Algorithms with Predictions: learning challenges

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Bocconi

Algorithms with Predictions (ALPS): Context

a.k.a. Learning-Augmented Algorithms

- Lykouris and Vassilvitskii (ICML'18, JACM)
- Kraska, Beutel, Chi, Dean, Polyzotis (SIGMOD'18)

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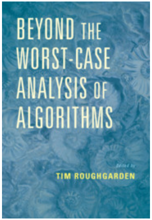

Beyond the worst-case analysis [Roughgarden 2021]

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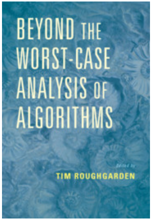


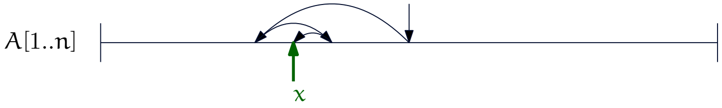
Beyond the worst-case analysis [Roughgarden 2021]

- What are the "typical" inputs?
  - stochastic properties, specific structure/patterns
- Chapter 30 on ALPS [Mitzenmacher, Vassilvitskii]

Example: Binary Search with Predictions

Classical Binary Search:

- Sorted array A
- Given x, find i s.t. $A[i] = x$



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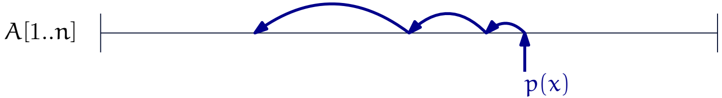

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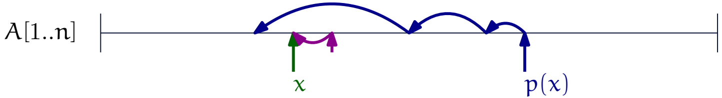

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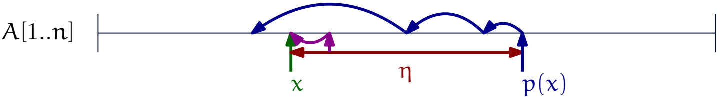

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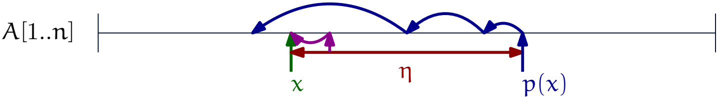

Properties

- consistency: $O(1)$ steps with perfect predictions
- smoothness: $O(\log \eta)$ steps with error η
- robustness: never worse than $O(\log n)$ steps $(\eta \leqslant n)$

Template

Pick up an algorithmic problem

- online: caching, scheduling, routing, ...
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Design algorithms

- for learning the predictions with small error (efficient)
- for using the predictions (consistent, smooth, robust)

Outline

(1) Learning the predictions

- PAC learning and beyond

(2) Achieving smoothness

- how to detect small errors?

(3) Adapting algorithmic strategy online

- learning from the best algorithm

(1) Learning the predictions

PAC learning paradigm

- $I_1, \ldots, I_k \sim \mathcal{D}$
- find prediction $\hat{p}$ s.t.

$$\mathbb{E}_{I \sim \mathcal{D}}[\eta(\hat{p}, I)] \leqslant \min_{p} \mathbb{E}_{I \sim \mathcal{D}}[\eta(p, I)] + \epsilon.$$

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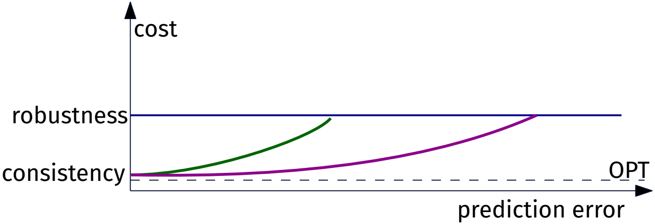
We want:

- k polynomial
- polytime algorithm for minimizing empirical error
  - often easy: scheduling [Lattanzi et al.'20], matching [Dinitz et al.'21]
  - but not always: shortest path [Lattanzi et al.'23]

Typical situation

- prediction error η small but non-zero



(2) Smoothness

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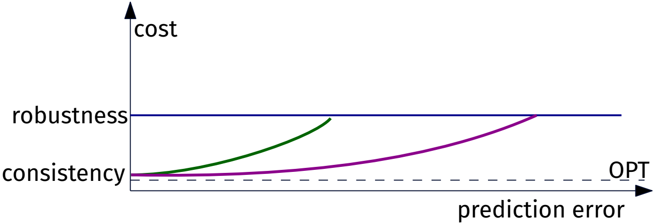


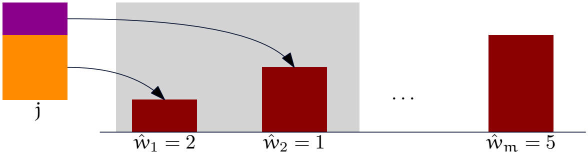
How to deal with imprecise predictions?

- $\hat{p}$ contains useful information but a few errors
- we need to find the errors (and correct them)

(2) Smoothness: Example

Online Load Balancing (Restricted Assignment) [Moseley et al. '20]

- predictions: machine weights $\hat{w}_1, \ldots, \hat{w}_m$,
  - jobs assigned to machines proportionally to the weights



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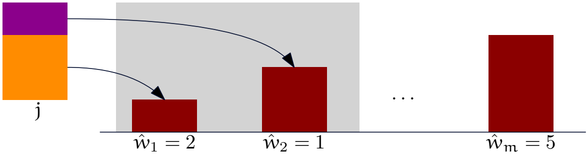

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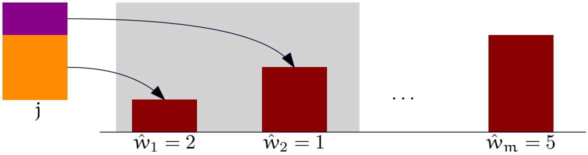

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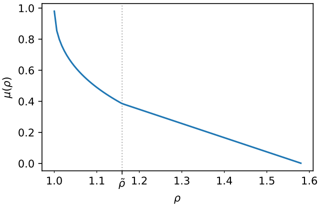
Smoothness bound [Moseley et al. '20]

There is an algorithm with competitive ratio $O(\log \eta)$ for the fractional restricted assignment.

Online Dynamic Power Management

- $\mathrm{ALG}_\rho \leqslant \rho\,\mathrm{OPT} + \mu(\rho)\eta$ [Antoniadis, Coester, Eliáš, Polak, Simon '21]



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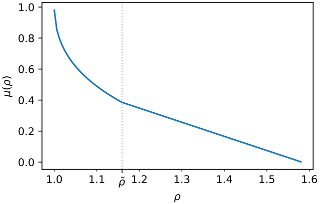

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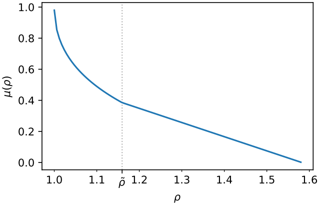

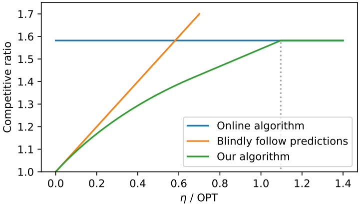

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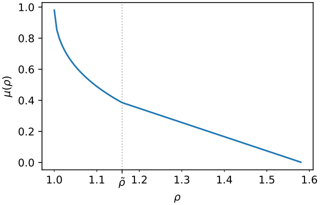

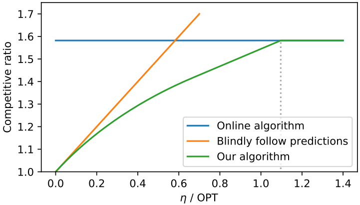

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Learning question

How much can we trust the prediction?

Algorithmic questions

How important is the current decision?

What is the alternative to following the prediction?

(3) Adapting strategy online

Two scenarios in online setting

- algorithm A uses predictor trained on distribution $\mathcal{D}$
- $\mathrm{I} \sim \mathcal{D} \Rightarrow A$ works great
- $\mathrm{I} \nsim \mathcal{D} \Rightarrow$ some worst-case A' still works ok
- I arriving online: we want cost $\approx \min\{A(\mathrm{I}), A'(\mathrm{I})\}$

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Multiple scenarios in online setting

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Challenge:

- past actions $\rightarrow$ current state of the algorithm
- if we take an action of a bad algorithm, we cannot take it back

Related works

Online choice of online algorithms

- k-server [Fiat, Rabani, Ravid 1990]
- caching [Fiat, Karp, Luby, McGeoch, Sleator, Young 1991]
- general MTS [Azar, Broder, Manasse 1993]

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Algorithms with multiple predictions

- Online Covering [Anand et al. 2022], [Kevi, Nguyen 2023]
- Offline Matching, Scheduling [Dinitz et al. 2022]
- Ski Rental [Gollapudi, Panigrahi 2019], [Wang et al. 2020]
- Online Facility Location [Almanza et al. 2021]
- offline algorithms [Srinivas, Blum 2024]

Metrical Task Systems (MTS)

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- $\min\{A_1, \dots, A_\ell\} + \text{Regret}$ [Blum, Burch '00]
- $O(\ell^2)$ dyn.comb$(A_1, \dots, A_\ell)$ [Antoniadis, Coester, Eliáš, Polak, Simon '23]
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Bandit setting

- $\min\{A_1, \ldots, A_\ell\} + \text{Regret}$ [Cosa, Eliáš '25]
- Dynamic combination: open

Further perspectives

Improve the current techniques to:

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End-to-end learning of online algorithms

- Algorithms with Explicit Predictors
  [Eliáš, Kaplan, Mansour, Moran '23]
- algorithm has direct access to the dataset of past inputs
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Offline and Approximation algorithms

- currently understudied
- many techniques do not translate to offline setting

Further information

Posters

- Xizhi Tan: Learning-Augmented Mechanism Design
- Kaito Fujii: The Secretary Problem with Predictions

https://algorithms-with-predictions.github.io/

- by Alexander Lindermayr
- database of the papers in the area