Assignment - I

solve LPP by II phase.

$\max Z = 5x_1 + 3x_2$

constrains

$3x_1 + 2x_2 \geq 3$.

$x_1 + 4x_2 \geq 4$.

$x_1 + x_2 \leq 5$

first convert it into a standard Lpp format

$Z^* = 0x_1 + 0x_2 + 0S_1 + 0S_2 + 0S_3 - A_1 - A_2$

$3x_1 + 2x_2 - S_1 + A_1 = 3$

$x_1 + 4x_2 - S_2 + A_2 = 4$.

$x_1 + x_2 + S_3 = 5$.

		C_j	0	0	0	0	0	-1	-1	
C_B	Basic var	X_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	Min
-1	A_1	3	3	2	-1	0	0	1	0	3/2
-1	A_2	4	1	4	0	-1	0	0	1	1
0	S_3	5	1	1	0	0	1	0	0	5
		$Z_j - C_j$	-4	-6	1	1	0	0	0	

$4 - \frac{6}{20}$ $\quad \frac{80-6}{20} = \frac{74}{20} = \frac{37}{10}$ $\quad \frac{37}{10} \Big/ \frac{1}{4}$

		C_j	0	0	0	0	0	-1	
C_B	Basic	X_B	X_1	X_2	S_1	S_2	S_3	A_1	
-1	A_1	1	5/2	0	-1	1/2	0	1	2/5
0	x_2	1	1/4	1	0	-1/4	0	0	4
0	S_3	4	3/4	0	0	1/4	1	0	5
		$Z_j - C_j$	-5/2	0	1	-1/2	0	0	

$3 - \frac{1}{2}$

$\frac{6-1-2}{2}$

$3 - 2 = 1$

$3 - \frac{2}{4} = \frac{1}{2} = \frac{6-1}{2}$

$2 - 2$

		C_j	0	0	0	0	0	
C_B	Basic	X_B	X_1	X_2	S_1	S_2	S_3	Min value
0	x_1	2/5	1	0	-2/5	1/5	0	
0	x_2	9/10	0	1	-1/10	-3/10	0	
0	S_3	37/10	0	0	3/10	3/10	1	
		$Z_j - C_j$	0	0	0	0	0	

-1 0

0 $-\frac{1}{2}$

0 0

4

1 0

1) $\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = 1 - \frac{1}{10} = \frac{9}{10}$

2) $\frac{1}{4}$

3) 0

4) $\frac{-2}{5} \times \frac{1}{4} = \frac{-2}{20}$

5) $\frac{1}{20} - \frac{1}{4} = \frac{-5+1}{20} = \frac{-4}{20} = \frac{-1}{5}$

Now phase II.

CB	Basic	C_j / X_B	5 x_1	3 x_2	0 S_1	0 S_2	0 S_3	Min
5	x_1	2/5	1	0	-2/5	1/5	0	∞
3	x_2	9/10	0	1	1/10	3/10	0	3
0	S_3	37/10	0	0	3/10	1/10	1	37/3
			0	0	-17/10	3/10	0	

C_B	Basic	x_B	5 x_1	3 x_2	0 S_1	0 S_2	0 S_3	Min
5	x_1	4/9	1	4	0	-1	0	-4/5
0	S_1	9	10	10	1	-3	0	-1/3
0	S_3	1	0	-3	0	1	1	34/10
		$Z_j - C_j$	0	7	0	-5	0	

$\frac{37}{10} - \frac{3}{10}$

C_B	Basic	X_B	5 x_1	3 x_2	0 S_1	0 S_2	0 S_3	Min
5	x_1	5	1	1	0	0	1	
0	S_1	11	0	1	1	0	3	
0	S_2	1	0	-3	0	1	1	
			0	2	0	0	5	

Min ratio

1	2/5
0	4
0	5
0	

Ex.

Max $Z = 3x_1 + 2x_2$

Subject to,

$x_1 + x_2 \leq 4$

$-x_1 + x_2 \leq -2$.

where $x_1, x_2 \geq 0$.

Solution:

The standard LPP format

$Z = 3x_1 + 2x_2$

subject to

$x_1 + x_2 \leq 4$.

$-x_1 + x_2 \leq -2$.

$x_1 + x_2 + s_1 \leq 4$

$Ax = B$. $-x_1 + x_2 + s_2 \leq -2$

$$\cancel{\begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}}.$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

		C_j	3	2	0	0
Basic	C_B	X_B	X_1	X_2	S_1	S_2
S_1	0	$\boxed{4}$	$\boxed{1}$	1	1	0
S_2	0	-2	$\boxed{-1}$	1	0	1
		$Z_j - C_j$	-3	-2	0	0

C_B	Basic var	X_B	x_1	x_2	s_1	s_2
0	x_1	21/13	1	0	-7/13	1/13
0	x_2	10/13	0	1	1/13	-2/13
		0	0	0	0	0

		c_j	1	1	0	0	
C_B	Basic	X_B	x_1	x_2	s_1	s_2	
1	x_1	21/13	1	0	-7/13	1/13	-3 ×
1	x_2	10/13	0	1	1/13	-2/13	10 ✓
			0	0	-6/13	-1/13	

			1	1	0	
C_B	Basic	X_B	x_1	x_2	s_1	s_2
1	x_1	7	1	7	0	-1
0	s_1	10	0	13	1	-2
	$z_j - c_j$		0	6	0	-1

unbounded solution.

$2x_1 + 3x_2 = 13$

put $x_2 = -1$

$2x_1 + 3\times(-1) = 13$

$2x_1 - 3 = 13$

$2x_1 = 13 + 3$

$x_1 = 16/2$

$\boxed{x_1 = 8}$

~~O (0,0) =~~

Feasible region is OABC

O $(0,0) =$ ~~OOO~~ Max $z = 9x_1 + 3x_2 = 0 + 0 = 0$

A $(2.5, 0) =$ Max $z = 9x_1 + 3x_2 = (9\times 2.5) + 0 = 22.5$

B $(8, -1) =$ Max $z = 9x_1 + 3x_2 = (9\times 8) + 3\times -1 = 69$

C $(0, 4.3) =$ Max $z = 9x_1 + 3x_2 = 0 + 4.3\times 3 = 12.9$

~~At~~ Point B Max $z = 69$

where $x_1 = 8$ and $x_2 = -1$

Q.4 write Short Note on.

A) Linear Programming Problem.

B) Simplex Method Algorithm.

A) Linear Programming Problem :-

1) LPP is a method to achieve the best outcome in a mathematical concept whose requirements are represented by linear relationships

2) LPP is a optimization problem for which :

a) we attempt to maximize a linear function of decision variab

b) The value of decision variable must satisfy a set of constraints each of which must be linear inequality or linear equality

3) Properties :-

a) Proportionality :-
If one item brings in profit of x then K items bring in a profit of Kx.

b) Additivity :-
The decisions made are independent excepts as noted in the constraints.

c) Divisibility :- Decision variables can take fract^n values

b) Simplex Method Algorithm :-
Simplex method is an approach to solving linear programming models by hand using slack variables & pivot variables as means to finding the optimal solution of optimization problem.

Step 1 :- check objective funct^n is in maximization form. If not then convert it by multiplying ~~&~~ -1

Ex :- $\text{Min } z = 3x_1 + 2x_2$

$\text{max } z^* = -3x_1 - 2x_2$

Step 2 :- If constraints are not $\leq$ then convert it by multiplying -1 on both sides.

Ex $x_1 + x_2 \geq -10$

$\longrightarrow -x_1 - x_2 \leq 10$

Slack Variable :- If a constraint has $<$ sign then in order to make an equality we have to add something positive to L.H.S then that variable is called slack variable.