Assignment 5

CS 310: Discrete Computational Structures University of Regina Department of Computer Science Fall 2018

1. (35 points) Solve these recurrence relations together with the initial conditions given.

(a)
$$a_n = a_{n-1} + 6a_{n-2}$$
 for $n \ge 2, a_0 = 3, a_1 = 6$

(b)
$$a_n = 7a_{n-1} - 10a_{n-2}$$
 for $n \ge 2, a_0 = 2, a_1 = 1$

(c)
$$a_n = 6a_{n-1} - 8a_{n-2}$$
 for $n \ge 2, a_0 = 4, a_1 = 10$

(d)
$$a_n = 2a_{n-1} - a_{n-2}$$
 for $n \ge 2, a_0 = 4, a_1 = 1$

(e)
$$a_n = a_{n-2}$$
 for $n \ge 2, a_0 = 5, a_1 = -1$

(f)
$$a_n = -6a_{n-1} - 9a_{n-2}$$
 for $n \ge 2, a_0 = 3, a_1 = -3$

(g)
$$a_{n+2} = -4a_{n+1} + 5a_n$$
 for $n \ge 0, a_0 = 2, a_1 = 8$

- 2. (7 points) Find the solution to $a_n = 2a_{n-1} + a_{n-2} 2a_{n-3}$ for $n = 3, 4, 5, \ldots$, with $a_0 = 3, a_1 = 6$ and $a_2 = 0$.
- 3. (16 points) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if
 - (a) a is taller than b.
 - (b) a and b were born on the same day.
 - (c) a has the same first name as b.
 - (d) a and b have a common grandparent.
- 4. (32 points) Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or ransitive, where $(x,y) \in R$ if and only if
 - (a) x + y = 0.

- (b) $x = \pm y$.
- (c) x y is a rational number.
- (d) x = 2y.
- (e) $xy \geqslant 0$.
- (f) xy = 0.
- (g) x = 1.
- (h) x = 1 or y = 1.
- 5. (10 points) Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.
 - (a) $\{(a,b) \mid a \text{ and } b \text{ are the same age}\}$
 - (b) $\{(a,b) \mid a \text{ and } b \text{ have the same parents}\}$
 - (c) $\{(a,b) \mid a \text{ and } b \text{ share a common parent}\}$
 - (d) $\{(a,b) \mid a \text{ and } b \text{ have met}\}$
 - (e) $\{(a,b) \mid a \text{ and } b \text{ speak a common language}\}$