2.9)

a)
$$P(C \mid A, B, D) = P(C \mid \{A\}, \{B, D\})$$

$$= \frac{P(B, D \mid A, C)\, P(C \mid A)}{P(B, D \mid A)}$$

$$= \frac{P(B, D \mid C)\, P(C \mid A)}{P(B, D \mid A)} \quad \text{... Case 1}$$

$$= \frac{P(D \mid B, C)\, P(B \mid C)\, P(C \mid A)}{P(B, D \mid A)}$$

$$= \frac{P(D \mid B, C)\, P(B)\, P(C \mid A)}{P(B, D \mid A)} \quad \text{... (i)} \quad \text{... Case 3}$$

$$P(B, D \mid A) = \sum_C P(B, D, C \mid A) \quad \text{marg.}$$

$$= \sum_C P(B, D \mid C, A)\, P(C \mid A)$$

$$= \sum_C P(B, D \mid C)\, P(C \mid A) \quad \text{... case 1}$$

$$= \sum_C P(D \mid B, C)\, P(B \mid C)\, P(C \mid A)$$

$$= \sum_C P(D \mid B, C)\, P(B)\, P(C \mid A) \quad \text{... case 3}$$

Putting in (i)

$$\Rightarrow P(C \mid A, B, D) = \frac{P(D \mid B, C)\, \cancel{P(B)}\, P(C \mid A)}{\sum_C P(D \mid B, C)\, \cancel{P(B)}\, P(C \mid A)}$$

$$= \frac{P(D \mid B, C)\, P(C \mid A)}{\sum_C P(D \mid B, C)\, P(C \mid A)}$$

b) $P(E|A,B,D) = \sum_c P(E,C|A,B,D)$

$= \sum_c P(E|A,B,D,C)\,P(C|A,B,D)$

$= \sum_c P(E|C)\,P(C|A,B,D)$

... (case 1 & 2)

c) $P(G|A,B,D)$

$= \sum_e P(G,E|A,B,D)$ $\boxed{\text{marg.}}$

$= \sum_e P(G|A,B,D,E)\,P(E|A,B,D)$ $\boxed{\text{P.R.}}$

$= \sum_e P(G|E)\,P(E|A,B,D)$ (case 1)

$= \sum_e \sum_f P(G,F|E)\,P(E|A,B,D)$ $\boxed{\text{marg.}}$

$= \sum_e \sum_f P(G|F,E)\,P(F|E)\,P(E|A,B,D)$ $\boxed{\text{P.R.}}$

$= \sum_e \sum_f P(G|F,E)\,P(F)\,P(E|A,B,D)$ $\boxed{\text{case 3}}$

d) $P(F|A,B,D,G)$

$= \dfrac{P(G|F,A,B,D)\,P(F|A,B,D)}{P(G|A,B,D)}$

$$= \frac{P(G|A,B,D,F)\,P(F)}{P(G|A,B,D)} \qquad \ldots \text{Case 3} \quad \ldots (i)$$

$$P(G|A,B,D,F) = \sum_e P(G,E|A,B,D,F)$$

$$= \sum_e P(G|A,B,D,E,F)\,P(E|A,B,D,F)$$

$$= \sum_e P(G|E,F)\,P(E|A,B,D)$$

$\ldots$ Case 1 & case 3.

$\Rightarrow$ Substituting in (i).

$\Rightarrow$

$$P(F|A,B,D,G) = \frac{P(F)\sum_e P(G|E,F)\,P(E|A,B,D)}{P(G|A,B,D)}$$

2.8

a) $<$

Substituting values from 2nd lecture's burglary example

b) $<$

c) $<$

d) $<$

e) $=$

f) $>$

g) $<$

2.7

1) $P(A|D) = P(A|S)$

$S = \{D, C\}$ case 1 & 3

2) $P(A|B, D) = P(A|S)$

$S = \{B, D, E, F, C\}$ case 1, 2, 3

3) $P(B|D, E) = P(B|S)$

$S = \{D, E, F\}$ case 1

4) $P(E) = P(E|S)$

$S = \{A\}$ case 2, 3

5) $P(E|F) = P(E|S)$

$S = \{F\}$

6) $P(E|D,F) = P(E|S)$

$S = \{D, F\}$

7) $P(E|B,C) = P(E|S\}$

$S = \{B, C, A, D\}$ case 2, 3

8) $P(F) = P(F|S)$

$S = \phi$

9) $P(F|D) = P(F|S)$

$S = \{D, A\}$ case 1

10) $P(F|D,E) = P(F|S)$

$S = \{A, B, C, D, E\}$ case 1

2.6)

1) False
2) True
3) True
4) False
5) True
6) True
7) False
8) True
9) True
10) False

2.5)

Case 1 :- $Y \in \{ child(Parent(child(X))) \}$

All such nodes Y are d-seperated from X on blanket nodes of X based on rule-2 of d-separation

Case 2 :- $Y \in \{ parent(Parent(child(X))) \}$

All such nodes of Y are d-separated from X on blanket nodes of X based on rule-1 of d-separation

Case 3 :- $Y \in \{ Parent(Parent(X)) \}$

All such nodes of Y are d-separated from X on blanket nodes of X based on rule-1 of d-separation.

Case 4 :- $Y \in \{child(Parent(X))\}$

All such ~~cases~~ nodes of Y are d-separated from X on blanket nodes of X based on rule -2 of d-separation

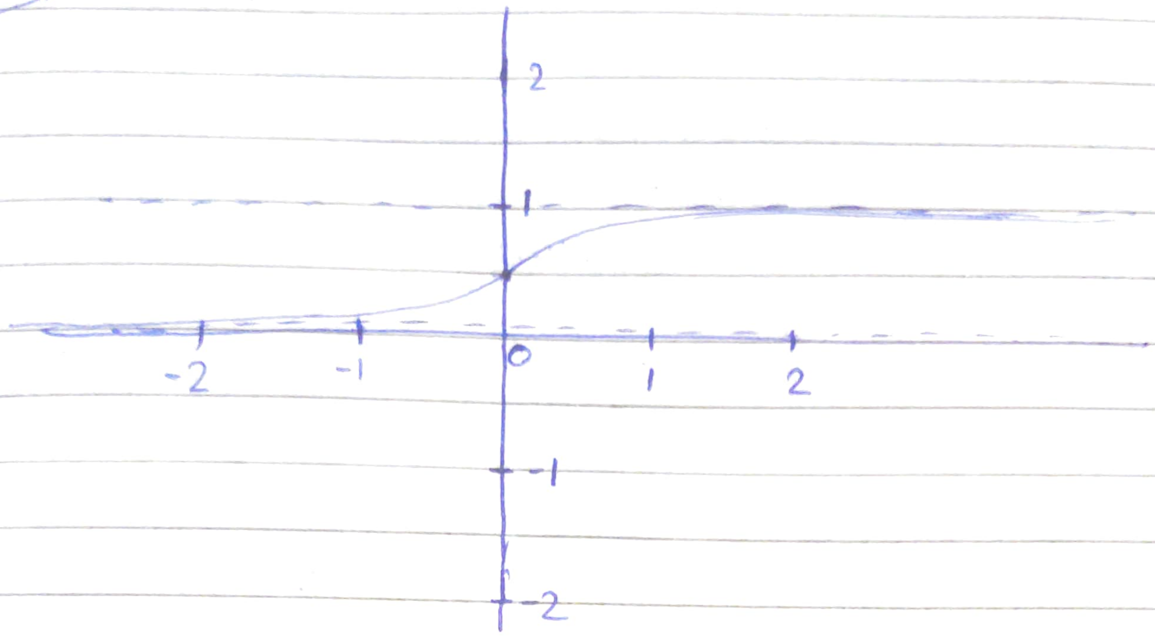
Case 5 :- $Y \in \{child(child(X))\}$

All such nodes of Y are d-separated from X on blanket nodes of X based on rule-1 of d-separation

2.4) M = month P = Puddle
S = Sprinkler F = Fall.
R = Rain

X	Y	E
S	R	M
M	F	P
M	F	P, R
M	F	P, S
M	F	P, R, S
M	F	R, S
M	P	F, R, S
M	P	R, S
R	F	M, P
R	F	M, P, S
R	F	P
R	F	P, S
S	F	M, P
S	F	M, P, R
S	F	P
S	F	P, R

2.3



a) $$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\frac{d(\sigma(z))}{dz} = \frac{d}{dz}\left(\frac{1}{1+e^{-z}}\right)$$

$$= \frac{d}{d(1+e^{-z})}\left(\frac{1}{1+e^{-z}}\right) \times \frac{d(1+e^{-z})}{dz}$$

$$= \frac{-1}{(1+e^{-z})^2} \times \frac{d\frac{1}{e^z}}{de^z} \; \frac{de^z}{dz}$$

$$= \frac{1}{e^z(1+e^{-z})^2} \quad \ldots (i)$$

$$\sigma(z)\,\sigma(-z) = \frac{1}{1+e^{-z}} \times \frac{1}{1+e^{z}}$$

$$= \frac{1}{(1+e^{-z})} \times \frac{1}{e^{z}(1+e^{-z})}$$

$$= \frac{1}{e^{z}(1+e^{-z})^2} \quad \cdots \text{(ii)}$$

from i) & ii)

$$\sigma'(z) = \sigma(z)\,\sigma(-z)$$

b)

$$\sigma(z) + \sigma(-z) = \frac{1}{1+e^{-z}} + \frac{1}{1+e^{z}}$$

$$= \frac{e^{z}}{1+e^{z}} + \frac{1}{1+e^{z}}$$

$$= \frac{1+e^{z}}{1+e^{z}}$$

$$= 1$$

c)

$$L(\sigma(z)) = \log\left(\frac{\sigma(z)}{1-\sigma(z)}\right)$$

$$= \log\left(\frac{1/1+e^{-z}}{1-\frac{1}{1+e^{-z}}}\right) = \log\left(\frac{1}{e^{-z}}\right)$$

$$= \log e^{z} = z$$

d) $\omega_i = \mathcal{L}(P_i)$ To prove.

$$P_i = P(Y=1 \mid X_1=0, X_2=0 \ldots, X_i=1, \ldots X_N)$$

$$= \sigma(\omega_1 \times 0 + \omega_2 \times 0 + \ldots \omega_i x_i + \ldots \omega_i \times 0)$$

$$= \sigma(\omega_i) \qquad \because x_i = 1$$

$$\Rightarrow \mathcal{L}(P_i) = \cancel{\sigma}\, \mathcal{L}(\sigma(\omega_i))$$

From (C)

$$\mathcal{L}(\sigma(\omega_i)) = \omega_i$$

2.2 $P(D \mid S_1, S_2 \ldots S_k)$

$$\Rightarrow \frac{P(S_1, S_2 \ldots S_k \mid D)\, P(D)}{P(S_1, S_2, \ldots S_k)}$$

$$\Rightarrow \frac{P(S_1, S_2 \ldots S_k \mid D)\, P(D)}{\sum_{d} P(S_1, S_2, \ldots S_k \mid D)\, \cancel{P(D)}}$$

$$\Rightarrow \frac{P(S_1 \mid D)\, P(S_2 \mid D) \ldots P(S_k \mid D) \times P(D)}{\sum_{d} P(S_1 \mid D)\, P(S_2 \mid D) \ldots P(S_k \mid D)} \quad \ldots (\text{Case } 2)$$

$$\Rightarrow \quad r_k = \frac{P(D=0 \mid S_1 = 1, S_2 = 1 \ldots S_k = 1)}{P(D=1 \mid S_1 = 1, S_2 = 1 \ldots S_k = 1)}$$

$$= \frac{\dfrac{P(S_1 \mid D_0)\, P(S_2 \mid D_0) \ldots P(S_k \mid D_0)\, P(D_0)}{\sum_{d} P(S_1 \mid D)\, P(S_2 \mid D) \ldots P(S_k \mid D)}}{\dfrac{P(S_1 \mid D_1)\, P(S_2 \mid D_1) \ldots P(S_k \mid D_1)\, P(D_1)}{\sum_{d} P(S_1 \mid D)\, P(S_2 \mid D) \ldots P(S_k \mid D)}}$$

$$= \frac{P(S_1 \mid D_0)\, P(S_2 \mid D_0) \ldots P(S_k \mid D_0)\, P(D_0)}{P(S_1 \mid D_1)\, P(S_2 \mid D_1) \ldots P(S_k \mid D_1)\, P(D_1)} \quad \ldots (i)$$

$$\because P(D_0) = P(D_1) = \frac{1}{2}$$

$$P(S_1=1 \mid D=0) = 1$$

$$P(S_k=1 \mid D=0) = \frac{f(k-1)}{f(k)}$$

$$f(k) = 2^k + (-1)^k$$

$$P(S_k=1 \mid D=1) = \frac{1}{2}$$

Substituting all this in (i)

$\Rightarrow$

$$r_k = \frac{1 \times \frac{f(1)}{f(2)} \times \frac{f(2)}{f(3)} \times \cdots \frac{f(k-1)}{f(k)}}{\left(\frac{1}{2}\right)^k}$$

$$= \frac{2^k}{2^k+(-1)^k}$$

$$\boxed{r_k = \frac{1}{1+\left(\frac{-1}{2}\right)^k}}$$

a) doctor's diagnosis alternates between even and odd days

b) $\lim_{k\to\infty} \frac{1}{1+\left(\frac{-1}{2}\right)^k} \approx 1$

$\Rightarrow$ Diagnosis becomes less certain

2.1) a) $$P(E|A) = \frac{P(A|E)P(E)}{P(A)}$$

$$= \frac{\sum_b P(A,B|E)\,P(E)}{P(A)}$$

$$= \frac{\sum_b P(A|B,E)P(B|E)P(E)}{P(A)}$$

$$= \frac{\sum_b P(A|B,E)P(B)P(E)}{P(A)}$$

$$P(A) = \sum_b P(A,B)$$

$$= \sum_b P(A|B)P(B)$$

$$= \sum_b \sum_e P(A,E|B)P(B)$$

$$= \sum_b \sum_e P(A|B,E)P(E|B)P(B)$$

$$= \sum_b \sum_e P(A|B,E)P(E)P(B)$$

$$P(A=1) = \sum_b \left(P(A=1|B,E=0)P(E=0)P(B) + P(A=1|B,E=1)P(E=1)P(B)\right)$$

$$= P(A=1|B=0,E=0)P(E=0)P(B=0) + P(A=1|B=0,E=1)P(E=1)P(B=0)$$
$$+ P(A=1|B=1,E=0)P(E=0)P(B=1) + P(A=1|B=1,E=1)P(E=1)P(B=1)$$

$$= 0.001 \times 0.998 \times 0.999 + 0.29 \times 0.002 \times 0.999$$
$$+ 0.94 \times 0.998 \times 0.001 + 0.95 \times 0.002 \times 0.001$$

$$= 9.97002 \times 10^{-4} + 5.7942 \times 10^{-4} + 9.3812 \times 10^{-4}$$
$$+ 0.019 \times 10^{-4}$$

$$= 25.16442 \times 10^{-4}$$

$$\Rightarrow P(A=1) = 0.002516442$$

$$\sum_b P(A=1|B, E=1) P(B) P(E=1)$$

$$= \left(P(A=1|B=0, E=1) P(B=0) + P(A=1|B=1, E=1) P(B=1)\right) P(E=1)$$

$$= (0.29 \times 0.999 + 0.95 \times 0.001) \times 0.002$$

$$= 5.8132 \times 10^{-4}$$

$$\Rightarrow \boxed{P(E|A) = 0.231008702}$$

b) $P(E|A,B) = \dfrac{P(A|E,B)\, P(E|B)}{P(A|B)}$ bayes

$= \dfrac{P(A|E,B)\, P(E|B)}{\sum_e P(A,E|B)}$ marg.

$= \dfrac{P(A|E,B)\, P(E)}{\sum_e P(A|E,B)\, P(E|B)}$ independence & P.R

$= \dfrac{P(A|E,B)\, P(E)}{\sum_e P(A|E,B)\, P(E)}$ Independence

$$P(E=1 | A=1, B=0) = \frac{P(A=1|E=1, B=0)\, P(E=1)}{\sum_e P(A=1|E, B=0)\, P(E)}$$

$$\Rightarrow \frac{P(A=1|E=1,B=0)P(E=1)}{P(A=1|E=0,B=0)P(E=0)+P(A=1|E=1,B=0)P(E=1)}$$

$$\Rightarrow \frac{0.29 \times 0.002}{0.001 \times 0.998 + 0.29 \times 0.002}$$

$$\Rightarrow \frac{0.58 \times 10^{-3}}{1.578 \times 10^{-3}}$$

$$\Rightarrow P(E=1|A=1,B=0) = 0.367553865$$

c) $$P(A|M) = \frac{P(M|A)P(A)}{P(M)}$$

From 2.1.a

$$P(A) = \sum_b \sum_e P(A|B,E)P(E)P(B)$$

$$P(M) = \sum_a P(M,A) = \sum_a P(M|A)P(A)$$

$$P(A=1) = 25.16442 \times 10^{-4}$$

$$P(A=0) = P(A=0|B=0,E=0)P(B=0)P(E=0) + P(A=0|B=0,E=1)P(B=0)P(E=1)$$
$$+ P(A=0|B=1,E=0)P(B=1)P(E=0)$$
$$+ P(A=0|B=1,E=1)P(B=1)P(E=1)$$

$$= 0.999 \times 0.999 \times 0.998 + 0.71 \times 0.999 \times 0.002$$
$$+ 0.05 \times 0.001 \times 0.002$$
$$+ 0.06 \times 0.001 \times 0.998$$
$$= 0.99748358$$

$\Rightarrow P(M=1) = P(M=1|A=0)P(A=0) + P(M=1|A=1)P(A=1)$

$= 0.70 \times 0.002516442 + 0.01 \times 0.997483\ldots$

$= 0.011736344$

$\cancel{= 0.69826 3655}$

$\Rightarrow P(A=1|M=1) = \dfrac{P(M=1|A=1)P(A=1)}{P(M=1)}$

$= \dfrac{0.70 \times 0.002516442}{\cancel{0.69826 3655}\ 0.011736344}$

$= 0.15009013$

$\cancel{= 2.522699538 \times 10^{-3}}$

$\boxed{P(A=1|M=1) = 0.15009013}$ – Answer

$\cancel{\boxed{P(A=1|M=1) = 0.002522699538}}$

2) $P(A|M,J) = \dfrac{P(M|A,J)\,P(A|J)}{P(M|J)}$ Bayes

$= \dfrac{P(M|A)\,P(A|J)}{P(M|J)}$ cond^n independence

$P(A|J) = \dfrac{P(J|A)\,P(A)}{P(J)}$

$P(J) = \sum_a P(J,A) = \sum_a P(J|A)\,P(A)$

$$P(J=0) = P(J=0|A=0)P(A=0) + P(J=0|A=1)P(A=1)$$

$$= 0.95 \times 0.997483558 + 0.1 \times 0.002516442$$

$$= 0.947861024$$

$$\Rightarrow P(A=1|J=0) = \frac{P(J=0|A=1)P(A=1)}{P(J=0)}$$

$$= \frac{0.1 \times 0.002516442}{0.947861024}$$

$$= 2.654863884 \times 10^{-4}$$

$$\Rightarrow P(A=1|M=1, J=0) = \frac{P(M=1|A=1)P(A=1|J=0)}{P(M=1|J=0)}$$

$$P(M|J) = \cancel{\sum P(M, A} \frac{P(J, M)}{P(J)}$$

$$= \sum_{a} \frac{P(M, J, A)}{P(J)}$$

$$= \sum_{a} \frac{P(M, J|A)P(A)}{P(J)}$$

$$= \sum_{a} \frac{P(M|J, A)P(J|A)P(A)}{P(J)}$$

$$= \sum_{a} \frac{P(M|A)P(J|A)P(A)}{P(J)}$$

$$\Rightarrow P(A|M,J) = \frac{P(M|A)\,P(A|J)}{P(M|J)}$$

$$P(A|J) = \frac{P(J|A)\,P(A)}{P(J)}$$

$$P(M|J) = \frac{\sum_{\alpha} P(M|A)P(J|A)P(A)}{P(J)}$$

$$\Rightarrow P(M=1|J=0) = \frac{\sum_{\alpha} P(M=1|A)P(J=0|A)P(A)}{P(J=0)}$$

$$= \frac{P(M=1|A=0)P(J=0|A=0)P(A=0)}{P(J=0)}$$

$$+ \frac{P(M=1|A=1)P(J=0|A=1)P(A=1)}{P(J=0)}$$

$$= \frac{0.01 \times 0.95 \times 0.99783558}{0.94861024}$$

$$+ \frac{0.70 \times 0.1 \times 0.00216442}{0.947861024}$$

$$= 0.010183185$$

$$\Rightarrow P(A=1 \mid M=1, J=0) = \frac{P(M=1 \mid A=1)\, P(A=1 \mid J=0)}{P(M=1 \mid J=0)}$$

$$= \frac{0.7 \times 2.659863884 \times 10^{-4}}{0.01018318S}$$

$$\boxed{P(A=1 \mid M=1, J=0) = 0.018249739}$$

8) $$P(A=1 \mid M=0) = \frac{P(M \mid A)\, P(A)}{P(M)}$$

$$P(M) = \sum_a P(M \mid A)\, P(A)$$

$$\Rightarrow P(M=0) = P(M=0 \mid A=0)\, P(A=0) + P(M=0 \mid A=1)\, P(A=1)$$

$$= 0.99 \times 0.997483558 +$$

$$0.3 \times 0.002516442$$

$$= 0.988263655$$

$$P(A=1 \mid M=0) = \frac{P(M=0 \mid A=1)\, P(A=1)}{P(M=0)}$$

$$= \frac{0.3 \times 0.002516442}{0.988263655}$$

$$\boxed{P(A=1 \mid M=0) = 7.638979701 \times 10^{-4}}$$

f) $$P(A|M,B) = \frac{P(M|A,B)\,P(A|B)}{P(M|B)}$$

$$P(M|A,B) = P(M|A)$$ rule-1 d-separation

$$P(M|B) = \sum_a P(M,A|B)$$ marg.

$$= \sum_a P(M|A,B)\,P(A|B)$$ P.R

$$= \sum_a P(M|A)\,P(A|B)$$ rule-1

$$= \sum_a \sum_e P(M|A)\,P(A,E|B)$$ Marg.

$$= \sum_a \sum_e P(M|A)\,P(A|E,B)\,P(E|B)$$ P.R.

$$= \sum_a \sum_e P(M|A)\,P(A|E,B)\,P(E)$$ independ

$$\Rightarrow P(M=0|B=1) = \sum_a \sum_e P(M=0|A)\,P(A|E,B=1)\,P(E)$$

$$= P(M=0|A=0)\,P(A=0|E=0,B=1)\,P(E=0) +$$

$$P(M=0|A=0)\,P(A=0|E=1,B=1)\,P(E=1) +$$

$$P(M=0|A=1)\,P(A=1|E=0,B=1)\,P(E=0) +$$

$$P(M=0|A=1)\,P(A=1|E=1,B=1)\,P(E=1)$$

$$= 0.99 \times 0.06 \times 0.998 + 0.99 \times 0.05 \times 0.002$$
$$+ 0.3 \times 0.94 \times 0.998 +$$
$$0.3 \times 0.95 \times 0.002$$
$$= 0.3413862$$

$$\Rightarrow P(A=1 \mid M=0, B=1) = \frac{P(M=0 \mid A=1)\, P(A=1 \mid B=1)}{P(M=0 \mid B=1)}$$

$$P(A \mid B) = \sum_{e} P(A, E \mid B) \qquad \text{marg.}$$

$$= \sum_{e} P(A \mid E, B)\, P(E \mid B) \qquad \text{P.R.}$$

$$= \sum_{e} P(A \mid E, B)\, P(E) \qquad \text{Indep.}$$

$$\Rightarrow P(A=1 \mid B=1) = P(A=1 \mid E=0, B=1)\, P(E=0) +$$

$$P(A=1 \mid E=1, B=1)\, P(E=1)$$

$$= 0.94 \times 0.998 + 0.95 \times 0.002$$

$$= 0.94002$$

$$\Rightarrow P(A=1 \mid M=0, B=1) = \frac{0.3 \times 0.94002}{0.3413862}$$

$$\boxed{P(A=1 \mid M=0, B=1) = 0.82606151}$$

(a) versus (b)

Probability that earthquake has occurred should increase given that alarm rang and burgulary didn't happen

This is depicted by the results as:-

$P(E=1/A=1) < P(E=1/A=1, B=0)$

Thus, result is consistent with commonsense pattern of reasoning.

(c) versus (d)

Probability that alarm has rang should decrease if only mary called and John didn't called, compared to if only John called since we don't know if Mary will call yet.

This is depicted by the result as:-

$P(A=1/M=1) > P(A=1/M=1, J=0)$

Thus, result is consistent with commonsense pattern of reasoning.

(e) versus (f)

Probability of alarm ringing should increase even if Mary didn't call compared to only if Mary didn't call

This, is depicted in the results as:-

$$P(A=1 \mid M=0, B=1) > P(A=1 \mid M=0)$$

Thus, the results are consistent with commonsense pattern of reasoning.