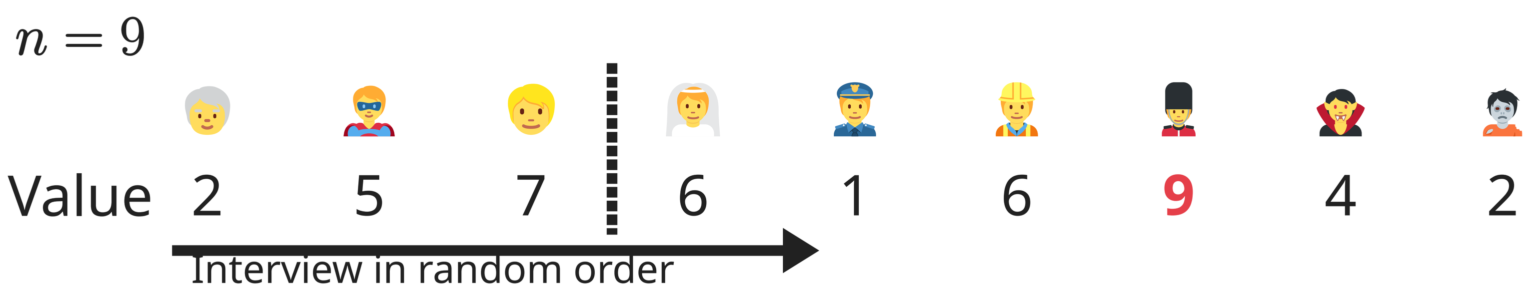
The secretary problem with predictions

Kaito Fujii and **Yuichi Yoshida** (National Institute of Informatics)

1. Classical secretary problem

Interview n candidates in **random order** (n is known)
Irrevocably decide whether to hire just after the interview

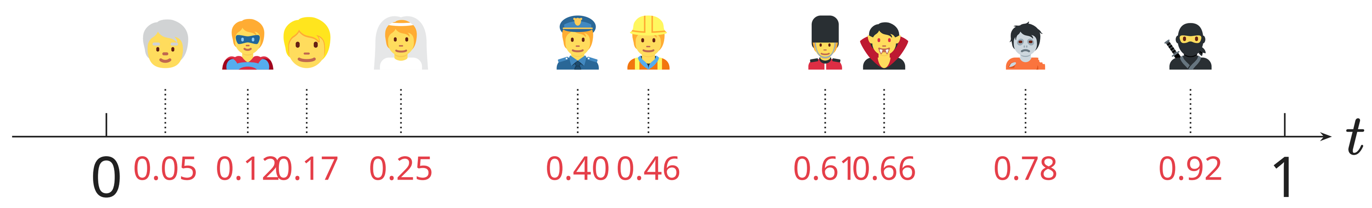
Dynkin's algorithm
Ignore the first $\approx n/\mathrm{e}$ candidates, and then
hire the first candidate who is best so far.
$\to$ The success probability is $\approx 1/\mathrm{e}$



Goal Maximize the **competitive ratio** CR $:= \mathbb{E}\left[\frac{\text{hired candidate's value}}{\text{best value}}\right]$

Continuous-time model

Each candidate is assigned an arrival time $\sim U(0,1)$



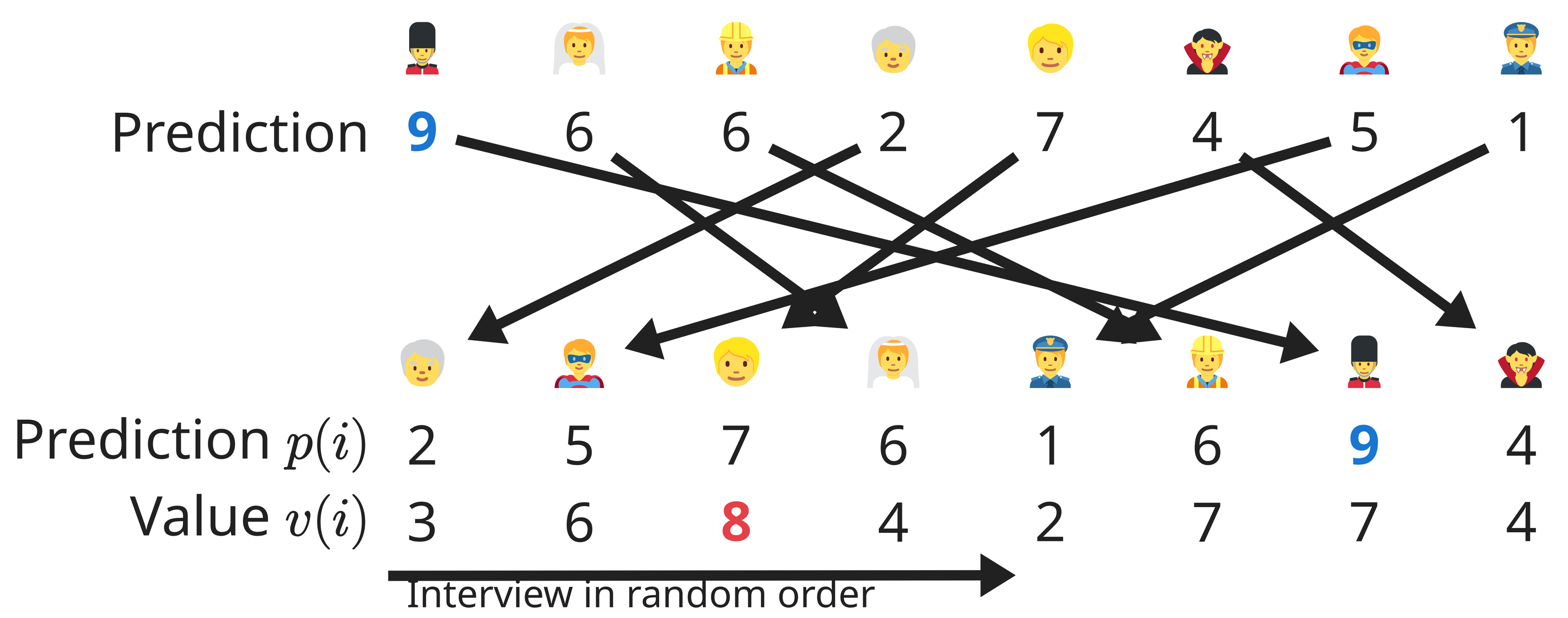
Obs This model is equivalent to random-order model

Related work

- The setting in which a predicted value of the maximum is given
  Antonios Antoniadis, Themis Gouleakis, Pieter Kleer, and Pavel Kolev: Secretary and Online Matching Problems with Machine Learned Advice. In *NeurIPS 2020*.
- Prophet secretary: The setting in which true distributions are given
  José R. Correa, Raimundo Saona, and Bruno Ziliotto: Prophet Secretary Through Blind Strategies. In *SODA 2019*.

2. Classical secretary with predictions

Given the predicted values of each candidate in advance



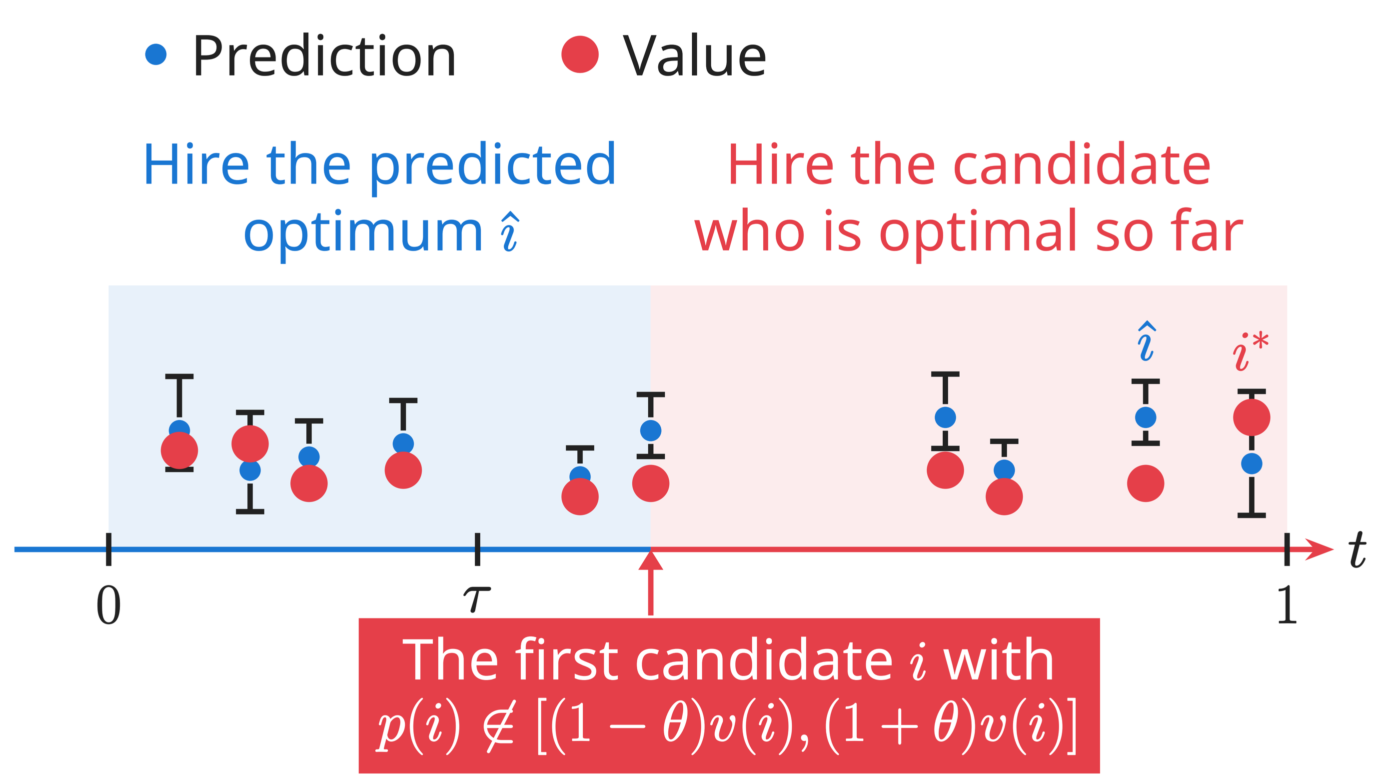
Prediction error $\epsilon := \max_{i \in [n]} \left| \frac{p(i)}{v(i)} - 1 \right|$

Goal Achieve the almost optimal $1 - O(\epsilon)$ if ϵ is small, and a constant competitive ratio if ϵ is large

3. Proposed approach

Initially, wait for the predicted optimum

$\to$ Switch to Dynkin's algo. if a large prediction error occurs



Theorem If $\theta = 0.646, \tau = 0.313$, then CR $\geq \max\{0.215, 1 - 2\epsilon\}$

Impossibility For any $C > 0$, if CR $\geq \max\{1 - C\epsilon, \alpha\}$, then
$\alpha \leq 0.25$ (deterministic) / $\alpha \leq 0.348$ (randomized)

4. Multiple-choice secretary

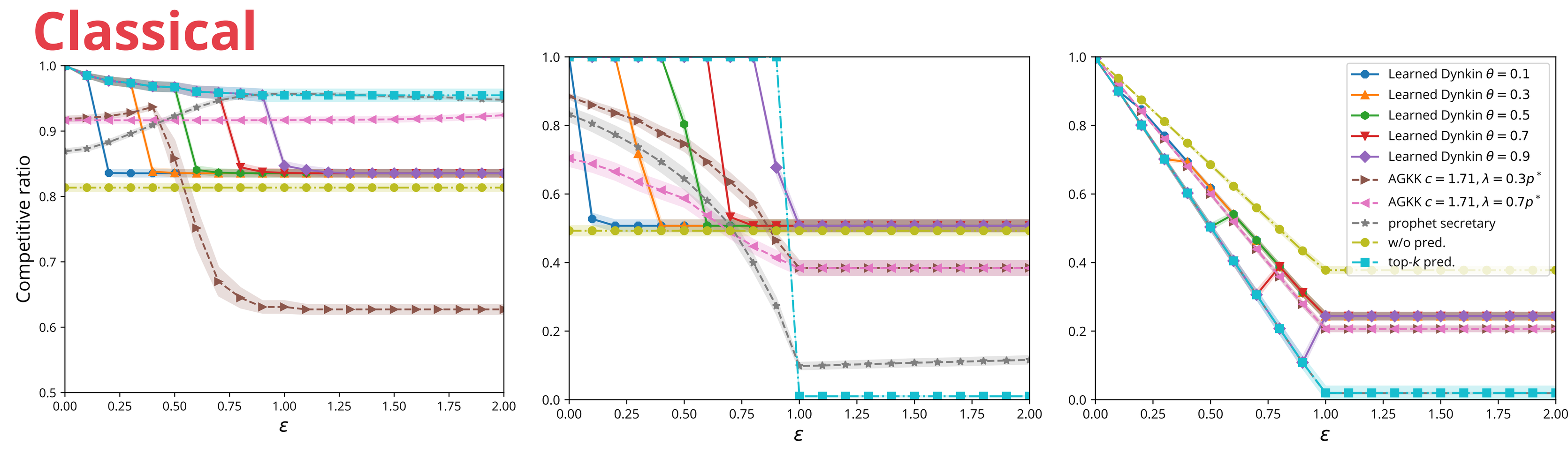
Hire k candidates to maximize the comp. ratio $\frac{\mathbb{E}[\sum_{i \in S} v(i)]}{\sum_{i \in S^*} v(i)}$

1: $\hat{S} \in \mathrm{argmax}_{S \subseteq [n]:\, |S| \leq k} \sum_{i \in S} p(i)$ Top-k predictions
2: **for** Each candidate $i \in [n]$ **do**
3: **if** $p(i) \notin [(1-\theta)v(i), (1+\theta)v(i)]$ **then**
4: Hire i, and then apply Kleinberg's algo.
5: with the remaining slots
6: **if** $i \in \hat{S}$ **then**
7: Hire i

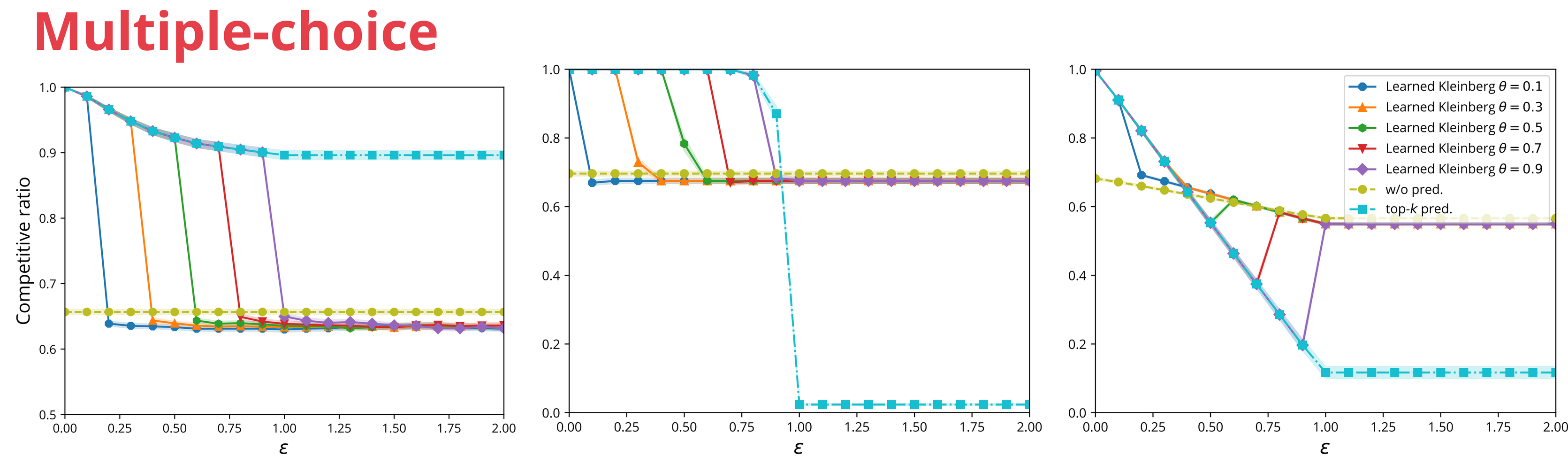
Theorem If $\theta = \frac{5 \ln k}{\sqrt{k}}$, then CR $\geq 1 - \min\left\{\frac{21 \ln k}{\sqrt{k}}, 5\epsilon\right\}$

5. Numerical experiments

Classical



Multiple-choice



Uniform	Adversarial	Almost-Constant
$v(i) \sim U(0,1)$ $p(i) = v(i) + U(-\epsilon, +\epsilon)$	$v(i) \sim \mathrm{Pareto}(1)$ $p(i) = (1-\epsilon)v(i)$ for top-$n/2$ candidates $p(i) = (1+\epsilon)v(i)$ for others	choose optimal $S^* \subseteq [n]$ s.t. $\lvert S^* \rvert = k$ randomly $v(i) = 1/\max\{1-\epsilon, 0.01\}$ for $i \in S^*$, $v(i) = 1$ for $i \notin S^*$, $p(i) = 1$, break ties randomly

compared with

- **w/o pred.**: Classical secretary or Kleinberg's algorithm
- **top-k pred.**: Choose top-k predicted values
- **AGKK**: The parameters are chosen such that the worst-case competitive ratio matches that of ours
- **prophet secretary**: Run Correa–Saona–Ziliotto's algorithm assuming uniform noise