Assignment 1

CS 310: Discrete Computational Structures

University of Regina

Department of Computer Science

Fall 2018

Due date: September 25, 2018 at the beginning of class

1. (16 points) List the first 10 terms of each the sequences.
   (a) the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term
   (b) the sequence whose nth term is the sum of the first n positive integers
   (c) the sequence whose nth term is $3^n - 2^n$
   (d) the sequence whose nth term is $\lfloor \sqrt{n} \rfloor$
   (e) the sequence whose first two terms are 1 and 5 and each succeeding term is the sum of the two previous terms
   (f) the sequence whose nth term is the largest integer whose binary expansion has n bits (Write your answer in decimal notation)
   (g) the sequence whose terms are constructed sequentially as follows: start with 1, then add 1, then multiply by 1, then add 2, then multiply by 2 and so on
   (h) the sequence whose nth term is the largest integer k such that $k! \leq n$

2. (10 points) Find the first six terms of the sequence defined by each of these recurrence relations and intital conditions.
   (a) $a_n = -2a_{n-1}, a_0 = -1$
   (b) $a_n = a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1$
   (c) $a_n = 3a_{n-1}^2, a_0 = 1$
   (d) $a_n = na_{n-1} + a_{n-2}^2, a_0 = -1, a_1 = 0$
   (e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}, a_0 = 1, a_1 = 1, a_2 = 2$

3. (8 points) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

   (a) $a_n = 0$

   (b) $a_n = 1$

   (c) $a_n = (-4)^n$

   (d) $a_n = 2(-4)^n + 3$

4. (24 points) For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

   (a) 3, 6, 11, 18, 27, 38, 51, 66, 83, 102, ...

   (b) 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, ...

   (c) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, ...

   (d) 1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, ...

   (e) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, ...

   (f) 1, 3, 15, 105, 945, 10395, 135135, 2027025, 34459425, ...

   (g) 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1,...

   (h) 2, 4, 16, 256, 65536, 4294967296, ...

5. (8 points) Compute each of these double sums.

   (a)
$$\sum_{i=1}^{3}\sum_{j=1}^{2}(i-j)$$

   (b)
$$\sum_{i=0}^{3}\sum_{j=0}^{2}(3i+2j)$$

   (c)
$$\sum_{i=1}^{3}\sum_{j=0}^{2} j$$

(d)

$$\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3$$

6. (18 points) Determine whether each of these sets is finite, countably infinite or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

   (a) the integers that are greater than 10

   (b) the odd negative integers

   (c) the integers with absolute value less than 1,000,000

   (d) the real numbers between 0 and 2

   (e) the set $A \times \mathbf{Z}^+$ where $A = \{2, 3\}$

   (f) the integers that are multiples of 10

7. (16 points) Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

   (a) integers not divisible by 3

   (b) integers divisible by 5 but not by 7

   (c) the real numbers with decimal representations consisting of all 1s

   (d) the real numbers with decimal representations of all 1s or 9s