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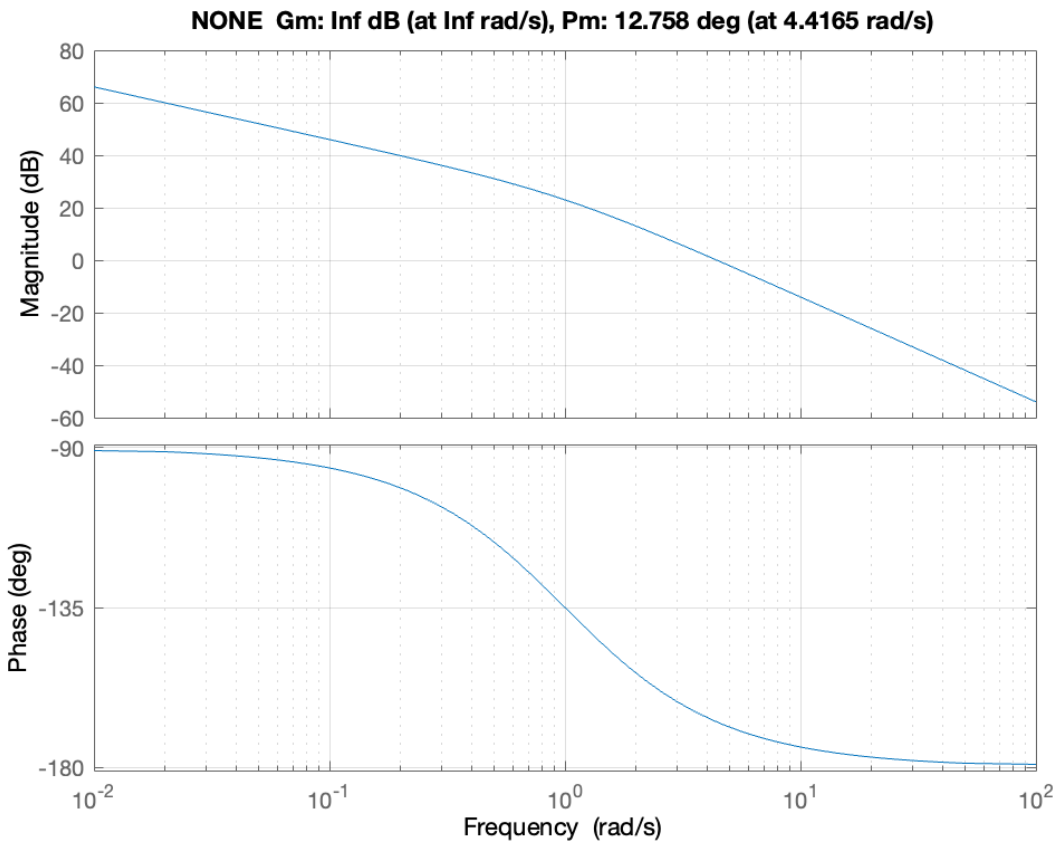
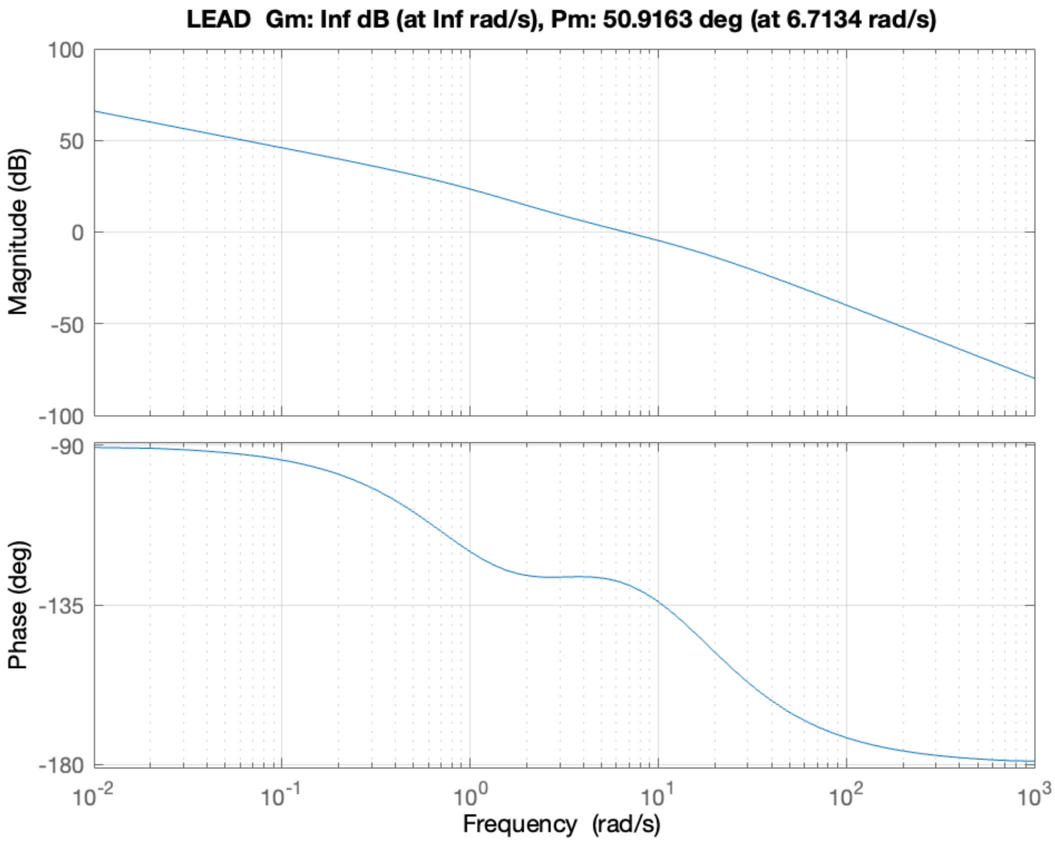
EE 3331C, HW3

RESULTS

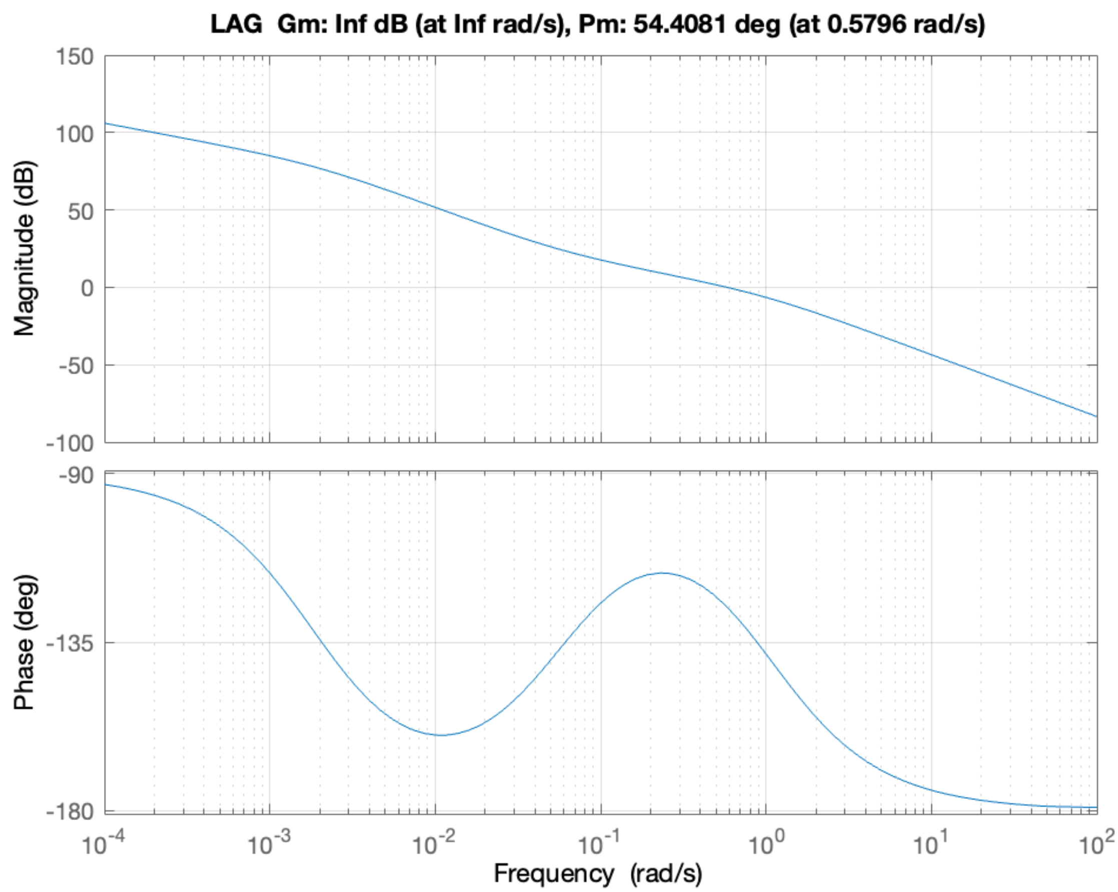
No compensation:



Lead compensation:



Lag compensation:



CODE

```
% A0271121X
% a = 1
%
% G(s) = 1/(s(s+a)), K(s) = K
% velocity error constant, Kv = 20
% phase margin of at least 50 deg

%% PART A - Lead compensator (see work in assignment)

D1 = tf([0.34, 1], [0.066, 1]); % +5 deg added to get >50 deg target

%% PART B - Design a lag compensator to meet the above specifications

D2 = tf([17.33, 1], [519.93, 1]); % +10 deg added to get >50 deg target

%% PART C - Simulate your results in Matlab and show the Bode plots

% Uncompensated system G(s)*K(s)
GK = tf(20, [1, 1, 0]); % numerator, denominator
figure;
h1 = bodeplot(GK);
grid on;

% Gain and phase margin analysis for G(s)*K(s)
[Gm, Pm, Wcg, Wcp] = margin(GK);
title(['{NONE}  Gm: ', num2str(Gm), ' dB (at ',num2str(Wcg),' rad/s), Pm: ', ...
    num2str(Pm), ' deg (at ',num2str(Wcp),' rad/s)']);



% Lead compensated system G(s)*D1(s)*K(s)
GD1K = GK*D1;
figure;
```

```
h2 = bodeplot(GD1K);
grid on;

% Gain and phase margin analysis for G(s)*D1(s)*K(s)
[Gm, Pm, Wcg, Wcp] = margin(GD1K);
title(['{LEAD}  Gm: ', num2str(Gm), ' dB (at ',num2str(Wcg),' rad/s), Pm: ', ...
   num2str(Pm), ' deg (at ',num2str(Wcp),' rad/s)']);


% Lag compensated system G(s)*D2(s)*K(s)
GD2K = GK*D2;
figure;
h3 = bodeplot(GD2K);
grid on;

% Gain and phase margin analysis for G(s)*D2(s)*K(s)
[Gm, Pm, Wcg, Wcp] = margin(GD2K);
title(['{LAG}  Gm: ', num2str(Gm), ' dB (at ',num2str(Wcg),' rad/s), Pm: ', ...
   num2str(Pm), ' deg (at ',num2str(Wcp),' rad/s)']);
```

MATH

Lead

1. Handle ess

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1 + KD_cG} \cdot R(s) \qquad D_c(s) = \frac{Ts+1}{\alpha Ts + 1} \qquad G(s) = \frac{1}{s(s+1)}$$

$$= \lim_{s \to 0} s \frac{1}{1 + K \frac{Ts+1}{\alpha Ts+1} \cdot \frac{1}{s(s+1)}} \cdot \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{s + K \frac{Ts+1}{\alpha Ts+1} \cdot \frac{1}{s+1}}$$

$$= \frac{1}{K_v} \longrightarrow e_{ss} = 20 = \frac{1}{K_v} \quad \therefore K_v = 20V$$

2. For the uncompensated system $KG(s)$, find PM

$$|KG| = \frac{K}{\omega\sqrt{\omega^2+1}} = 1$$

$$\omega^2(\omega^2+1) = 20^2$$

$$\omega_{cg} = 4.42 \text{ rad/s}$$

crossover frequency

$$PM = 180 + \angle KG$$
$$= 180 + \left(-90 - \tan^{-1}(\omega_{cg})\right)$$
$$= 12.748°$$
$$\approx 12.75°$$

no phase

$$\angle KG = \frac{K}{s(s+1)}$$

$-90°$ $\quad$ $-\tan\left(\frac{Im}{Re}\right)$

3. Use lead compensator for $PM \geq 50°$

a. Calculate ϕ, make sure add 5–10 deg. for shift

want $\quad$ a bit help $\quad$ current

$$\phi = (50° + 5°) - 12.75°$$
$$= 42.25° \longleftarrow \text{amt to boost by}$$

b. Solve for α

$$\alpha = \frac{1 - \sin\phi}{1 + \sin\phi}$$
$$= 0.196$$

d. Use new $\omega_{cg,n}$ to get T_D

$$\omega_{cg,n} = \frac{1}{\sqrt{\alpha}\, T_D}$$

$$T_D = \frac{1}{\omega_{cg,n}\sqrt{\alpha}}$$

$$T_D = 0.338$$

c. Get new $\omega_{cg,n}$, where $D_c(s) = \frac{1}{\sqrt{\alpha}}$

$$|KG(s)| = \frac{20}{\omega\sqrt{\omega^2+1}} = \sqrt{\alpha}$$

$$\omega\sqrt{\omega^2+1} = 45.186$$

$$\omega^2(\omega^2+1) = 2041.755$$

$$\omega^4 + \omega^2 - 2041.755 = 0$$

$$\omega_{cg,n} = 6.685 \frac{rad}{s}$$

e. Use values to get $D_c(s)$ eqn

$$D_c(s) = \frac{Ts+1}{\alpha Ts+1}$$

$$D_c(s) = \frac{0.34s+1}{0.066s+1}$$

f. New PM?

$$PM = 180 + \angle KD_cG$$

$$= 180 + \sim\sim$$

$$= \text{close to } 45°$$

$$KD_cG = \frac{K(Ts+1)}{s(s+1)(\alpha Ts+1)}$$

$$\angle KD_cG = \tan^{-1}(T\omega) - 90° - \tan^{-1}(\omega) - \tan^{-1}(\alpha T\omega)$$

= some negative angle value

Lag

target PM

extra for safety

3. Find ω where $\angle KG = -125°$ for PM of $50+10$ deg (5° increase wasn't enough)

$$PM = 180° + \angle KG$$

$$\angle KG = -120°$$

$$KG = \frac{K}{s(s+1)}$$

$$\angle KG = -90 - \tan^{-1}(\omega)$$

$$-90 - \tan^{-1}(\omega_{cg}) = -120$$

$$\omega_{cg} = \tan(30°)$$

$$\omega_{cg} = 0.58 \text{ rad/s}$$

4. At ω_{cg}, we want $|KD_cG| = 1$

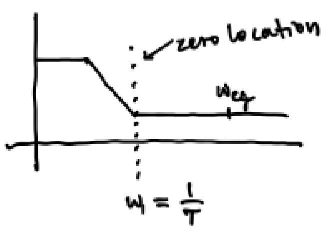
$$|D_c|\Big|_{\omega_{cg}} = \frac{1}{|KG|} = \frac{1}{\alpha}$$

$$|KG| = \alpha$$

$$\frac{20}{\omega_{cg}\sqrt{\omega_{cg}^2+1}} = \alpha$$

$$\alpha = 30$$

5. Find the zero location



$\omega_1 \ll \omega_{cg}$ (about 10 times)

$$\therefore 10\omega_1 = \omega_{cg}$$

$$\omega_1 = \frac{\omega_{cg}}{10}$$

$$\omega_1 = 0.0577$$

$$T = \frac{1}{\omega_1}$$

$$= 17.331$$

$$D_T(s) = \frac{Ts+1}{\alpha Ts+1}$$

$$D_T(s) = \frac{17.331s+1}{519.93s+1}$$