a) S \Rightarrow aSb \mid bAA$$

$$\text{FIRST}(aSb) = \{a\}$$

$$\text{FIRST}(b) = \{b\}$$

$$\text{FIRST}(bAA) = \{b\}$$

Intersection of $\text{FIRST}(aSb) \cap \text{FIRST}(b)$

So,

$$\text{FIRST}(aSb) \cap \text{FIRST}(b) = \emptyset$$

$$\text{Intersection of } \text{FIRST}(aSb) \cap \text{FIRST}(bAA) = \emptyset$$

$$\text{Intersection of } \text{FIRST}(b) \cap \text{FIRST}(bAA) = \emptyset$$

$\text{FIRST}(b)$ and $\text{FIRST}(bAA)$ have element 'b' in common, So, the grammar fails

$$b) A \Rightarrow b \{aB\}^* \mid a$$

$$Ab \{aB\}^* \mid a$$

$$\text{FIRST}(b \{aB\}^*) = \{b\}$$

$$\text{FIRST}(\{a\}) = \{a\}$$

Intersection of $\text{FIRST}(b \{aB\}^*) \cap \text{FIRST}(a)$

So,

$$\text{FIRST}(b \{aB\}^*) \cap \text{FIRST}(a) = \emptyset$$

The grammar passes the test as the right hand side or Non terminal A satisfies always disjoint

$$c) B \rightarrow aB \mid a$$

$$BaB \mid a$$

$$\text{First}(aB) = \{a\}$$

$$\text{FIRST}(a) = \{a\}$$

$$\text{Intersection } \text{FIRST}(aB) \cap \text{FIRST}(a) = a$$

~~The~~ The grammar fails to pass the test as the right hand side of non terminal symbol B does not satisfy pairwise disjoint

4 The trace for recursive descent parsing

1) Next token is 11, Next lexeme is a
Enter
Enter
Enter (<factor>)

2) Next token is 23, Next lexeme is *
Exit

3) Next token is 25, Next lexeme is (<
Enter

Next token is 11: Next lexeme is b
Enter
Enter
Enter <factor>

Next token is 21: Next lexeme is +
Exit
Exit

Next token is 11: Next lexeme is c
Enter
Enter <factor>

Next token is 26: Next lexeme is)
Exit
Exit
Exit

Next token is -1: Next lexeme is EOF
Exit
Exit
Exit