Assignment 2
CS 310: Discrete Computational Structures
University of Regina
Department of Computer Science
Fall 2018

Due date: October 10, 2018 at 1:30 pm

1. (10 points) Devise an algorithm to compute x^n, where x is a real number and n is an integer. [*Hint:* First give a procedure for computing x^n when n is non-negative by successive multiplication by x, starting with 1. Then extend this procedure, and use the fact that $x^{-n} = 1/x^n$ to compute x^n when n is negative.]

2. (10 points) Describe an algorithm for finding the smallest integer in a finite sequence of natural numbers.

3. (10 points) Specify the steps of an algorithm that locates an element in a list of increasing integers by successively splitting the list into four sublists of equal (or as close to equal as possible) size, and restricting the search to the appropriate piece.

4. (18 points) Determine whether each of these functions is $O(x^2)$.
   (a) $f(x) = 17x + 11$
   (b) $f(x) = x^2 + 1000$
   (c) $f(x) = x \log x$
   (d) $f(x) = x^4/2$
   (e) $f(x) = 2^x$
   (f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

5. (4 points) Use the definition of "$f(x)$ is $O(g(x))$" to show that $2^x + 17$ is $O(3^x)$.

6. (6 points) Show that $(x^3 + 2x)/(2x + 1)$ is $O(x^2)$.

7. (12 points) Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

   (a) $f(x) = 2x^2 + x^3 \log x$
   (b) $f(x) = 3x^5 + (\log x)^4$
   (c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$
   (d) $f(x) = (x^3 + 5 \log x)/(x^4 + 1)$

8. (10 points) Arrange the function $(1.5)^n$, n^{100}, $(\log n)^3$, $\sqrt{n} \log n$, 10^n, $(n!)^2$, and $n^{99} + n^{98}$ in a list so that each function is big-O of the next function.

9. (10 points) Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$ whenever n is a positive integer.

10. (10 points) Prove that for every positive integer n,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4.$$