Let $K_n$ be number of moves to move n disks from peg 1 to peg 3 under given conditions.

In a system of n pegs, lets label disks

$d_1, d_2, d_3 ... d_n$

To move $d_n$ from peg 1 to peg 3, the only possible way involves

| | | Moves Involved |
|---|---|---|
| i. | moving the first n-1 disks to peg 3 | $K_{n-1}$ |
| ii. | moving $d_n$ from peg 1 to peg 2 | 1 |
| iii. | moving $d_1 – d_{n-1}$ from peg 3 to peg 1 | $K_{n-1}$ |
| iv. | moving $d_n$ from peg 2 to peg 3 | 1 |
| v. | moving $d_1 – d_{n-1}$ from peg 1 to peg 3 | $K_{n-1}$ |

$K_n \quad = \quad 3K_{n-1} + 2$

**The three states that are mentioned occur at**

1. **Initially, i.e. before step 1. We already got stacked arrangement of n disks on peg 1.**
2. **While doing step 3, we will get a proper stack on peg 2. (proved later)**
3. **Final is the initial state, when all the disks are on peg 3.**

Prove for (2):

Lets define a boolean function $J_n$ that is *true* for a system of n disks if the system atains (atleast once) the state when all the disks are stacked on peg 2, while performing abhove 5 moves (i. to v.)

$J_1$ = *true*, as to move disk 1 from peg 1 to peg 3, first move will be to move it to peg 2.

Let $J_{n-1}$ *be true*

For n disks system, after step ii. , i.e., moving $d_n$ from peg 1 to peg 2, we will have all $d_1 – d_{n-1}$ on peg 3. Now to move $d_n$ from 2 to 3, we will first need to move all other disks from 3. And only option is to move them to peg 1. Now this is same as moving n-1 disks from peg 1 to peg 3, only here we are moving disks from peg 3 to peg 1. Now we know for n-1, $J_{n-1}$ is true. We will get n-1 disks arranged on peg 2 which already had $d_n$ at bottom.

Therefore, $J_n$ is *true.*

By principal of induction, since $J_1$ is *true* and $J_n$ *is true whenever* $J_{n-1}$ is true, we will have

$$J_k = true, \qquad k \in \mathbb{Z}$$

Therefore proved