

## Assignment 5

1

(a) characteristic equation:  $\lambda^2 - \lambda - 6$

$$\lambda = -5 \text{ and } 1$$

$$\alpha_1 + \alpha_2 = a_0 = 3$$

$$-5\alpha_1 + \alpha_2 = a_1 = 6$$

$$\text{solving } \alpha_1 = -\frac{1}{2}, \alpha_2 = 3\frac{1}{2}$$

(b) characteristic equation  $\rightarrow \lambda^2 - 7\lambda + 10$

$$\lambda = -5 \text{ and } -2$$

$$\alpha_1 + \alpha_2 = a_0 = 2, -5\alpha_1 - 2\alpha_2 = a_1 = 1$$

$$\alpha_1 = -\frac{5}{3}, \alpha_2 = \frac{11}{3}$$

$$\text{Therefore } a_n = -\frac{5}{3}(-5)^n + \frac{11}{3}(-2)^n$$

c

characteristic equation

$$r^2 - 6r + 8$$

$$r = -4 \text{ and } -2$$

$$\alpha_1 + \alpha_2 = \alpha_0 = 4, -4\alpha_1 - 2\alpha_2 = \alpha_1 = 10$$

$$\text{Solving } \alpha_1 = -9, \alpha_2 = 13$$

$$a_n = -9(-4)^n + 13(-2)^n$$

d

characteristic equation

$$r^2 - 2r + 1$$

$$r = -1 \text{ and } -1$$

$$\alpha_1 + 0\alpha_2 = \alpha_0 = 4$$

$$-\alpha_1 - 1\alpha_2 = \alpha_1 = 1$$

$$\alpha_1 = 4, \alpha_2 = 5$$

$$a_n = 4(-1)^n - 5n(-1)^n$$

c

characteristic equation

$$r^2 - 1$$

$$r = -1 \text{ and } 1$$

$$\alpha_1 + \alpha_2 = a_0 = 5$$

$$-\alpha_1 + \alpha_2 = a_1 = -1$$

$$\alpha_1 = 3 \quad \alpha_2 = 2$$

$$a_n = 3(-1)^n + 2$$

f

characteristic equation

$$r^2 + 4r - 5$$

$$r = -5, 1$$

$$\alpha_1 (-5)^n + \alpha_2 1^n = \alpha_1 (-5)^n + \alpha_2$$

$$2 = \alpha_1 + \alpha_2$$

$$-5\alpha_1 + \alpha_2 = 8$$

$$\alpha_1 = -1, \quad \alpha_2 = 3$$

$$a_n = (-5)^n + 3$$

2

The characteristic equation

$$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$$

$$\lambda^2(\lambda - 2) - 1(\lambda - 2) = 0$$

$$\Rightarrow (\lambda - 1)(\lambda + 1)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = -1, 1, 2$$

$$a_n = \alpha_1(-1)^n + \alpha_2(1)^n + \alpha_3(2)^n$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 3 \quad \text{--- (i)}$$

$$-\alpha_1 + \alpha_2 + 2\alpha_3 = 6 \quad \text{--- (ii)}$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 0 \quad \text{--- (iii)}$$

from (i) & (iii)

$$2\alpha_2 + 6\alpha_3 = 6 \quad \text{--- (iv)}$$

from (i) & (ii)

$$2\alpha_2 + 3\alpha_3 = 9 \quad \text{--- (v)}$$

from (iv) & (v)

$$3\alpha_3 = -3 \Rightarrow \alpha_3 = -1$$



$$\alpha_2 = 6, \alpha_1 = -2$$

$$a_n = (-2)(-1)^n + 6(1)^n + (-1)(2)^n$$

$$a_n = \text{odd } 2(-1)^{n+1} + 6(1)^n + (-1)(2)^n$$

3

(a) a is taller than b

Not Reflexive  $\rightarrow$  a is not taller than A

Not Symmetric  $\rightarrow$  If a is taller than B then B is not taller than A

Antisymmetric  $\rightarrow$  Yes

transitive  $\rightarrow$  Yes  $\begin{matrix} \text{if } x > y \\ y > z \\ \text{then } x > z \end{matrix}$

① ~~and~~ A and b were born on the same day

Reflexive  $\rightarrow$  Yes      a is born on the same day as a

Symmetric  $\rightarrow$  Yes      a and b both born on the same day

Asymmetric  $\rightarrow$  No      ~~can~~ a and c can be born on different days

transitive  $\rightarrow$  Yes

if  $x = x$   
 $y = 2$   
then  $x = 2$

② ~~a~~ a has the same first name as b

reflexive  $\rightarrow$  Yes  
Symmetric  $\rightarrow$  Yes  
Asymmetric  $\rightarrow$  NO  
transitive  $\rightarrow$  Yes

① A and b have common grandparent

Reflexive  $\rightarrow$  yes

Symmetric  $\rightarrow$  yes

Antisymmetric  $\rightarrow$  NO

transitive  $\rightarrow$  NO

4

②  $x + y = 0$

Reflexive  $\rightarrow$  NO

$$x + x \neq 0$$

Symmetric  $\rightarrow$  yes

$$x + y = y + x$$

~~Antisymmetric~~ Antisymmetric  $\Rightarrow$  NO

transitive  $\Rightarrow$  NO

(b)  $x = \pm y$

reflexive : Yes  
symmetric : Yes  
antisymmetric : No  
transitive : Yes

(c)  $x - y$  is a rational number

reflexive : yes  
symmetric : yes  
anti-symmetric : ~~yes~~ No  
transitive : yes

(d)  $x = 2y$

reflexive  $\Rightarrow$  No  
symmetric  $\Rightarrow$  No  
antisymmetric  $\Rightarrow$  Yes  
transitive  $\Rightarrow$  No

(e)  $xy > 0$

reflexive  $\Rightarrow$  Yes  
symmetric  $\Rightarrow$  Yes  
antisymmetric  $\Rightarrow$  No  
transitive  $\Rightarrow$  No



5

(a) reflexive  $\rightarrow$  Yes  
symmetric  $\rightarrow$  Yes  
transitive  $\rightarrow$  Yes

Equivalence relation  $\rightarrow$  true

(b) reflexive  $\rightarrow$  Yes  
symmetric  $\rightarrow$  Yes  
transitive  $\rightarrow$  Yes

Equivalence relation  $\rightarrow$  true

(c) reflexive  $\rightarrow$  Yes  
symmetric  $\rightarrow$  Yes  
transitive  $\rightarrow$  No

Equivalence relation  $\rightarrow$  false

(d) Same answer as option c

(e) Same answer as option c